

PRACTICAL TREATISE
ON FINDING THE
LATITUDE AND LONGITUDE
AT SEA.

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NAUTICAL ASTRONOMY.

Fig. 1.

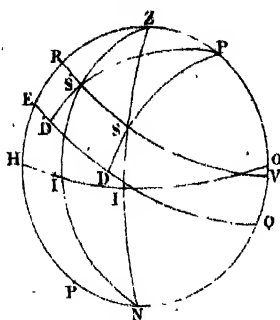


Fig. 2.

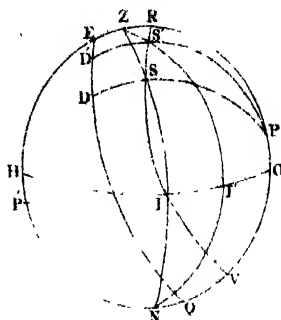


Fig. 3.

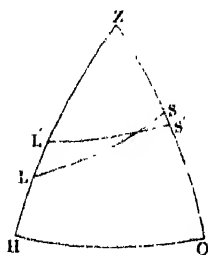


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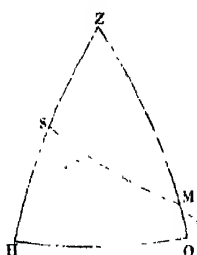


Fig. 5.

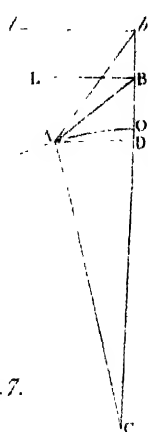


Fig. 6.

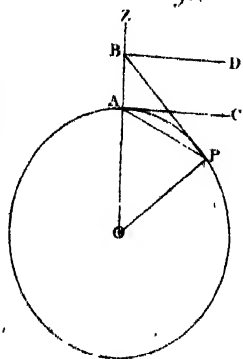
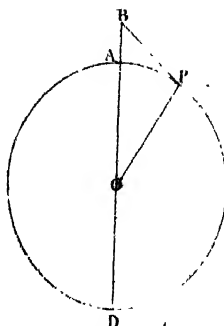


Fig. 7.



PRACTICAL TREATISE
ON FINDING THE
LATITUDE AND LONGITUDE,
AT SEA;

WITH TABLES DESIGNED TO FACILITATE THE
CALCULATIONS.

TRANSLATED FROM THE FRENCH OF
M. DE ROSSEL,
Member of the French Board of Longitude, late Captain in the Navy,
3c 4c 5c.

BY **THOMAS MYERS, A.M.**
OF THE ROYAL MILITARY ACADEMY, WOOLWICH,
AND
HONORARY MEMBER OF THE PHILOSOPHICAL SOCIETY OF LONDON

TO WHICH ARE SUBJOINED, AN EXTENSIVE SERIES OF

Practical Examples,
AN
INTRODUCTION TO THE TABLES,
AND
SOME ADDITIONAL TABLES,
BY THE TRANSLATOR.

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PREFACE.

IN a country whose political and commercial interests are so inseparably connected with her naval prosperity, as in Britain, an attempt to render a correct knowledge of Navigation more easy and accessible to her mariners, merits encouragement rather than demands apology. Daily experience also proves that numbers of young men, after having spent several years in the service, are but very imperfectly acquainted with the scientific principles of their profession. Under the influence of these impressions, united with a desire to remove this defect as much as possible, the subsequent work was undertaken. With respect to the Treatise on Nautical Astronomy which forms its bases, the learned French astronomer, M. Biot, to the second edition of whose "*Traité élémentaire d'Astronomie Physique*" it forms an important addition, thus describes the nature of the work, and the qualifications of its author.

"There is one branch of Astronomy (says he) which has never been treated in a convenient manner in elementary works, because this required great accuracy and simplicity joined to an experience beyond what most men have an opportunity

of acquiring. This is Nautical Astronomy, which has either been treated too superficially or in much too scientific a manner for mariners. I have, however, been very fortunate in having this part added to my work, by one who ranks among those who are best qualified to write on the subject. This is M. de Rossel, late Captain in the French Navy, coadjutor in and writer of the voyage of *d'Entrecasteaux*. The observations made by M. de Rossel and the other officers, during the voyage, have generally been regarded as the most accurate ever made in any French maritime expedition; and M. de Rossel's discussion of them as constituting an excellent Treatise on Nautical Astronomy. It is a Treatise of this kind, but more simple and concise, which this author has added to my work. It will be found to contain all the methods of calculation requisite at sea, and, what is not less valuable, they are given under the most simple and commodious forms that can be employed in their application. Mariners will not fail to remark the ingenious tables which M. de Rossel has calculated for facilitating the use of Douwe's method of finding the latitude from two observations of the sun taken out of the meridian. This method, which may frequently be of great utility, is rendered so easy and convenient, by these tables, that its use will doubtless become familiar to all mariners."—It is but justice to MM. Biot and Rossel, to add, that the Translator

has been favoured with a confirmation of this statement from a gentleman whose personal knowledge afforded him many opportunities of appreciating the talents and qualifications of M. de Rossel, during the period he was in the service of the British Admiralty.

To render the work more complete, and better adapted for perfecting the young mariner in the most difficult branches of his art, the Translator has added an extensive series of practical examples, and an Introduction to the Tables, explanatory of their construction and use; with a Table of the Right Ascensions and Declination of the principal fixed stars, used in finding the longitude at sea, and another of the logarithms of numbers and their complements, to an extent sufficient for the work. To these he has likewise subjoined a Table, the logarithmic sines and cosines with their complements, and differences for every 10 seconds of a degree, and also the logarithmic tangents and cotangents, with their differences corresponding to every 10 seconds. These, he trusts, will be found more convenient than the logarithmic tables in common use. A new and easy method of clearing the distance, lately published by the Rev. Dr. Brinkley, Professor of Astronomy in the University of Dublin, has likewise been added to the present work, and accompanied by a Table of Natural Versed Sines, by means of which the solution of this troublesome problem is greatly facilitated.

From this brief explanation, it will readily be perceived that the object of this Treatise is two-fold. First; to furnish mariners with an accurate work, containing the most simple and commodious methods of calculating their position on the globe at any given instant, with the assistance of the Nautical Almanac ONLY. The second is that of supplying the young navigator with an extensive series of *new* and practical examples, the solutions of which will gradually unfold the scientific principles of his profession, and familiarize him with their application. With this view, the work of one of the examples corresponding to each rule, has been inserted at full length, as a specimen of the method of working those to which the answers only are given. These examples have also been principally adapted to the years 1814 and 1815; by which means, a Nautical Almanac of a proper date will, for a considerable time, be constantly at hand.

Great care has been taken to avoid errors, both in the formation and solution of these examples; and they are now submitted, with greater confidence, to those who are accustomed to such calculations, from a firm persuasion that, should any error be discovered, the liberal and enlightened British mariner will ever be more ready to *correct* than to condemn.

Royal Military Academy, Woolwich,

April, 1815.

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ERRATA.

The word logarithm, instead of logarithmic, having been printed in several places, by mistake, the reader is requested to make the necessary correction mentally.

Page 40, line 2, *dele the*.

69, — 3, *for* logariths *read* logarithms.

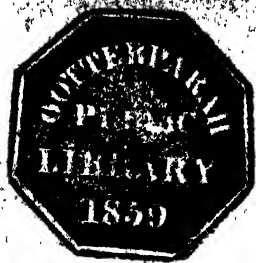
220, — 10 from bottom, *for* 5 *read* 5th. *

250, last line of the note, *for* 8 *read* 8"; and add
the word *Translator* to the end of the note.

INTRODUCTION TO THE TABLES.

Page i line 4 from bottom, *for* BP *read* BD.

iii — 3 from top, *for* fig. 6 *read* fig. 7.



A TREATISE,

&c. &c.

CHAPTER I.

Preliminary Observations, and Methods of finding the given Quantities of the Calculations in the Nautical Almanac, or The Connaissance des Temps.

1 ASTRONOMY teaches us to calculate the motions of the heavenly bodies; and to ascertain the places which they ought to occupy in the heavens at any given instant. Nautical Astronomy is one of the most useful branches of this vast science; its object is to furnish navigators with the means of knowing the position which their zenith ought to have in the heavens, with respect to those heavenly bodies, the situations of which have been made known by astronomers. It prescribes simple and easy rules, by the assistance of which they may ascertain their position on the globe, or their latitude and longitude.

2. Latitude is an arc of the terrestrial meridian comprised between any place and the equator; it is, consequently, the distance of the place from the equator measured in degrees; and is called *North latitude* when the place is situated in the northern hemisphere; and *South latitude* when it is situated in the southern hemisphere.

3. Longitude is an arc of the equator contained between the meridian of any place and that of another, which is called the *first meridian*. It is generally reckoned towards the east and west, from 0° to 180° through each half of the equator. The longitude of all the meridians situated eastward of the first meridian is styled *East longitude*; and that of those meridians on the west of it, is called *West longitude*.

There is not any circle on the earth's surface, the position of which is fixed like that of the equator, and from which the commencement of longitude can be reckoned; and therefore any meridian may be taken at pleasure for the first. The different nations of Europe have adopted the meridian of the principal place where they observe the motions of the heavenly bodies, and to which each is accustomed to refer their positions: it is generally this meridian for which their Ephemerides are computed. The French reckon their longitude from the meridian of Paris, and the English from that of the Royal Observatory, at Greenwich. There is not, therefore, any absolute longitude, properly speaking: it is only the difference of longitude that can be ascertained, which, as already observed, is equal to the arc of the equator comprised between the meridians of the two places, the positions of which are compared; or, which is still the same, to the spherical angle formed by the meridians of these places.

4. Astronomers generally calculate the situations of the heavenly bodies with respect to the ecliptic; but observations can only give them directly in relation to the equator; they are equally obliged, in calculating from observations, to employ the elements, which serve to fix these positions with regard to the equator; and, in Nautical Astronomy, the declinations and right ascensions of those bodies only are used.

5. Declination is the distance of a heavenly body from the equator, measured on a great circle perpendicular to the equator, which is called the circle of declination. It may be

considered as a *celestial latitude*, and, consequently, might be called by that name. Declination is north, when the body is in the northern hemisphere; and south, when in the southern hemisphere.

The circles of declination, being perpendicular to the equator, ought to pass through the poles of that circle, and to have analogous and corresponding positions in the heavens to those of the meridians on the globe. Thus, when a heavenly body passes the meridian of any place its circle of declination is immediately above that meridian, and in the same plane with it. If, at that instant, the arc of the circle of declination, or of the celestial meridian, comprised between the body and the zenith of the observer, be measured, or otherwise, if the altitude of the body, which is the complement of the zenith distance, be observed, it will be easy to ascertain the latitude. In fact, the declination of the body, or its distance from the equator, being given in the Ephemerides, it is evident that the distance of the observer from the same circle, or his latitude, will be equal to that declination plus or minus the distance of the body from the zenith of the same observer. The altitude, which is directly obtained from observation, may also be employed, instead of the zenith distance: the calculation is a little different, as will be subsequently seen, but the result is the same.

6. Right ascension is an arc of the celestial equator, comprised between the circle of declination of any heavenly body and the point where the ecliptic cuts the equator, and the sun commences his revolution: this is called the vernal equinoctial point. Right ascension may, therefore, be regarded as a *celestial longitude*, with this difference, that, in the heavens, there is a point fixed by nature, from which to begin the reckoning; but, on the earth, the first meridian must be arbitrarily assumed, from which the computation of terrestrial longitude commences. But the analogy is accurate between the difference of longitude and that of right

ascension; for this last is equal to an arc of the equator comprised between two circles of declination, or *celestial meridians*, or to the spherical angle formed by those circles. The difference of longitude of any two places on the surface of the earth, is, therefore, equal to the difference of right ascension of the two circles of declination which correspond to the meridians of those two places, and which are, consequently, in the same planes with them.

7. This last consideration furnishes a new means of measuring terrestrial longitudes; it is derived from the diurnal motion of the earth, or its revolution on its axis. The duration of a day, or twenty-four hours, is the time in which the earth makes one revolution with respect to the sun, which is equal to the time that elapses between the passage of the sun over any meridian, and his return to the same meridian: twenty-four hours, therefore, corresponds to 360° of longitude or right ascension. Now, supposing the sun to be on the first meridian, all places situated on that meridian reckon noon at the same time; but those places on the other half of the same great circle diametrically opposite to the first meridian, that is, the places situated on another meridian 180° distant from the first, reckon midnight, or twelve hours less, at the same instant: therefore, 180° of longitude correspond to twelve hours of time. The great circle which passes through the poles, and has its plane perpendicular to that of the first meridian, forms two other meridians; one of which 90° to the east, and the other 90° to the west of the first. All places situated on that to the west, reckon six hours less than those on the first meridian; the astronomical day will therefore not have commenced at them, and it will be effectively only the 18th hour of the preceding day. Those places that are situated on the meridian 90° east of the first, reckon six hours more than at this last, and have the sixth hour of the day which has commenced at them: 90° of longitude, therefore, answer to six hours of time. These 90° ,

or a fourth of the equator, may be supposed to be divided into six equal parts, each of which will be 15° , and from this, it is concluded, that 15° of longitude correspond to one hour of time; hence 1° answers to the fifteenth part of an hour, or four minutes. By continuing the subdivision, it will be found, that $15'$ of a degree answer to $1'$ of time, and $15''$ of a degree to $1''$ of time. Thus, longitude, or rather the difference of longitude, may be reckoned in time, at the rate of $15'$ to an hour.

8. Those places which are situated on a meridian 90° west of the first meridian, reckon, as already observed, six hours less than those in the first meridian; those that are 75° west, reckon five hours less; and those at 15° count one hour less. Generally, at all places of west longitude, the time is less than on the first meridian, by a number of hours and minutes equal to the longitude of those places converted into time. Hence, whenever we wish to know the hour which ought to be reckoned on any meridian west of that where we are, the difference of longitude reduced into time, must be subtracted from the hour at this latter place.

9. Those places that are situated on a meridian 90° eastward of the first meridian, reckon six hours when it is noon at this last, they, therefore, reckon six hours more. Thus, in order to obtain the hour at any meridian 90° east of the first, the longitude, reduced into time, must be added to the hour at this last. Generally, when the time is required that ought to be reckoned at places situated to the east of the meridian where we are, the time answering to the difference of the longitude corresponding to the two places must be added to the hour at this last meridian.

10. The problem of longitude, therefore, consists in finding directly by observation, the hour at the place where we are, and the hour which is reckoned on the first meridian, or on any other meridian, of which the longitude is known. It is easy to obtain the hour at any place by means of the alti-

tudes of the heavenly bodies; the hour at the first meridian is obtained by observations of the distances between the moon and the sun or the stars. The time at the first meridian, or at any other, is also obtained by marine chronometers; but as these machines are liable to experience slight derangements in their movements, they can only be depended upon during a certain lapse of time, and should be verified as often as possible. In general, they are more proper for ascertaining the difference of longitude of two places not far distant from each other, than for determining absolute longitudes.

The detail of the various operations which are necessary for calculating the latitude, the hour at any place where observations have been made, and its longitude, will be subsequently given; and the methods of obtaining the azimuths, which serve to make known the variation of the magnetic needle, will also be treated of. The altitudes of the heavenly bodies, and their distances, are the only data which can be obtained in a direct and precise manner by observation; but they are not sufficient for calculating the required quantities: the declinations and right ascensions, which fix the positions of these bodies in the heavens, must also be used, as well as several other elements which are found in Ephemerides. It will, therefore, be necessary, first, to show the methods of calculating these elements. The values of these different quantities change every instant, and are only predicted for the time at the first meridian; that time must therefore be calculated. It ought to be remarked, according to what has been said, that, previous to entering upon the calculations, we are obliged to suppose, that the longitude of the place of the observation is known. The declinations and right ascensions, with the other elements that are taken from the Nautical Almanac, or the *Connaissance des Temps*, partake, indeed, of the error of the longitude that has been employed in calculating the time at the first meridian; but that

of the results will always be so small, that it may be regarded as nothing. The rules that have been given in articles 8 and 9, for calculating the time at the first meridian, are therefore to be followed; and easy methods of converting longitude into time shall also be given, which will greatly facilitate their application. Other rules shall likewise be given, for converting time into degrees of longitude, or parts of the equator. This last operation is as useful as the first, and is often put in practice.

Method of reducing Degrees of Longitude into Time.

11. When the number of degrees exceeds 100, make use of the tables calculated for that purpose.*

Reduce $133^{\circ} 17' 30''$ of longitude into time,

Take successively

For 133°	-	-	-	-	$8^h 52' 0''$
For $0 17'$	-	-	-	-	$0 1 8$
For $0 0 30''$	-	-	-	-	$0 0 2$
Sum					$8^h 53' 10''$

This sum is the time required.

12. When the number of degrees is less than 100, it is more convenient to make use of the following rule:

Multiply the seconds, minutes, and degrees, by four, and reckon the seconds of the product for thirds, the minutes seconds, and the degrees for minutes.

Let it be required to reduce into time $43^{\circ} 17' 33''$

Multiplying by 4

Product $- 2^h 53' 10'' 12'''$

* This, however, may be readily done by the rule given in art. 12; and the use of the tables altogether avoided. *Trans.*

Divide the thirds by 6, and it will give a decimal fraction of a second, which, in the present case, is $.0^{\circ}2$; the time will therefore be $2^{\text{h}} 53' 10^{\circ}2$. *

Method of reducing Time into Degrees of Longitude.

13. This reduction may be made by the assistance of the proper tables.

Let it be required to reduce into degrees $5^{\text{h}} 53' 3''$
Take successively

For 5^{h}	-	-	-	75°	$0'$	$0''$
For $0 53'$	-	-	-	13	15	0
For $0 0 3''$	-	-	-	0	0	45
Sum	-	-	-	88°	$15'$	$45''$

This sum is the required reduction.

When the proposed number contains tenths of a second, multiply the tenths by 6, and the product will be thirds, with which the corresponding parts of the degree is to be sought.

If it were necessary to reduce into degrees $3^{\text{h}} 21' 11^{\circ}7$

Take for 3^{h}	-	-	-	45°	$0'$	$0''$
for $0 21'$	-	-	-	5	15	0
for $0 0 11''$	-	-	-	0	2	45
for $0 0 0.7 \times 6 = 42'''$	-	-	-	8	0	10.5
Reduction required, Sum	-	-	-	50°	$17'$	$55^{\circ}5$

14. In the case where the proposed number contains only minutes and seconds, it will be most expeditious to follow the reverse of the second method which has been given for reducing degrees into time : viz.

* For practical examples of this and the subsequent rules, see the APPENDIX, by the TRANSLATOR.

Divide the minutes and seconds by four, and reckon the minutes in the quotient as degrees, and the seconds as minutes. *

Reduce into degrees of longitude - - - 59' 44"

The fourth is - - - - - 14° 56'

If the proposed number contain tenths of a second, convert them into thirds, by multiplying them by 6; and reckon the fourth of the thirds for seconds.

Let it be required to reduce into degrees of

longitude - - - - - 45' 35"·4

Write - - - - - 45' 35" 24"

The fourth of which is - - - - - 11° 23' 51"

Method of calculating the given Quantities that are found in the Nautical Almanac, or the Connaissance des Temps, for any proposed instant.

15. When the quantities contained in the Nautical Almanac, or Connaissance des Temps, change slowly, they are

* The most expeditious method of converting time into longitude, and which is applicable to all cases, is to divide the minutes, seconds, &c. by four, as above directed, and then to add the product arising from multiplying 15 by the number of hours in the given time, to the degrees in the quotient. By this method, the whole calculation may generally be performed in less time than the several parts of the given quantity can be taken separately from a table; besides the great advantage of not requiring any table. Thus, if it were required to find the longitude answering to 9h. 24' 50"·4 of time:

$$\begin{array}{r} \text{First, } 4'' \times 6 = 24'' \\ \text{Then dividing by 4) } 24^{\circ} 30'' 24'' \\ \hline \text{Quotient} \quad - \quad - \quad 6^{\circ} 13' 51'' \\ 15 \times 9 = 135 \end{array}$$

Longitude required 141° 13' 51"

It should be remarked, that as the multiplier for converting the decimals of a second into thirds is 6, and the number of hours in the given time, in almost all practical cases, does not exceed 12, these multiplications may always be performed mentally, which will greatly facilitate the whole operation.

calculated for every twenty-four hours; those which change more rapidly, are calculated for every twelve hours: the declination of the moon, given in the *Connaissance*, is calculated for every six hours.* It would be useless to give a particular example for each of the quantities necessary to be obtained, because all the operations are the same, and are comprehended in the following rules. It will, therefore, be sufficient to unite, in several examples, the principal difficulties that occur in practice.

16. Calculate, according to the rules given in articles 8 and 9, the time at the first meridian corresponding to the proposed instant, or the time of observation: then, take in the *Nautical Almanac* the declination, right ascension, or any other element corresponding to the nearest epoch preceding that instant, also take the same element corresponding to the next following epoch. The difference of the two quantities thus found will be the change which the declination, right ascension, or other element, has experienced in the interval between the two epochs for which this element has been calculated. Subtract the time of the first epoch from the time at the first meridian, which will give a second interval; then find, by proportional parts, the change which corresponds to it. If the quantities in the tables are increasing, add the calculated change to the quantity cor-

* The sun's longitude, right ascension in time, and declination, are given in the *Nautical Almanac* for every 24 hours, or at noon for every day in the year; and his semi-diameter for every sixth day of the month. The moon's right ascension, declination, semi-diameter, and horizontal parallax, are also given for every 12 hours, or at both noon and midnight, with the time of her passage over the meridian at the Royal Observatory, Greenwich, for every day, are also given in the same Ephemeris. The latitudes and longitudes of nine of the principal fixed stars are likewise given in the last page of the *Nautical Almanac* for every year; the longitude for the beginning, and the latitude for the middle of the year; with the annual increase of the former, and the variation of the latter. *Time.*

responding to the first epoch; but if they are diminishing, subtract it from the quantity corresponding to the same epoch.

EXAMPLE I.

On the 15th of March 1810, being in $51^{\circ} 13'$ east longitude, required the declination of the sun, at the time of his passing the meridian.

Reduce, by the rules of article 12, the longitude into time, which will give $3^h 24' 52''$, or by taking the nearest minute, $3^h 25'$. The first meridian is west of that of observation, and it is not yet noon there; hence, subtract $3^h 25'$, or the difference of longitude, from the time at the first meridian, which is 0 or 24 hours. The remainder, $20^h 35'$, is the time for which the declination of the sun is to be calculated. But the 15th of March has not yet commenced at the first meridian; the calculation must, therefore, be made for the 14th, at $20^h 35'$. The nearest preceding epoch is that of the 14th at noon, and the next following one is that of the 15th, at the same hour.

The 14th March at noon, declination				$2^{\circ} 40' 10''$ S.
The 15th		-	-	$2^{\circ} 16' 32''$ S.
Change in 24 hours, difference		-	-	$23' 38''$
24^h	$23' 38''$	For 12^h	-	$11' 49''$
12	11 49	For 6	-	$5' 54.5$
6	5 54.5	For 1	-	0 59
3	2 57.2	For 1	-	0 59
1	0 59	For 0 35'	-	0 34.4
		For $20^h 35'$	Sum	$20' 15.9$

Make a small table, similar to that on the left hand above, in the following manner: — say, the half of 24^h is 12^h ; the half of the change in 24^h is $11' 49''$, which answers to 12^h . The half of 12^h is 6^h , and that of $11' 49''$, or $5' 54.5$ is the change in 6^h . By following the same method, we shall have

the change in 3^h , which is $2' 57'' \cdot 2$. The change for one hour will be the third of this number. It may be seen from the above table, what quantities it is necessary to add together to obtain the change of declination which answers to $20^h 35'$; it is $20' 16''$ nearly, which ought to be subtracted from the declination corresponding to the first epoch, or from $2^\circ 40' 10''$, because the declination of the sun is decreasing, and we shall have the declination required.

The 14th of March at noon	-	-	-	$2^\circ 40' 10''$ S.
Change in $20^h 35'$	-	-	-	$0 20 16$

DECLINATION 14th March at $20^h 35'$, diff. $2^\circ 19' 54''$ S.

If the declinations taken from the Nautical Almanac have not the same denomination, that is, if one is north and the other south, it will be a proof that the sun has passed the equator between the two epochs to which the declinations correspond. Then the change in declination in 24^h , instead of being equal to the difference of the two declinations, will be equal to their sum. The following example will show the manner of proceeding under this circumstance.

EXAMPLE II.

On the 21st of March 1810, at $7^h 12'$ in the morning, civil time, or the 20th March, at $19^h 12'$, astronomical time, being in $41^\circ 22'$ of west longitude, it is required to calculate the declination of the sun at that moment.

The longitude reduced into time is $2^h 45'$, neglecting the seconds: the first meridian is east of the place of observation, and, at the former, it is more than $19^h 12'$; therefore, if to this hour there be added the difference of longitude of the meridians, $21^h 57'$ will be obtained for the time at the first meridian.

Declination \odot , 20th March, at noon	-	$0^\circ 18' 8''$ S.
Declination 21st March, at noon	-	$0 5 33$ N.
Change in 24 hours	-	Sum $0^\circ 23' 41''$

24 ^h	23' 41"	For 12 ^h	-	11' 50.5
12	11 50.5	For 6	-	5 55.2
6	5 55.2	For 3	-	2 57.6
3	2 57.6	For 0 57'	-	0 56.2
1	0 59.2	For 21 ^h 57'	-	Sum 21' 39.5

From the 20th of March at noon to the 21st at the same hour, the declination at first diminished progressively, until it became nothing, then it changed its denomination, and increased until it became equal to $0^{\circ} 5' 33''$ N., which is that of the second epoch. Since the change which has taken place between the 20th of March at noon, and the required instant, is greater than the declination at the first epoch, it is a proof that, at that instant, the sun had crossed the equator, and the declination had changed its name. In this case, subtract the declination of the 20th of March, from the calculated change in declination, which will have a different denomination from that of the first epoch.

Declination on the 20th of March, at noon $0^{\circ} 18' 8''$ S.

Change in declination for 21^h 57' - $0 21 39$

DECLIN. \odot 20th of March, at 21^h 57' - $0' 3' 31''$ N.

If the change in the calculated declination had been less than that of the first epoch, the sun would not then have been in the northern hemisphere; therefore, the change in declination must have been subtracted from declination of the first epoch, and the remainder would have been the declination required, of the same denomination as that of the first epoch.

EXAMPLE III.

On the 10th of April 1810, being in $161^{\circ} 31'$ east longitude, required the declination of the moon at 15 minutes past 8 at night, civil time, or 8^h 15' astronomical time.

The hour at the place is $8^h 15'$, or, by adding 24 hours, it is $32^h 15'$: subtract $10^h 46'$ from this, which is the longitude reduced into time, and the hour at the first meridian will be $21^h 29'$; but as it was necessary to add 24^h to the proposed time, the 10th of April had not then commenced at the first meridian, the required epoch is the 9th of April, at $21^h 29'$.

Declin. of the ζ the 9th of April, at 18^h $18^\circ 19' N.$

Declin. of the ζ on the 10th of April, at noon $18^\circ 12'$

Change in 6^h :—Difference - - - $7'$

		18^h	
6^h	$7'$	For 3	$0^\circ 3' \cdot 5$
3	$3 \cdot 5$	$0 \cdot 29'$	$0 \ 0 \cdot 6$
1	$1 \cdot 2$	For $21^h 29'$	$0^\circ 4' \cdot 1$

Declin. of the ζ the 9th of April, at 18^h $18^\circ 19' N.$

Declination diminishes, *subtract* - - - 4

DECLINATION, the 9th of April, at $21^h 29'$ $18^\circ 15' N.$

EXAMPLE IV.

On the 13th of March, at $4^h 30'$ at night, being in $91^\circ 49'$ of west longitude, required the moon's right ascension.

The longitude reduced into time, is $6^h 7'$; this is to be added to the hour at the place, and the time proposed at the first meridian, is the 13th of March, at $10^h 37'$

Moon's right ascension, the 13th at noon - $16^h 12'$

Right ascension, the 13th at midnight - $94 \ 27$

Change in 12 hours:—Difference - - - $8^\circ 15'$

12^h	$8^\circ 15'$	For 6^h	$4^\circ 7' \cdot 5$
6^h	$4^\circ 7' \cdot 5$	3	$2 \ 3 \cdot 7$
3	$2 \ 3 \cdot 7$	1	$0 \ 41 \cdot 2$
1	$0 \ 41 \cdot 2$	$0 \ 37'$	$0 \ 25 \cdot 4$
		For $10^h 37'$	$7^\circ 17' \cdot 8$

Right ascen. of the ☾, the 13th at noon	96° 12'
Right ascension increasing; add	7 18
RIGHT ASCENSION REQUIRED	93° 30'

The declination and right ascension of the moon are given in the Nautical Almanac only in degrees and minutes. It will be sufficient, in calculating the proportional parts, to take into the account tenths of minutes, and to employ the sum without the fractions. Below 0.5 the tenths are to be neglected; and above that quantity, as in the last case, one minute more is to be taken.

The preceding examples are sufficient to show the method of calculating the quantities that serve to fix the positions which the sun, moon and planets occupy in the heavens. The other elements which experience changes may also be calculated by methods altogether analogous to these, as the mean time at true moon, the semidiameters of the sun and moon, and the moon's parallax.* The time of the moon's culminating, for any other place than those on the first meridian, may also be calculated in the same manner.

17. In the calculations of Nautical Astronomy, it may be supposed that the stars have not any apparent motion, and that they always preserve the same position with respect to each other; or, that their respective distances remain the same. It will, therefore, not be necessary to have any regard to the small periodic changes, denominated nutation and aberration, which amount only to a few seconds. But an attention to the annual variation of the stars in right ascension and declination is indispensable. These last changes do not arise from their proper motion, but from another cause, which shall be explained. It should be recollected, that, in article 6, right ascension has been defined to be an arc of the equator comprised between the circle of declination

* For examples of these calculations, see the Appendix. *Trans.*

of any star, and the point of the ecliptic, where it cuts the equator, and the sun commences his revolution. This point, which is called the vernal equinoctial point, has a very slow retrograde motion, by which it is removed from east to west, or in a contrary direction to that in which right ascension is estimated: this last ought therefore to be progressively augmented by a certain quantity; consequently, the annual variation is always additive. The motion of the equinoctial point appears to be made on the ecliptic; but it really arises from a motion of the earth's axis, by which the plane of the equator, which preserves nearly the same degree of inclination to that of the ecliptic, and has the same motion as the axis, is slightly displaced with respect to the stars; and this always takes place in the same direction: the plane of the equator, therefore, approaches certain stars while it removes from others. The declination of some of the stars ought, on this account, to increase, and their annual variation in declination to be additive; while the annual variation in the declination of those stars, to which the plane of the equator approaches, is subtractive. In catalogues of stars, their right ascensions and declinations are generally given for an epoch but little distant from the time of their publication; the annual variations are found in the column which immediately follows that containing these quantities. These variations in right ascension are always additive, as already stated, for any periods of time posterior to those in the catalogue, and subtractive for the periods anterior to them. The annual variations in declinations, which are additive for the epochs posterior to those in the catalogues, are preceded by the sign +, and those which are subtractive, by the sign —. Whenever the declination of a star is calculated for any epoch anterior to that of the catalogue which is used, the annual variation must be employed with a contrary sign

18. When it is required to calculate the right ascension of a star for any period posterior to that of the catalogue, * multiply the annual variation by the number of years since the time for which the catalogue was calculated.

The proportional parts for the months and days may then be found in the following manner:—Reduce the days into decimals of a month, by dividing them by thirty, and multiply the twelfth part of the annual variation by the number of months and decimal parts thus found. The sum of this product, and the right ascension for the years, is the quantity to be added to the right ascension of the catalogue, in order to obtain the right ascension corresponding to the time proposed.

The same method of operation must be used for finding the declination, with this difference, that the sum for the years and months must be added to the declination of the catalogue, when that declination is preceded by the sign +, but subtracted when it is preceded by the sign —.

EXAMPLE.

Required the right ascension and declination of *Antares*, for the 16th of April 1808.

Right ascension, Jan. 1st 1805	-	244° 22' 6"
Declination, Jan. 1st 1805	- -	25 59 0 S.
Annual variation in right ascension	- -	54".6
From the 1st of Jan. 1805 to Jan. 1st 1808		3 years
Product. Proportional parts for the years		2' 43".8
Annual variation	- - -	54".6
Twelfth part	- - -	4".5
The 16th April	- -	3.5
		<u>13".5</u>
		2.3
Proportional parts for the months		<u>15".8</u>

* See TABLE XVI, at the end of this volume. *Trans.*

Proportional parts for the years	-	-	2' 43" 8
Proportional parts for the months	-	-	15 8
	Sum	-	3' 0"

Right ascension of the catalogue	-	244° 22' 6
RIGHT ASCENSION required	-	244° 25' 6"

Annual variation in declination	-	-	+ 8" 8
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From Jan. 1st 1805 to Jan 1st 1808	-	3 years.
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Product, Proportional parts for the years	-	+ 26" 4
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Annual variation	-	+ 8" 8
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Twelfth part	-	+ 0' 7
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The 16th of April	-	3 5
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2 1

0 4

Proportional part for the months	2" 5	-	+ 2 5
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Sum - 28" 9

Declination of the catalogue	-	25° 59' 0 S.
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DECLINATION required	-	25° 59' 29" S.
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CHAPTER II.

Corrections which ought to be made in all the observed Altitudes of the Sun, Moon, and Stars.

19. OBSERVED altitudes should be subjected to several corrections before they are employed in calculations. They must first be corrected for the depression of the horizon, and by subtracting or adding the semi-diameter according as the upper or lower limb of the sun and moon has been observed; then they must be corrected for the effects produced by refraction and parallax. The observed altitudes of the sun and moon should almost always be subjected to these corrections. The stars having neither diameter nor parallax, their observed altitudes should only be corrected for the depression of the horizon, and the effect of refraction. The principal causes which render these corrections necessary shall be explained, and the methods of making them shown.

On the Depression of the Horizon.

20. The altitudes observed at sea are arcs of the vertical circles comprised between the heavenly bodies and the visual horizon. They would be the same as the true altitudes, abstracting the other quantities above mentioned, if the visual rays directed to the circle that terminates the visible part of the sea's surface coincided with the

horizontal plane; then they would not require any correction. But these rays are inclined below the horizontal plane, and form an angle with it, called the depression of the horizon, which increases as the observer is more elevated above the surface of the sea.* All observed altitudes are, therefore, too great, and the depression of the horizon must be subtracted from them. This depression is contained in Table I, at the end of this volume for different heights, from one to 100 feet. The height of the observer's eye above the surface of the sea is expressed in feet, and the corresponding depression of the horizon is inserted in the adjoining column on the right hand, and the differences in the next column. When the height of the eye falls between two of the consecutive numbers in the first column, the depression for the proportional parts may be calculated in the following manner :—

EXAMPLE.

Required the depression of the horizon, when the eye of the observer is elevated 15·7 feet above the surface of the sea.

Difference between 15 and 16 feet, in the

Table	-	-	-	-	-	-	-	7"
Depression for 15 feet	-	-	-	-	-	-	-	3' 48"
Depression for the proportional parts, 0·7 feet	-	-	-	-	-	-	-	5
DEPRESSION for 15·7 feet	-	-	-	-	-	-	-	<u>3' 53"</u>

21. The visual rays which meet the horizon of the sea are tangents drawn from the eye of the observer to the surface of the earth; but, the points of contact of these tangents are more distant, as the eye is more elevated: the visual horizon will, therefore, be as much more distant from

* For the proof of this, and the method of calculating the quantity of the depression, see the Introduction to Table I. of this volume. *Trans.*

the observer, as his height is greater. If an observation be made from the most elevated part of the deck, or of a large ship, its distance would be between five and six miles, or nearly two marine leagues. Thus, in navigating near the land, it may happen that the shore is nearer the vessel than the circle which terminates the horizon ought to be; and this is what mariners express, when they say the horizon is bounded by the land. Then the visual rays that meet the shore are more inclined below the horizontal plane than those by which the horizon would have been perceived: the depressions of Table I, are then too small, and only a part of the corrections can be applied to the altitudes. The fourth column contains the distances corresponding to the different heights. When the estimated distance from the shore is either greater than or equal to the distance in the Table, which answers to the height of the eye, the depression which is found in the same Table may be employed for correcting the altitude. It is essential to remark, that an error of a mile, committed in estimating the distance of the shore, ought not generally to occasion an error in the altitude of more than a quarter of a minute, and never more than a minute. When the depression of the horizon is affected by an error of this kind, the corrected altitude will always be too great. If the distance taken from the Table exceed the estimated distance between the observer and the shore by more than a mile, it will be a proof that the horizon is bounded by the land: then the depression of Table I, cannot be employed for correcting the altitude. It would be useful to ascertain this some time before the observation is to be made, in order to preserve a convenient distance from the shore. In general, when the elevation of the eye does not much exceed 26 feet, there is not any fear of committing an error of more than a minute, at least at a league or three miles from the land.

22. Several much esteemed works on navigation contain methods of ascertaining directly from observation, the inclination of the visual ray that meets the shore by which the horizon is bounded. The directions there given are to observe, at the same instant, the altitudes of the sun, from two places situated exactly in the same vertical line, but of very different elevations. But the methods of calculating the corrections are either long and troublesome, or only approximations, by which sufficient accuracy is not obtained. It would not be difficult, however, to give great precision to the approximating methods, by means of a small table which would not add much to the length of the calculations; nevertheless it has been suppressed, because the methods that are here given for avoiding the errors in the depression of the horizon, are not only sufficiently accurate, but much more convenient in practice. When altitudes are to be obtained with all the accuracy of which these observations are susceptible, it will always be best to remove from the land, and to preserve the distance indicated in the first Table.

23. The depressions in this Table have been calculated from the dimensions of the terrestrial globe *, concluded from the new measure of an arc of the meridian, taken for the purpose of fixing the length of the metre. To correct them for the effects of refraction, which generally increases the apparent elevations of objects, they have been diminished by $\frac{8}{100}$ ths, a quantity or co-efficient which Delambre has found by numerous observations, and which has since been

* The mean radius of the earth, or that of 45° , considering it as an ellipsoid, employed in these calculations, is therefore equal to 3,266,611 toises, 6,366,745 metres, 6,964,837 English yards, or 3957.3 miles nearly; the French metre being equal to 1.09394 Eng. yards. *Trans.*

confirmed by M M. Biot and Arago, by observations made in Spain, for extending the measure of the meridian.

24. The variations which common refractions cause in the depressions of the horizon, are so small, that they may be neglected in the practice of navigation. We shall, therefore, content ourselves with mentioning, in this place, some extraordinary phenomena which M. Biot has proved by the most delicate observations, and of which he has given the first satisfactory explanation, by subjecting them to the most rigorous calculations of analysis. The limits within which it is necessary to comprise this treatise, do not permit us to follow his learned researches; we shall, therefore, only extract the most useful results. Their importance cannot fail of being felt by mariners, to whom they will afford new means of perfecting their art.

The great errors which refraction may occasion in the depression of the horizon, arise from the difference which almost always subsists between the temperature of the water at the surface of the sea, and that of the air at several yards above it. Experience has shown, that the region where these errors are the most sensible, is from the surface of the water to 10 or 11 yards in height. Therefore the visual rays from the eye of an observer on the deck of a vessel, by which the altitudes of the heavenly bodies are referred to the horizon, always traverse this region; and it is important to know the circumstances under which the greatest errors take place, in order to guard against those by which the observations ought then to be affected. These errors are subject to frequent variations, occasioned by the changes which the rays of the sun suddenly effect in the temperature of the atmosphere, either when he emerges from behind a cloud, or becomes hidden by one. It is probable that we shall never obtain an exact knowledge of their value; or at least, very minute attentions would be requisite to obtain it; and,

if it were known, it might not be of great use in Nautical Astronomy; we shall therefore ~~not~~ be satisfied with giving an approximative value of these errors, and showing in what manner they ought to affect the altitudes: attention shall also be paid to ~~the~~ such indications ~~only~~ as are easy to be comprehended, and may be understood by all.

25. The causes which give rise to the variations in the extraordinary refractions of the visual horizon are the same that produce those phenomena which the French mariners call *Mirage*, and the English, *Looming*; thus, whenever the phenomena of *looming* are manifest, the depression of the horizon will be very uncertain during their whole continuance.

The direction in which the errors in the depression of the horizon, and consequently, those of the observed altitudes, take place, depend upon the temperature of the sea being greater or less than that of the incumbent atmosphere.

1st. If the sea be warmer than the air, the altitude corrected by the depression taken in the Table will be too great.

2nd. If the sea be colder than the air, the corrected altitude will be too little.

3rd. When the temperature of the sea is from 7° to 10° of Fahrenheit's thermometer, different from that of the air at the height of one or two yards above the surface, the error in the altitudes may be from $3'$ to $4'$; a difference of from 4° to 6° of temperature may occasion an error of $1'$ or $2'$.

4th. The water of the sea is heated much more slowly by the presence of the sun than the atmosphere, it will therefore be colder than the air for some time after the rising of that luminary; then the altitudes corrected by the depressions in the Table will be too little, and will continue to be all other things remaining the same, until the heat of the

day is considerably augmented. In the evening, the contrary takes place; the altitudes corrected for depression will begin to be too great as the heat of the day diminishes, and their errors will continue to increase until the sun has set. The depressions in the Table are corrected for the effects of common refraction; thus, whenever extraordinary refractions depress the horizon, instead of elevating it, the altitudes will be too great; and this is the reason why they should be a little more at night than in the morning.

Those accidental and extraordinary refractions may serve to explain, why certain latitudes observed at sea by navigators, equally careful and experienced, sometimes differ several minutes from each other, while in general, their observations are found to agree.

On the Semi-diameters of the Sun and Moon.

26. The altitudes of the upper or lower limbs of the sun and moon only can be obtained immediately from observation; the semi-diameters of these bodies must therefore be added to, or subtracted from these observed altitudes, in order to obtain those of their centres. These semi-diameters are not the same at all times of the year or month, but it will be easy to calculate them, from the *Nautical Almanac* or the *Connaissance des Temps*, for any proposed instant.

27. When the lower limb of the sun or moon has been observed, the semi-diameter must be added to the observed altitude; but if the upper limb, it must, on the contrary, be subtracted.

When the supplement of the sun's altitude is observed, by bringing that edge of his image into contact with the horizon, which appears to be nearest it, but which is effectively the most distant, the semi-diameter must be subtracted from the supplement of the altitude which has been observed.

Several examples of these operations will be subsequently given, which are so simple, that it has been thought proper to dispense with them here.

28. The semi-diameter of the moon appears to be increased by a small quantity as she becomes more elevated. There will be found in Table II, entitled, *Augmentation of the Moon's Semi-diameter*, the quantity that must be added to the true or horizontal semi-diameter, in order to obtain that which agrees with the observed altitude. Thus, when the apparent altitude of the moon's centre is to be calculated, the semi-diameter corrected for this augmentation, or the apparent semi-diameter, is to be employed.

Astronomical Refraction.

29. Astronomical refraction is the quantity by which the heavenly bodies, after their luminous rays have traversed the atmosphere, appear to be more elevated than they really are. It ought always to be subtracted from the observed altitudes. The greatest refraction takes place when the bodies are in the horizon; it diminishes as their altitudes increase; and becomes nothing when they have arrived at the zenith.

30. Refraction is not always the same at the same altitudes; it varies on account of the greater or less density of the atmosphere. In general, the more dense the atmosphere is, the greater is the astronomical refraction; it also diminishes as the density of the air decreases. Cold has the property of condensing the air, and heat of rarifying it; the density of the air is, therefore, increased by cold, and diminished by heat. It follows from this, that the variation in the height of the mercury in the thermometer may be employed in calculating the corresponding changes which the astronomical refractions ought to experience. The atmo-

sphere is also more dense when its weight is greater, or when it sustains a longer column of mercury in the barometer; and a less elevation of that column indicates a diminution in the density of the atmosphere. The changes of atmospheric refraction depend, therefore, upon the height of the mercury in the barometer. These refractions will be greater as the column of mercury is more elevated, and less as the height of the column is diminished.

31. The numbers in the third column of Table V, intitled, *Refraction of the *, or of the stars*, are the refractions of all the heavenly bodies; but, for reasons that shall be explained, they are to be used only in correcting the altitudes of the stars. These refractions have been extracted from the Tables published by the French Board of Longitude, and reduced to those that take place when the mercury, in the centigrade thermometer, stands at 14° above zero, or, in Fahrenheit's thermometer, at $57^{\circ} \cdot 2$; and the height of the mercury in the barometer is $\cdot 76$ of a metre, or $29 \cdot 92$ English inches.

The numbers in the second column, entitled, *Refraction less Parallax of \odot* , or of the sun, are those of the third column, from which the parallax of the sun, agreeing with the altitudes opposite the corresponding numbers, has been subtracted. They are only to be used for correcting the altitudes of the sun; with respect to the altitudes of the moon the numbers in Table VIII, which are the refractions of the moon diminished by her parallax, should be employed. When the calculations do not require a very great degree of precision, the numbers in Tables V, and VIII, may be used without any regard to the variations experienced by the refractions in consequence of the changes in either the temperature or weight of the atmosphere.

32. But when it is required to correct the apparent distance between the moon and the sun or a star, the cor-

rections corresponding to the heights of the mercury in the barometer and the thermometer must be applied to the numbers of the Tables V, and VIII. These corrections are to be found in Tables VI, and VII, the use of which shall be shown by an example.

EXAMPLE

The apparent altitude of the sun's centre being $17^{\circ} 45'$, Fahrenheit's thermometer $82^{\circ} 4$, and the barometer 29.53 inches nearly, required, the refraction diminished by the parallax.

In the second column of Table V, we find, at $7^{\circ} 40'$ of altitude, that the refraction diminished by the parallax is $6' 35''$. The column of *Differences*, which is common to the refractions of the stars and those of the sun, shews that the refraction diminishes $8''$ for an increase of $10'$ in altitude; for $5'$, it will therefore decrease $4''$; and the calculation is to be performed in the following manner:—

Apparent altitude \odot	$7^{\circ} 40'$	refraction	- - -	$6' 35''$
Proportional parts for	- $5'$	subtract,	- - -	- $4''$
Apparent altitude \odot	$7^{\circ} 45'$	refraction	- - -	$6' 31''$
Thermometer	- $82^{\circ} 4$	} Table VI, Subtract 21		
Apparent altitude \odot	$7^{\circ} 45'$			
				$6' 10''$
Barometer	29.53	- - -	} Table VII, Subtract 5	
Apparent altitude \odot	$7^{\circ} 45'$			
Corrected REFRACTION	- - -	- - -	- - -	$6' 5''$

Parallax.

33. The positions of all the heavenly bodies is given in the Nautical Almanac, or *Connaissance des Temps*, relatively to an observer supposed to be situated at the centre of the earth; this is, therefore, the point to which all the lines employed in measuring angular distances should be

referred. The altitude of any heavenly body observed at the surface of the globe, can only be equal to that which would have been observed at the centre, when the heavenly body is very distant, and when the distance of the two places of observation, or the radius of the earth, may be regarded as comparatively nothing with respect to the distance of that body. In fact, the line supposed to be drawn from that point of the earth's surface, where the observation is made to the heavenly body, would then be parallel to that supposed to be drawn from the centre of the earth to the same body; or, at least, the angle which these lines would form would be so small, that it might be considered as nothing. This is what takes place in observations of the stars, the distances of which from the earth are very great; their positions as determined by an observer placed at the surface of the globe, are the same as would have been observed at the centre of the same sphere: consequently, the stars have not any parallax. But when the altitudes of the moon, which is the nearest of all the heavenly bodies, are observed, the line supposed to be drawn from the point of the earth's surface where the observation is made, to the moon, will make an angle with that supposed to be drawn from the centre of the earth to the same heavenly body: then the altitude observed at the surface will not be equal to that which would have been measured at the centre. The difference of these two altitudes is, what is called Parallax of Altitude.

It ought to be remarked, that the vertical line is the prolongation of the radius of the earth, considered as spherical, through the point where the observation is made; consequently, whenever the moon is in the zenith, the two lines supposed to be drawn to her, the one from the centre, the other from the point of observation, will form only one: then the parallax is nothing. When the moon begins to

depart from this vertical line, her altitude decreases, and the two lines form an angle between them, which increases in proportion as the altitude diminishes. Finally, when the moon has arrived at the horizon, the line supposed to be drawn from the eye of the observer to that body is perpendicular to the radius of the earth, which joins the centre and the place of observation; and the parallax ought therefore to have its greatest value: hence this value depends upon the apparent altitude. Since the place of observation, and the centre of the earth, are always in the same vertical line, it is evident that the observer is always situated at a greater elevation than the centre; hence the height of the moon will appear to him to be too little: the parallax ought, therefore, to be added to the observed altitudes.

The greatest parallax takes place when the altitude is equal to nothing, and is called the Horizontal Parallax; it is that which is given in Astronomical Tables, and in the Nautical Almanac, or *Connaissance des Temps*. Its value varies rapidly; it frequently increases to $60'$ and some seconds; then it diminishes to less than $54'$. It is usually calculated for every 12 hours. That which corresponds to any proposed instant may be found by rules analogous to those which have been given for obtaining the different elements relative to the positions of the heavenly bodies.

34. When the sun is above the horizon, his parallax of altitude varies according to the same laws as that of the moon; but his horizontal parallax is much less, and experiences only very small changes. It is never more than $8''.95$, nor less than $8''.65$. It is therefore supposed to be constantly equal to $8''.8$; and the value which it ought to have at different altitudes have been subtracted from the corresponding refractions at those altitudes; by which means, the numbers in the second column of Table V, entitled,

Refraction diminished by the Parallax of \odot have been obtained. They give the correction of the sun's altitude, for refraction and parallax at once.

35. It is evident, from what has been said above, that the moon's parallax ought to be greater, as the place of observation is more distant from the centre of the earth; and that it should be the same at all places equally distant from this centre. If the earth were spherical, the horizontal parallax would be the same in all places; but as its form is that of a spheroid slightly compressed at the poles, the equatorial radii are the greatest, and its radii decrease successively in approaching the poles: the parallax ought, therefore, to diminish at the same time, in a very small degree. When the parallax is taken from the *Connaissance des Temps*, it is that which takes place at the equator; and, to obtain that which corresponds to the latitude of the place of observation, it must be subjected to a slight correction. Before calculating the parallax of altitude, we should search in Table III, entitled, *Diminution of the Equatorial Parallax*, for the quantity which is to be subtracted from the parallax given in the Ephemeris.

36. The numbers in Table VIII, are the parallaxes of the moon diminished by refraction, for every 10' of altitude, and for every minute of the horizontal parallax. The proportional parts for the seconds of the parallax are found in the continuation of the table. When the altitude of the moon is below 10°, the proportional parts for the minutes of altitude must be calculated by means of the difference of the numbers corresponding to the two heights, between which the observed altitude is found. Above 10°, the proportional parts are immediately found in the last column of the table.

37. When the apparent distance of the moon from the sun or a star is to be corrected, the numbers in Table VIII,

must be increased or diminished, by the value of the corrections which ought to be made in the refractions on account of the temperature and weight of the atmosphere. It is essential to remark, that in this case, the numbers ought to be employed in a contrary sense to that denoted at the head of the Tables VI. and VII; in fact, the numbers of Table VIII, being the parallax of the moon diminished by refraction, the greater the refraction is, the more the number in the table is to be diminished: an increase of refraction therefore diminishes them; and, for the same reason, a decrease of refraction increases them.

EXAMPLE.

On the 23d of April 1810, at 21^h past 1 in the morning, civil time, or the 21st, at 13^h 21', astronomical time, being at 43° 36' of north latitude, and 31 7' of east longitude, the altitude of the moon's centre, corrected for the depression of the horizon, was 23 44'. Required its true altitude.

The hour at the first meridian corresponding to the proposed hour is the 21st at 11^h 17'

Horizontal parall. at the equator	(21st at noon	59' 21"
	(21st at midnight	59 29
Change in 12 hours	- - - -	- 8"

12 ^h	8"	For 6 hours	- - - -	4"
6	4"	3	- - - -	2
3	2	1	- - - -	0.6
1	0.6	1	- - - -	0.6
		0 17'	- - - -	0.2
		11 ^h 17'	- - - -	7 ^h 4

Horizontal parall. at the equator, 21st at noon	59' 21"
Proportional parts for 11 ^h 17'	- - - - 7
	Sum 59' 28"
Diminution of the equatorial parallax	- - 6
Horizontal parallax for the latitude	- - 59' 22"

Refraction —	51	56
for 22° of altitude	—	1
for 22° of hor. parallax	—	4 30
Parallax of altitude — refraction	52	12
Apparent height of C's centre	23	44 0
True altitude of the C	24	00 12

Correction of the Less of Two Altitudes taken out of the Meridian, for obtaining the Latitude.

38. The method given in this Treatise for calculating the latitude from two altitudes of the sun, taken out of the meridian, and the interval of time elapsed between the observations, requires these observations to be made at the same place; but, as it almost always happens, that the altitudes are taken in two different places, it becomes necessary to correct one of these data of the calculations, in order to obtain that which would have been found if both the observations had been made at the same point of the globe. These corrections depend upon the direction and length of the ship's course during the interval between the observations. The difference of latitude and longitude answering to the length and direction of the course must first be found by the known means, which will, at the same time, be the difference of latitude and longitude of the two places of observation. It will be easy to have respect to the difference of longitude, as will be subsequently shown. It is only required in this place to take into the account the way made in latitude, in order to correct the less of the two observed altitudes.

The calculation should give the latitude of the place where the greater altitude was observed; and the less altitude is always to be corrected. Tables XII and XIII, afford an easy method of finding this correction; which appears to be

so much the more advantageous, as it renders the uncertain observation of the sun's azimuth unnecessary.

39. The operations necessary for this purpose may be divided into two parts; in the first it is required to find, by means of Tables XII and XIII, a number which is called the multiplier of the difference of latitude; the second part consists in the manner of employing this multiplier in obtaining the correction of the less altitude. The rules which should be followed shall first be explained; and then several examples for facilitating their application given.

40. Search, in one of the left-hand pages of Table XII, with the latitude, which is inserted at the head of each column, and the less altitude, which is contained in the first column of the table, a number which is the first term, and write it down separately; then, with the same data, look in the right-hand page of the same table for the argument, which write opposite the first term.

With the argument thus found and the declination of the sun; according as it is of the same or a different denomination with the latitude, search, in Table XIII, for the second term, and write it below the first.

Subtract the first term from the second, increased by two units if necessary; and the remainder will be the multiplier sought.

This rule holds good in all cases, except that in which the latitude and declination are of the same denomination, and the declination greater than the latitude. Then the second term must be subtracted from the first, and the required multiplier will be obtained. It must be observed, that in this circumstance only, the sun passes the meridian between the observer's zenith and the elevated pole; and then the second term is always less than the first.

41. The less altitude will be corrected by attending to the following rules.

* When the meridian altitude ought to be greater in the place of the greater observed altitude than in that of the less, add the difference of latitude to the less altitude; and then subtract the product of this difference of latitude and the multiplier already found from the sum.

If the meridian altitude should be less in the place of the greater observed altitude than at that of the less, subtract the difference of latitude from the less observed altitude; and then add to the remainder the product of the same difference of latitude and the multiplier found by means of Tables XII and XIII.

To render the application of these rules more easy, it must be observed, that the product of the difference of latitude and the calculated multiplier, should always be employed in a contrary sense to the difference of latitude itself; that is, the product must always be subtracted when the difference of latitude has been added; on the contrary, the product must be added when the less altitude has been diminished by the difference of latitude.

EXAMPLE I.

Being in estimated north latitude $33^{\circ} 19'$, the altitude of the sun was observed to be $31^{\circ} 12'$. Some hours afterwards, the altitude of the sun was taken again, and found to be $75^{\circ} 22'$. In the interval of these observations, the vessel had sailed 40 leagues, or 31.5 miles to the S.W. $\frac{1}{4}$ S. 5° S. The declination of the sun at the instant of the first observation was $20^{\circ} 41' N$. It was required to determine what would have been the least of these altitudes, if it had been made in the same place as the greater.

The known method of reducing the courses, shows that the difference of longitude of the two places of observation is $18^{\circ} 1'$, and the difference of latitude $27^{\circ} 6'$, of which the place of the greatest altitude is more to the south than that

of the less. As it is only necessary to make use of the way made in latitude, the last quantity only will be employed.

In the first place, there must be sought in the page of Table XII, which is entitled, *First Term*, the number corresponding to $33^{\circ} 19'$ of latitude, and to $31^{\circ} 12'$ of altitude, and there will be found 1.99, which is to be written down as in the following calculation. Then, in the page entitled, *Argument*, which is on the right of the preceding one, it will be seen that the argument corresponding to the same latitude and altitude, is 1.40, and this is to be written opposite the former number. This argument and the declination, which is of the same denomination as the latitude, serve to find, in Table XIII, the second term, which is 0.50, and which is to be written below the first. As the latitude of the place is of the same denomination, and greater than the declination of the sun, the first term 1.99, is to be subtracted from the second 0.50, increased by two units, or 2.50; the remainder, 1.11, is the required multiplier. The product arising from the difference of latitude of the two places of observation multiplied by 1.11, is $30^{\circ} 7'$.

It should be observed, that the latitude being N. as well as the declination, but the former greater than the latter, the sun passes the meridian to the south of the observer. Since the place of the greater altitude is south of that of the less, the meridian altitude of the former ought to be greater; the way made in latitude, which is $27^{\circ} 6'$, must therefore be added to the less altitude of the sun, $31^{\circ} 12'$, and we shall have $51^{\circ} 39' 6''$, from which there must be subtracted, according to the preceding rules, $30^{\circ} 7'$, or the product of the difference of latitude, by the number which has been found by the assistance of Tables XII and XIII; the less altitude reduced to the place of the greater will then be $31^{\circ} 8' 9''$, or,

by neglecting the seconds above $59^{\circ} 31' 8'' 40''$. The operations may be arranged in the following manner:—

Alt. \odot . $31^{\circ} 12'$	} 1st term	1.39.	Argum. 1.40		
Lat. N. $33^{\circ} 19'$					
Declin. $20^{\circ} 41'$					
Argum. 1.40	} 2d term		$2 + 0.53$	Differ. of lat. $27^{\circ} 6'$	
2nd term — 1st term. 1.11			Multiplier	1.11	
				<u>27.6</u>	
				2.8	
				<u>0.3</u>	
			Product	<u>30.7</u>	

The sun passes the meridian to the south of the observer, and its meridian altitude ought to be greater in the place of the greatest altitude than in that of the less.

Less altitude of the \odot	-	-	-	-	$31^{\circ} 12'$
Add the difference of latitude	-	-	-	-	<u>27.6</u>
				Sum	$31^{\circ} 39.6'$
Subtract the product	-	-	-	-	<u>30.7</u>
LESS ALTITUDE reduced to the place of the greater					$31^{\circ} 8' 9''$
				or	$31^{\circ} 8' 50''$

As the detail of the operations for finding the less corrected altitude, given in the preceding example, will be sufficient to show the manner to be followed in all other cases, the greater part of this detail is suppressed in the two following examples:—

EXAMPLE II.

Being in $48^{\circ} 10'$ of N. latitude, the less altitude of the sun was observed to be $12^{\circ} 26'$; some hours afterwards, the greater altitude was found to be $28^{\circ} 15'$. The vessel had sailed $11\frac{1}{2}$ leagues, or 34 miles to the N.E. and the declination of the sun was $4^{\circ} 32' S$

The place of the greater altitude was therefore 24' north of that of the less.

Alt. of the ☉	12° 26'	} 1st term 1.25. Argum. 1.53.	
Lat. North	48° 10'		
			Diff. of lat. 24' 0
Declin. S.	4° 32'	} 2d term 1.89. Multiplier	0.64
Argument	1.53		
2nd term — 1st term, Multiplier 0.64			14' 4
			0.9
Product			15' 3

The sun passes the meridian to the south of the observer, and his meridian altitude, at the place of the greater altitude, ought to be less than that at the place of the less.

Less altitude of the sun	-	-	-	12° 26'
Subtract the difference of latitude	-	-	-	24
		Difference	-	12° 2'
Add the product	-	-	-	15' 3
LESS ALT. reduced to the place of the greater				12° 17' 3
or, by taking the nearest tens of seconds				12° 17' 20"

EXAMPLE III.

Being in S. latitude 3° 42', the greater altitude of the sun was found to be 70° 31'. After his passage over the meridian, the less altitude was observed to be 50° 22'. The vessel had sailed 4½ leagues, or 13½ miles to N.N.W. in the interval between the observations. The declination of the sun was 22° 50' S.; consequently, it was of the same denomination with the latitude, but greater.

The place where the greatest altitude was observed was 12' 5 to the south of that where the less was taken.

Alt. of the ☉.	5° 22'	} 1st term' 1.06. Argum. 1.57.
Lat. South	3 42	
Decln S.	22 30	} 2nd term 0.60. Diff. of lat. 12.5
Argument	1.57	
1st term — 2nd term, 0.46.		Multiplier 0.46
		5.0
		0.8
		Product - 5.8

The sun crosses the southern part of the meridian; the meridian altitude at the place of the greater altitude should therefore be greater than at the place of the less. Hence, the

Less altitude of the sun	-	-	-	5° 22'
Add the diff of latitude	-	-	-	12.5
Sum				50° 34.5
Subtract the product	-	-	-	5.8
LESSALT reduced to the place of the greater	50° 28.7			
or, by taking the nearest tens of seconds	50° 28' 40"			

CHAPTER III.

On the Latitude.

42. THE latitude may be found at sea by three different kinds of observations. The most common and the most simple, is an observation of the meridian altitude; the second consists in observing several altitudes near the meridian, and concluding the meridian altitude from them; this is that by which the greatest degree of accuracy is obtained, but, as the calculation is rather long and requires a knowledge of the time corresponding to each observation, it is *only* necessary to employ it in ascertaining the latitudes of places, the exact positions of which are essential to be known. Lastly, the latitude may be obtained from the observation of two altitudes taken out of the meridian and the interval of time elapsed between them. Though this last method may not be susceptible of giving the latitude with as much precision as the others, it is of great use in the practice of navigation, when the sun is obscured at noon and it is impossible to observe his meridian altitude. The rules proper for each of these methods shall be given.

To find the Latitude by the Meridian Altitude of any of the heavenly Bodies.

43. The latitude may be calculated, as explained in article 5, by adding the meridional zenith distance of a hea-

venly body, of which the altitude has been observed, to its declination; or else by subtracting these quantities from each other. In the following rules, the altitude itself, which is obtained directly by observation, is to be employed. The operations resulting from them differ from those in common use; but the following explanations will make their application more easy.

44. When we are at the terrestrial equator the latitude is nothing; then the celestial equator passes through the zenith, and the two poles are in the horizon. If we advance along the meridian into either hemisphere, the pole of that hemisphere will appear to rise above the horizon by an arc equal to the latitude passed through; and the latitude is equal to the altitude of this pole. The pole of the other hemisphere, on the contrary, descends on the opposite side below the horizon, and the celestial equator is on that side depressed the same number of degrees. The celestial equator is therefore towards the depressed pole, and its inclination to the horizon is equal to the complement of the latitude. From this last principle, the following rules for calculating the latitude directly by means of the meridian altitude are obtained. It is easy to perceive, that the inclination of the equator to the horizon is measured by an arc of the meridian comprised between these two circles; this arc is called the altitude of the equator; but it should be understood, that it is effectively the altitude of that point only of the celestial equator which is cut by the meridian.

45. First, calculate the time at the first meridian corresponding to the instant at which the observed body passes the meridian, and look in the Nautical Almanac for its declination at that time; then correct the observed altitude for the depression of the horizon and refraction: if the altitude of the sun or moon have been taken, regard must be had to semi-diameter and parallax; which last quantity will be

obtained at the same time as the refraction, by taking, if required for the sun, the numbers from the second column of Table V; if for the moon, the numbers must be taken from Table VIII. When the true altitude and declination are obtained, the latitude may be calculated.

1st. Remark towards what pole the heavenly body was, when the meridian altitude was taken; that is, on what side of you it passed the meridian.

2nd. If the declination has a different denomination from that of the pole towards which the altitude was observed, subtract the declination from the true altitude; the remainder will be the altitude of the equator, the complement of which is equal to the latitude. The heavenly body, in this case, has passed the meridian towards the depressed pole, and the latitude will have a denomination different from that of the pole towards which the altitude of the body was observed: this rule is without exception.

3rd. If the declination be of the same denomination as the pole towards which the meridian altitude of the heavenly body was observed, the declination must be added to the true altitude; the sum will be the altitude of the equator. When this sum is less than 90° , the sun has been observed towards the depressed pole, as in the preceding case; and its complement will be the required latitude, the denomination of which is different from that of the depressed pole.

When the sum of the declination and the true altitude, or the altitude of the equator, is greater than 90° , it is a proof that the celestial equator was behind the observer, or on the contrary side of his zenith, at the time the altitude was observed; then the body has passed the meridian towards the elevated pole. Subtract 90° from the sum of the declination and the true altitude, the remainder will be the latitude; the denomination of which ought to be the same as that of the pole towards which the meridian altitude was observed.

EXAMPLE I.—*Altitude of the Sun.*

On the 29th of April 1810, being in $31^{\circ} 10'$ of west longitude, the sun passed the meridian towards the south; the meridian altitude of his lower limb was observed to be $51^{\circ} 25'$; the elevation of the eye above the surface of the sea was $8\frac{1}{2}$ yards, or $26\frac{1}{2}$ feet; required the latitude of the place of observation.

By the rules in art. 9, the hour at the first meridian at the time of the observation, is found to be $2^h 5'$; this time is therefore the 29th at $2^h 5'$; by following the directions given in art. 16, the declination of the sun will be found to be $14^{\circ} 21' 57''$ N.

Observed altit. of the lower limb of the \odot .		$51^{\circ} 25' 0''$
Elevation of the eye $26\frac{1}{2}$ feet	Depression	- 5 1
	Remainder	$51^{\circ} 19' 59''$
Semi-diameter of the sun "	- - -	+ 15 54
	Sum	$51^{\circ} 35' 53''$
Refraction—Parallax of the \odot	- - -	- 0 40
True altit. of the \odot . towards the S.	-	$51^{\circ} 35' 13''$
Declination of the \odot . towards the N.	-	14 21 57
Altitude of the equator—Difference	-	$37^{\circ} 13' 16''$
Complement LATITUDE N.	- - -	$52^{\circ} 46' 44''$

EXAMPLE II.—*Altitude of the Moon.*

On the 26th of March 1810, being in $13^{\circ} 7'$ west longitude, and $25^{\circ} 36'$ of north latitude, the moon passed the meridian towards the south at $4^h 20'$ in the morning, or the 25th at $16^h 20'$, the meridian altitude of her upper limb was $46^{\circ} 19'$; and the elevation of the eye 23 feet above the level of the sea. Required the latitude.

The time of the moon's passage over the meridian was, at the first meridian, $19^h 12'$; when she had $17^{\circ} 43'$ of south declination.

Altitude of the upper limb of the ☾	-	46° 19' 0"
Elevation of the eye 23 feet	-	— 4 41
Remainder	-	46° 14' 19"
$\frac{1}{2}$ horiz. diam. 16' 5"	} Semi-diameter of the ☾	— 16 17
Augmentation + 12		
Apparent altitude of the ☾'s centre	-	45° 58' 2"
Horiz. Parallax 58' 57"	} Parallax—refraction	+ 40 ' 2
Diminished 58 55		
True altitude of the moon towards the S.	-	46° 38' 4"
Declination of the ☾ towards the S.	-	17 43 0
Altitude of the equator	-	Sum - 61° 21' 4"
Complement. LATITUDE N.	-	45° 38' 56"

EXAMPLE III.—*Altitude of a Star.*

On the 16th of April 1808, Antares passed the meridian towards the south, his observed altitude was 64° 30', and the elevation of the eye was 21·3 feet.

It has been found, art. 18, that the declination of Antares, on the 16th of April 1808, was 25° 59' 29" S.

Observed altitude of Antares towards the S.	64° 30' 0"
Elevation of the eye 21 3 - - Depression	— 4 31
Remainder	64° 25' 29"
Refraction - - - - -	— 28
Altitude of Antares towards the S.	64° 25' 1"
South declination - - - -	25 59 29
Altitude of the equator - Sum - -	90 24' 30"
Subtract 90°. - LATITUDE S. - -	0° 24' 30"

To find the Latitude by several Altitudes of the Sun, taken very near the Meridian.

46. When we wish to find the latitude from several altitudes taken near the meridian, the greatest possible number of altitudes must be observed in the interval of 14' or 16';

the observations are to be commenced 7' or 8' before the passage of the sun over the meridian, and continued 7' or 8' after that time. Astronomers calculate by a direct and rigorous method, the quantities that must be added to each of the observed altitudes, in order to obtain from it the meridian altitude; but as in the practice of navigation, an error of 2" or 3" is of little importance, we shall give a method of approximation which is more generally used, because the calculations are rather more simple. By this method, a number of meridian altitudes, equal to that of the observations, is obtained; and the latitude is reduced from them with much greater precision than could possibly be done from a single observation of the meridian altitude. It is unnecessary, as will shortly appear, to calculate the correction for each observed altitude; it is sufficient to find the correction of the mean altitude which results from all the observations, and, by this means, the calculations are much abridged.

It may be supposed, without apprehending any sensible error, that in the interval of 7' or 8' before the passage of the sun over the meridian, and the same time after, the changes in the altitudes are proportional to the squares of the times elapsed before or after this passage. Now, it is possible to calculate the quantity which the sun ought to ascend during the last minute of his approach to the meridian, and the first of his departure from it; it is therefore easy to conclude from this, how much he ought to ascend or descend in any other interval, provided that interval does not exceed 7' or 8' minutes. It is only required to multiply the change in altitude which answers to the last minute before his passing the meridian, or the first after it, by the square of the interval corresponding to each observation, or by the square of horary angle. Such is the fundamental principle of this method of approximation. It requires a knowledge of the time of each observation; a seconds watch

must therefore be used, or, what is still better, a marine chronometer, the rate of which shall have been calculated from observations taken in the morning or evening of the same day. The method of finding the time at the place where we are, and of ascertaining the quantity which a watch or chronometer, going either too fast, or too slow, differs from the true time, will subsequently be given; in the following rules, it is supposed that this quantity is known.

47. The corrections of the observed altitudes will be greater or less as the observations have been made farther from or nearer to the meridian, or as the corresponding horary angles are greater or less. If the time of the sun's passage over the meridian, as marked by the watch, is affected with an error, and this error is of such a nature as to increase the horary angles of the observations taken before the sun's passage, it will follow, that the corrections of the corresponding altitudes will be too great. The same error will diminish the horary angles of the observations taken after the passage, and the corrections of these last altitudes will be too small; in the case in which the horary angles of the former observations are too little, those of the latter will be too great. An error in the time by the watch will therefore have an influence, in a contrary sense, upon the corrections of the altitudes observed before and after the passage of the sun over the meridian; and, consequently, upon the meridian altitudes deduced from them. Hence it follows, that if an arithmetical mean of all the calculated meridian altitudes be taken, the errors, in our sense, will either wholly, or in part, compensate for those of the contrary or opposite kind; and the error in the mean altitude, or of the latitude itself, will always be less than the greatest errors above mentioned. It ought, therefore, to be regarded as a general rule, that the same number of altitudes should,

as often as possible, be observed before and after the passage of the sun over the meridian.

48. Calculate, before-hand, within nearly half a minute, the time which the watch ought to give at the instant of noon. Commence the observations 7 or 8 minutes before this instant, and mark the hour, the minute and the second, corresponding to each; cease to observe 7 or 8 minutes after the sun has passed the meridian. If a sextant is used, the arc indicated by the index on the limb of the instrument must be read off at every observation; and its value written down opposite the corresponding time. When the reflecting circle is used, the arc passed over by the index of the great mirror, must be read off at the end of every second or even observation. This method of reckoning will enable the observer to reject those observations which he may judge defective, either from their differing too much from others, or because of some unforeseen accident during the observation itself.

Take the sum of all the altitudes, if they were observed with a sextant, or the whole arc passed over by the index, if the reflecting circle was used, and divide this sum or arc by the number of observations, which will give the mean apparent altitude. Correct this altitude for the depression of the horizon, the diameter of the sun, and the effects of refraction and parallax, and the true mean altitude will be obtained, which is only to be augmented by the quantity found by the following rules, in order to conclude from it both the meridional altitude and the latitude of the place of observation.

1st. It is supposed, that the time the watch ought to give at noon has been calculated from observations made in the morning or evening, and at a place little distant either to the east or west from that where the altitudes near the meridian are to be observed. Correct this time (art. 8. and 9.), by

means of the way made in longitude during the interval between the observations; and the time of the sun's passage over the meridian, at the place where the altitudes are observed, will be obtained.

2^d. Search in Table IX, with the estimated latitude and the declination, for the quantity which the sun ought to ascend or descend in the minute before and after his passage over the meridian. This quantity is expressed in seconds, and fractions of a second; write it down as in the following example.

3^d. Take the difference between the time as marked by the watch at the instant of each observation, and that of the passage over the meridian, which will give the horary angle corresponding to each altitude. Find in Table X, opposite each of the horary angles, a number which is its square, expressed in minutes and decimals of a minute, and write it on the right hand of the horary angle to which it belongs. Add together the squares of all the horary angles, then divide their sum by the number of observations, and the quotient will be the number by which the quantity found in Table IX, is to be multiplied. This product will be the correction to be added to the true mean altitude of all the observations, in order to obtain the true meridian altitude; which is to be used in the same manner as if it had been found directly by observation, and the required latitude will be obtained.

The following example will illustrate the preceding rules:—

EXAMPLE.

On the 17th of June 1793, being in south latitude $9^{\circ} 52'$, and east longitude $148^{\circ} 55'$, the altitudes of the sun near the meridian were observed, for the purpose of ascertaining the latitude. It had been found from observations made in

the morning, that at 7^h 50', the watch was 1^h 22' 34".2 behind true time. The place where we were at noon was 4' 50" of a degree, or 19".3 of time, to the west of the place where the time had been observed in the morning. The elevation of the eye was 20 feet: what was the correct latitude?

The time by the watch at noon, at the place where the horary angles were observed, would be 10^h 37' 25".8; but as the place of observation was west 19".3 of time, the passage of the sun over the meridian would take place later by the same quantity. The 19".3 must therefore be added to 10^h 37' 25".8; and the time of passage by the watch, neglecting the fractions of a second, would be 10^h 37' 45". The details of the following calculation shall not be specified: the operations which ought to be performed will easily be perceived by inspection.

The time reckoned at the first meridian, corresponding to the instant of the passage over the meridian of the place of observation, was 14^h 4', but the 17th had not commenced, and the time of the passage was therefore the 16th of June, at 14^h 4'; the corresponding declination of the sun was 23° 24' 29" N.

Time of passing the Meridian 10^h 37' 45".

Time by the Watch.	Intervals.	Squares of the Intervals, or Multipliers.
10 ^h 35' 47" - -	1' 58" - - -	3.9
36 21 - - -	1 24 - - -	2.0
38 9 - - -	0 24 - - -	0.2
39 10 - - -	1 25 - - -	2.0
Sum - -		8.1

The fourth. Multiplier - - 2.02

Quantity ascended by the sun in 1' before passing the meridian	} 3.3
Multiplier	2.02
	6.1
	0.6
Number to be added to the mean altitude	6.7
Sum of the altitudes of the ☉'s lower limb	226° 1' 40"
<i>The fourth.</i> Mean apparent altitude of the ☉ at noon	} 56 30 25
Elevation of the eye 20 feet. Depression	— 4 23
Remainder	56° 26' 2"
Semi-diameter of the ☉	+ 15 46
	56° 41' 48"
Refraction—Parallax	— 33
True mean altitude of the ☉.	56° 41' 15"
Add	+ 7
Meridian altitude North	56° 41' 22"
Declination North	23 24 29
Sum. Altitude of the equator	80° 5' 51"
Complement. LATITUDE SOUTH	9° 54' 9"

49. In the interval of 14', during which the observations may be continued, it is possible to take eight or ten, or even a greater number of altitudes; therefore, the errors in the calculated latitude will be greatly attenuated. The latitude may be obtained in a single day with as much accuracy as by the observations of eight or ten meridian altitudes, which require as many days as there are observations; and, if the altitudes are taken with a circle, the accuracy will be still greater.

The greatest errors arise from the uncertainty of astronomical refractions, and principally from those which influence the depression of the horizon, treated of in art. 24

and 25; but in common cases, there can be little doubt of obtaining the latitude to a minute, or even to nearly half a minute, and sometimes with much greater accuracy.

To find the Latitude from Two Altitudes of the Sun, taken out of the Meridian, and the Interval of Time elapsed between the Observations.

50. The calculation of the latitude from two altitudes taken out of the meridian, and the time elapsed between the observations, is very complicated. The limits to which this treatise is confined, do not permit us to give the demonstration in this place, but it will be found at the end of the work. The object which is here proposed, is to explain the operations which are proper to be performed in each of the methods that can be employed at sea, in order to render their application easy and familiar. This reason has induced us to give separately, at the end of this treatise, the demonstrations of all the methods which depend upon the resolution of spherical triangles.

51. This method requires us to know, whether the sun ought to pass the meridian towards the elevated or depressed pole; but such a condition cannot be productive of any inconvenience in practice. In fact, it is impossible to obtain the latitude from altitudes taken before and after noon, whenever the sun's meridian altitude exceeds 84° , that is, when his meridional zenith distance is less than 6° ; but, as we can never have so great an uncertainty in the estimated latitude, whenever this kind of observation is practicable, we shall never be liable to mistake the denomination of the pole towards which the sun passes the meridian.

52. All altitudes that can be observed while the sun is above the horizon, are not equally proper for giving the latitude with that precision which the safety of navigation

requires; those might be observed from which the results would be very defective, and even among those altitudes that might be taken in favourable circumstances, there are some from which the calculated latitudes would admit of greater precision than from others. These circumstances depend, in general, upon the interval of time elapsed between the observations, with respect to that of the horary angle, corresponding to the altitude taken nearest the meridian. The probability of an error with which the latitude may be affected, may also be estimated by the ratio which exists between the azimuths corresponding to each observation; it is these last angles of which the greatest use will be made in the following rules. The method here treated is discussed with the greatest detail in the second volume of *d'Entrecasteaux's Voyage*; and it is from this work that the following precepts have been extracted.

53. When the two altitudes have been taken on the same side of the meridian, that is, when they have been both observed in the morning or the evening, they are said to be of the same kind. When one of them has been observed before the sun passed the meridian, and the other after, they are of a different kind.

The azimuth corresponding to the altitude which has been observed nearest the meridian, or to the greater altitude, is called the less azimuth; that which corresponds to the less altitude, is called the greater azimuth.

GENERAL PRECEPTS,

For finding the Latitude from Two Altitudes taken out of the Meridian.

54. When the meridian altitude would exceed 84° , this method cannot be employed.

The less altitude should be more than 6° or 7° .

The way made by the vessel in the interval between the observations should not exceed 12 leagues.

The watch with which the interval of time between the observations is measured, should not vary from mean time more than 3 minutes in 24 hours.

OBSERVATIONS OF THE SAME KIND.

Rules for the Altitude nearest the Meridian.

55. The nearer the greater altitude is taken to the meridian, the greater precision will be obtained in the result.

If the interval be measured with a marine chronometer, the least azimuth ought not to be greater than 40° or 45° . In the case where this measure can only be taken with a common watch, susceptible of a variation of 3 minutes in 24 hours, the least azimuth ought never to exceed 15° .

Rules for the Altitude farthest from the Meridian.

56. The interval of time elapsed between the observations should always be greater than that corresponding to the least horary angle; but as the ratio of these two quantities is subject to a variation, according as the meridian altitude of the sun is greater or less, general rules can only be deduced from the values of the azimuths corresponding to the two altitudes.

The value of the azimuth corresponding to the less altitude, or the greater azimuth, ought not to be less than about $2\frac{1}{2}$ times the value of the less azimuth. When a marine chronometer is used, the larger the former of these azimuths is, the greater precision will be obtained in the result, provided the sun has always more than 6° or 7° of altitude; and the way made in the interval between the

observations is not more than 4 leagues. With a common watch, the greater azimuth should not exceed 75° .

By following these rules, the latitude may be obtained to within about 3 minutes of the truth.

OBSERVATIONS OF A DIFFERENT KIND.

57. The nearer the two altitudes are observed to the meridian, the greater precision may be obtained in the result.

Rules for the Altitude nearest the Meridian.

58. If the interval of time be measured with a marine chronometer, the less azimuth ought never to exceed 45° ; with a common watch, it should not be more than 30° .

Rules for the Altitude furthest from the Meridian.

59. The supplement of the greater azimuth, or of the azimuth corresponding to the less altitude, ought not to be less than two and a half times the value of the less azimuth. This rule is without exception when the interval between the observations is measured with a marine chronometer; but it is to be recollected, that the sun must not be below 6° or 7° of altitude, and that the way made in the interval is not to exceed 12 leagues. When a common watch is used, the less azimuth may be between 15° and 30° , and the sum of the azimuths corresponding to the two altitudes, or the azimuthal interval may be 60° . When the less azimuth is not more than 15° , the greater should not surpass 75° .

Whenever these rules are complied with, the latitude may be obtained to nearly 3 minutes of the truth.

Remark on the Application of the preceding Rules.

60. It is not necessary to know the azimuth corresponding to each altitude with a great degree of accuracy, in order to be able to judge of the precision of which the observation is susceptible; it will be sufficient to obtain it within 2 or 3°. Tables XII and XIII, the use of which has been explained in art. 40. and 41, will give these azimuths with the necessary accuracy, at least with a very simple operation, as shall be shown.

61. When the multiplier for the correction of the less altitude, on account of the way made in latitude between the observations, has been calculated, enter Table XIV, with this number, and there will be found in the same line, on the left hand, the azimuth corresponding to the less altitude. The multiplier answering to the greater altitude being calculated in a similar manner, by means of Tables XII and XIII, the azimuth that answers to it will be found in Table XIV. The two azimuths being known, it will be very easy, according to the preceding rules, to ascertain whether the circumstances of the observation are favourable, and if the result will be comprised in the limits of precision already indicated. It is essential that the proportional parts should be taken with accuracy in the Tables XII and XIII, whenever the greater altitude corresponds to an azimuth less than 30°. The same tables are no longer proper for giving the value of the azimuth, even by approximation, when it is less than 15°; but in this case, the value will be very small, and the azimuth corresponding to the less altitude may always be from 40 to 45 degrees.

CALCULATION OF LATITUDE.

62. The latitude of the place where the greater altitude

has been observed cannot be directly obtained ; but several other quantities must first be calculated. 1st. It is necessary to ascertain the distance of the two places which the sun occupied in the heavens with respect to the meridian and horizon, at the time the altitudes were observed ; this is called the distance of the sun's places. 2nd. The angle formed by the arc of the great circle that measures this distance and the circle of declination corresponding to the least altitude, is to be calculated ; this is the first angle at the sun. 3rd. The second angle at the sun, which is formed by the arc of the distance, and the vertical circle of the less altitude, must also be calculated. 4th. These two angles, added together or subtracted from each other, will give the angle that the circle of declination makes with the vertical circle of the sun, at the moment of observing the less altitude, or the angle of variation. Lastly, by means of this last angle, the latitude may be directly calculated, which will be that of the place where the greater altitude was observed.

63. Previous to entering upon the calculations which have been specified, it will be necessary to obtain the data that are to be employed. The time at the first meridian corresponding to the two instants of observing the altitudes, must first be found by means of the estimated longitude ; then the two declinations answering to these instants are to be taken from the Nautical Almanac. Half the sum of these declinations taken from 90° , when the sun is in the same hemisphere with the elevated pole, will give the polar distance, which is to be used in the calculation. When the sun is in the other hemisphere, 90 degrees is to be added to half the sum of the declinations corresponding to observations of altitude.

64. The interval of time elapsed between the observations, as obtained by the watch, is the same as would have

been measured if the vessel had not changed its place; in short, whether we remain at rest, or move with great velocity, provided the instants indicated by the watch are the same, the elapsed time will always be equal to the difference of the times corresponding to the observations. But the difference of the times reckoned at the places, at the instants of the two observations, must be used in the calculations; thus, if the place where the less altitude was observed is to the eastward of that of the greater, the difference of longitude of the two places, reduced into time, must be added to the time of observing the less altitude; on the contrary, if the place of the less altitude is to the west of that of the greater, the difference of longitude must be subtracted. The difference which exists between the time of the least altitude so corrected, and the time of the watch corresponding to the greater altitude, will give an interval of time, the half of which, reduced into degrees, will be the half interval with which the calculation is to be performed. When the observations are of the same kind, that is, when both have been made in either the morning or the evening, subtract the less time, as given by the watch, from the greater, and it will give the interval of time which separated them. If the observations are of a different kind, subtract the time of the observation made before noon, from that which corresponds to the observation made after noon, increased by 12 hours.

65. The two observed altitudes should be corrected for the depression of the horizon, the semi-diameter of the sun, and the effects of refraction and parallax, according to the rules already given; there must also be another correction applied to the less altitude, for the purpose of taking into the account the way which the vessel has made in latitude during the interval between the two observations; this is to be found by the methods explained in art. 40 and 41.

66 The multiplier which serves to calculate the correction of the less altitude, will give, with the assistance of Table XIV, the azimuth corresponding to that altitude. The multiplier that agrees with the greater altitude, and also its corresponding azimuth, are to be found in the same manner; the two azimuths must then be compared together, and the ratio of their values will enable us to judge (see art. 35, and following), whether the observations have been made under favourable circumstances or not.

67, When the given quantities have been collected, and it has been ascertained that the result ought to be within the limits of the requisite precision, the latitude may be calculated according to the following rules.

1st. *Distance of the two places of the sun.* Add the logarithm sine of half the interval to the logarithm sine of the polar distance; the sum will be the logarithm sine of the half distance of the sun's places: double this, and it will give the whole distance.

2nd. *First angle at the sun.* Add the logarithm of the cotangent of half the interval to the complement of the logarithm cosine of the polar distance; the sum will be the logarithm tangent of the first angle at the sun. Half the corresponding arc will be half the first angle at the sun.

The arc answering to the logarithm tangent of the first angle at the sun should be less than 90° , if the distance from the sun to the elevated pole is less than 90° ; but greater than 90° , if the polar distance exceeds 90° : thus, in the first case, the arc found in the Tables will be the first angle; in the second, it must be subtracted from 180° to obtain this angle.

The data employed in the calculation of these two quantities are the same; and they may be disposed as shown in the following Table. Immediately after having taken the logarithm sine of half the interval, the logarithm cotangent is to

be taken; and written opposite the former. The same is also to be done with respect to the polar distance; after having found the logarithm of its sine, the arithmetical complement of its cosine is to be taken.

3rd. *Second angle at the sun.* Write one above another, and in the following order; the greater altitude, the less corrected altitude, and the distance of the sun's places. Add these three quantities together; and take half their sum; from which subtract the greater altitude.

Search then, in the Tables, for the arithmetical complement of the logarithm cosine of the less altitude, and that of the sine of the distance of the sun's places. Take from the same tables the logarithm cosine of the half sum, and the sine of the difference between this half sum and the greater altitude. Add the two arithmetical complements to the two logarithms: half their sum will be the logarithm sine of half the second angle at the sun; which is to be written below that of the first, which has already been found.

4th. *Angle of variation.* When the sun passes the meridian towards the depressed pole, take half the difference of the first and second angles at the sun. But when this passage is made towards the elevated pole, take half their sum: which will be half the angle formed by the sun's vertical circle, and his circle of declination, at the instant of observing the less altitude; or half the angle of variation.

5th. *Latitude.* Below the half angle of variation, write the distance of the sun from the elevated pole; and immediately under it the less corrected altitude. Subtract the less altitude from the polar distance, and take the difference between the remainder and 90° ; write half this difference below the other two numbers.

Find the logarithm cosine of half the angle of variation; add to it, first, half the logarithm sine of the polar distance, then half the logarithm cosine of the less

altitude, and, lastly, the arithmetical complement of the logarithm cosine of the half difference, referred to at the end of the last paragraph. The sum of these four numbers will be the logarithm sine of an auxiliary arc. Take the logarithm cosine of this arc, and subtract from it the arithmetical complement of the logarithm cosine of the half difference between the remainder and 90° , above found; which will give the logarithm cosine of half the sum of the latitude plus 90° . Multiply the corresponding arc by 2, and subtract 9 from the tens and hundreds of the degrees in the product; and the remainder will be the latitude of the place where the greater altitude was observed.

EXAMPLE.

On the 17th of July 1809, about $6^h 40'$ in the morning, being in $43^\circ 6'$ of north latitude by account, and $148^\circ 56'$ of east longitude; when the watch was $6^h 44' 20''$, the altitude of the sun's lower limb was observed to be $21^\circ 34' 50''$; and when the same watch indicated $11^h 12' 36''$, a second altitude of the same limb was taken, and found to be $65^\circ 18' 58''$. The elevation of the eye at these two observations was about $21\frac{1}{2}$ feet. In the interval between the observations, the ship had advanced $25' 3''$ of a degree in longitude towards the west, and $27' 26''$ in latitude towards the north. The latitude of the place where the greater altitude was observed is required.

The rules already given are sufficient for finding the elements of the calculation for this example; and, from an inspection of the following table, it will be easy to understand the operations which are to be performed; all details on the subject will therefore be dispensed with. It should now be remarked, however, that the common denomination of first and second observation, have not been used; it appeared that those of the greater and less altitudes would

Calculation of Latitude by two Altitudes taken out of the Meridian.

July 17th, 1809.

TIME BY THE WATCH.

At the place of the less alt.	- 6° 44' 20" 0
Less alt. taken E. of the greater	+ 1 40 2
Less alt. taken to the W.	- - - -
Time of the less alt. corrected	- 6 46 0 2
Time at the place of the gr. alt.	11 12 56 6
Interval in time	- 4 26 36 4
Interval in degrees	- 26° 39' 10"
Half interval	- 13° 19' 30"

LESS ALTITUDE.

Observed altitude of ☉	- 21° 34' 50"
Height of the eye 21 1/2 ft. depression	- 4 31
Semi-diameter of ☉	- 21 30 19
Refraction - Parallax	- 2 15
True altitude of ☉	- 21 43 50
Meridian alt. less at the place of the greater alt.	- 27 24

Product	- 21 36 26
Less altitude corrected	- 21 47 56
Half interval	- 33° 19' 30"
Polar distance of ☉	- 68 49 0
Distance two places of ☉	- 30 40 50
Double Distance	- 61 37 40

When the polar distance is > 90°, subtract the 1st angle at the ☉ from 180°

Greater altitude of ☉	- 68° 29' 50"
Less altitude of ☉	- 21 48 0
Distance of the ☉'s places	- 61 37 40
Sum	- 148 55 30
Half-sum	- 74 27 40
Half-sum - greater alt.	- 8 37 50
Sine	- 18 7083627
Half-sum	- sin. 9 3541813
Half 2d angle at the ☉	13° 3' 50"

GREATER ALTITUDE.

Observed altitude of ☉	- 65° 18' 58"
Height of the eye 21 1/2 ft. depression	- 4 31
Semi-diameter of ☉	- 65 14 27
Refraction - Parallax	- 2 22
True altitude of ☉	- 65 30 43
Meridian alt. less at the place of the greater alt.	- 27 24

Product	- 65 30 43
Less altitude corrected	- 65 29 50
Half interval	- 32° 59' 50"
Polar distance of ☉	- 21 48 0
Distance two places of ☉	- 10 54 00
Double Distance	- 21 48 00

When the polar distance is > 90°, subtract the 1st angle at the ☉ from 180°

Greater altitude of ☉	- 65° 18' 58"
Less altitude of ☉	- 21 48 0
Distance of the ☉'s places	- 61 37 40
Sum	- 148 55 30
Half-sum	- 74 27 40
Half-sum - greater alt.	- 8 37 50
Sine	- 18 7083627
Half-sum	- sin. 9 3541813
Half 2d angle at the ☉	13° 3' 50"

Estimated time of the less alt.	- 13° 40' 8"
Longitude East 148° 56' or -	- 9 54 0
Time at the first Meridian	- 9 46 0
Less alt.	- 21 48 0
Declination of ☉	21° 12' 58" N.
Mean Declination	- 21 10 28 N.
Dist. of ☉ from the elevated pole	68 49 3
Estimated lat. Less alt.	- 43 6 N.
Estimated lat. Greater alt.	- 43 58 6 N.
Difference of latitude	- 27 4 N.

Calculation of the multiplication of the difference of latitude.

Table XII. 1st term	157
Table XIII. 2d term	0.55 + 2
Multiplicand	- 1.16
Diff. of latitude	- 27.4
Product	- 31.5

When the polar distance is > 90°, subtract the 1st angle at the ☉ from 180°

Greater altitude of ☉	- 68° 29' 50"
Less altitude of ☉	- 21 48 0
Distance of the ☉'s places	- 61 37 40
Sum	- 148 55 30
Half-sum	- 74 27 40
Half-sum - greater alt.	- 8 37 50
Sine	- 18 7083627
Half-sum	- sin. 9 3541813
Half 2d angle at the ☉	13° 3' 50"

comp. cos. 0.0312972	-
sin. 9.9563698	-
cos. 9.6300783	-
cos. 9.5957811	-
66° 36' 30" N.	-
43° 18' 0" N.	-

Auxiliary angle	-
Comp. cos. 1/2 difference	-
1/2 sum of the latitude + 90°	-
Latitude of the place of the	-
Double - 90°	-
greater altitude	-

If the ☉ pass the meridian towards the depressed pole, subtract the 2d angle at the ☉; add it if he pass towards the elevated pole.

render the application of the rules more uniform, and the distinction of cases more easy. Care has also been taken to specify, in the same table, the quantities which are additive, and those that are subtractive. When the same quantities may have, in different cases, either sign, the circumstances that determine in what sense they are to be used, have been written opposite them. Thus, without any other assistance than this table, any observations may be calculated, whatever may be the circumstances under which they are made.

CHAPTER IV.

Calculation of the Hourly Angle, and of the Altitude of any of the heavenly Bodies.

68. It has already been shown, that a knowledge of the time at the place where we are is necessary, for obtaining the latitude from several altitudes of the sun taken near the meridian, and it is equally essential in calculating the longitude by means of marine chronometers, and the distances of the moon from the sun or the stars. This problem may, therefore, be regarded as one of the most important in Nautical Astronomy. We shall therefore give, in this chapter, the means of finding the true time at the vessel, then treat of the inverse method, which consists in calculating the altitude of any heavenly body, from knowing the time at the place of observation. The calculation of the altitude is useful in certain cases, where it is required to find the true distance of two of the heavenly bodies when the apparent distance has been observed. These two problems will then be applied to that of longitude, in the two following chapters, in which the methods to be used in calculating the longitude by marine chronometers, and the distances of the heavenly bodies, are explained.

Calculation of the Horary Angle.

69. It was said at the commencement of this Treatise, that the astronomical day is the interval of time elapsed between the passage of the sun over any meridian, and his return to the same meridian. This interval is divided into 24 equal parts, which are called hours, and are so reckoned, that when the circle of the sun's declination, by virtue of its diurnal motion, has passed over 15° of the equator, we reckon one hour, and when it has passed over 30° , we count two hours. It follows from this, that when the circle of declination is at 180° from the meridian taken for the first, we reckon 12 hours; and, lastly, at the moment of the sun's return to the same meridian, the circle of declination has passed over 360° of the equator, and then the 24 hours of the day are elapsed. Those parts of time that have the same denomination, answer to equal parts of the equator; they may therefore be valued in Degrees. It also follows, from what has been said, that the time at any place is equal to the difference of right ascension between the celestial meridian of that place and the circle of the sun's declination, at the given instant, or to the spherical angle formed by the meridian and the circle of declination.

70. From noon, or the moment of the sun's passage over the meridian to his setting, and even to the moment of his arrival at the meridian, 180° from that of the place, the circle of declination becomes more distant from the meridian of that place; after which, it approaches it until the sun return to the meridian again. The least distance of the circle of declination from the meridian, at any given time, is called the Horary Angle. In the first half of the astronomical day, that is, from noon to midnight, the horary angle is equal to the hour itself; but in the latter half, or

from midnight to noon, the time is the difference between the horary angle and 360° , or 24 hours, when the day is reckoned astronomically; or to the difference between the same angle and 180° , or 12 hours, when the day is taken in a civil sense: this is the angle which is directly given by the following calculations.

71. As the altitude of the sun varies every moment he is above the horizon, the time, or the horary angle, may therefore be ascertained by observation of his altitude. From the rising of the sun, to his passage over the meridian, his altitude increases at first very rapidly, afterwards his movement in altitude becomes slower; and lastly, when he has arrived at the meridian, this motion ceases. When the sun begins to descend towards the horizon, his motion in altitude increases in the same proportion as it diminished before he arrived at the meridian; that is, the corresponding motions, at the same altitudes, are always equal to each other, or, may be considered as being so. The circumstances in which observations on the sun's altitude give the horary angle with the greatest accuracy, are those in which his motion in altitude is the most rapid; when the sun is near the meridian, observations of his altitude are not proper for ascertaining the horary angle. According to theory, the greatest motion in the sun's altitude is at the instant of his passage over the prime vertical, or when the azimuth of the sun attains its greatest value. Observations of altitude intended for the calculation of the horary angle, should therefore be made as near this instant as possible. By means of the sun's declination, and the latitude of the place, there may be found, in Table XV, the altitude which the sun has on the prime vertical, or when his azimuth is the greatest; the observations should, therefore, be made when the sun has nearly attained the altitude given by this table.

This table must be used only when the sun and the observer are both in the same hemisphere, that is, when the declination of the sun, and the latitude of the place of observation, have the same denomination; for, in the contrary case, or where the declination of the sun and the latitude have different names, the sun can never arrive at the prime vertical. Then the moment when the sun's azimuth is the greatest, is that of his rising or setting; the observations should therefore be made when the sun is near the horizon. But altitudes less than 6° or 5' must not be used, as below this altitude, the refraction is very uncertain, and might occasion sensible errors in the time which results from the calculation.

72. The latitude by account is one of the data necessary in the calculation of the horary angle; and this may be affected with errors sufficiently great to have a sensible influence on the result. The case in which the influence of this error is the least possible, also takes place when the sun passes the prime vertical, or attains his maximum azimuth. The error in the horary angle arising from latitude, will therefore be diminished the most, when the rules are followed which have been given relative to the circumstances in which the motion in altitude is the greatest.

73. In general, the nearer the azimuth corresponding to the observed altitude approaches to 90°, the less error there will be in the result. The error which may be apprehended in the horary angle will, on the contrary, be greater, as the observations are made nearer the meridian, and as the corresponding azimuth is less. Hence, observations of altitude are not proper for ascertaining the time at the place where they are made, during some time before and after the sun's passage over the meridian. But the results will always have the precision which the safety of navigation requires, if the altitudes are observed before half past 10 in the morning,

and after half past one in the afternoon. Then the time at the place may be obtained to about 8" or 10" of time.

When the day is cloudy, it may happen, that the observations cannot be made under the most favourable circumstances; and that an altitude taken between half past 10 and noon, or else between noon and half past one, may still be proper for ascertaining the time with sufficient accuracy. Then the azimuth corresponding to the altitude must not be less than 20° ; but in this last case, there cannot be any certainty of ascertaining the time within less than $20''$ or $25''$. It will be easy to ascertain if the corresponding azimuth is 20° , by the assistance of the Tables XII and XIII. These Tables have, therefore, the advantage of showing the precision of which the observations of the horary angles are susceptible; as well as that of giving the latitudes obtained from two observations taken out of the meridian. The multiplier proper for the observed altitude may be obtained by the rules given in art. 40. and 41; and in Table XIV there will be found the azimuth that corresponds to it. From the magnitude of this azimuth, we may judge of the degree of confidence that should be placed in an observation made near the limits within which the result may be defective. If the calculated azimuth is below 20° , the observations should be entirely rejected: even in the case where it does not exceed 30° , and where the latitude could not be observed, it will be necessary to conduct ourselves with circumspection, with respect to the result of the observation.

74. Whenever it is required to obtain the time at any place by observations of the sun's altitude, these altitudes should be taken as near as possible to the most favourable circumstances. Several altitudes of the sun may be observed in succession, and the hour, minute, and second corresponding to each observation, noted down. It will then be easy to deduce from them the mean time corres-

ponding to the mean altitude; after which, the calculation is to be performed in the following manner.

75. First, find by means of the estimated time, at the place, and the longitude by account, the time at the first meridian at the moment of the observation. This time will serve to find, in the Nautical Almanac, the sun's declination, from which his distance from the elevated pole is derived, which should be employed in the calculation. Then the necessary corrections must be made in the observed altitude for obtaining the true altitude of the sun's centre.

Then write in the following order, the sun's true altitude, the latitude, and the polar distance: take the sum of these three quantities, and half this sum; next, from this half sum subtract the true altitude. Search in the Tables the arithmetical complement of the logarithm cosine of the latitude, and the arithmetical complement of the logarithm sine of the polar distance. Add these two arithmetical complements to the logarithm cosine of the half sum, and to the logarithm sine of the half sum minus the true altitude, and half the sum thus obtained will be the logarithm sine of half the horary angle. Find in the Tables the corresponding arc, which will be half the horary angle reckoned in degrees. This arc, multiplied by two, will therefore give the horary angle, which will be reduced into time by multiplying the product by four. The calculation will be abridged, if the arc found in the Tables be multiplied at once by eight; and by reckoning the seconds of the product for thirds, the minutes for seconds, and the degrees for minutes, the horary angle of the sun will be had in time. If the observation was made in the afternoon, this horary angle will be the hour at the place; if in the morning, its complement to 24 hours will be the time reckoned astronomically, and its complement to 12 hours will be the civil time: but, in this latter case, care must be taken to specify whether the observation was made in the morning or evening.

76. The time thus found is called true time, because it is immediately concluded from the actual position of the sun, with respect to the place of observation. It is the sun which, by virtue of the earth's diurnal motion, causes the successive return of day and night; his annual motion also regulates the periodic return of the seasons; it is from this body that the most remarkable divisions of time are derived, and those which regulate the transactions of civil life.

EXAMPLE.

The 14th of July 1792, at about 8^h 18' in the morning, being in 5° 55' 45" of South latitude, and 152° 3' of East longitude, the following observations were made, from which it is required to reduce the time at the place of observation. The elevation of the eye was 14 feet.

Time by the watch,			
8 ^h	8' 48"	}	Sum of the ☉'s altitudes - 172° 46' 20"
9	19		Mean altitude of the ☉ - 28 47 43
10	3 5		Depression - - - - - 3 40
			28° 44' 3"
11	7		Semi-diam. of the ☉ - + 15 47
			28° 59' 50"
12	8		Refract. — Parallax - - - 1 36
12	58		True altitude - - - 28° 58' 14"
Sum	- - 64' 23".5		
Mean time	- 8 ^h 10' 43".9		

The time at the first meridian, concluded from the estimated time at the place and the longitude by account, is 10^h 10'. The corresponding declination is 21° 39' 30" N.: as the sun is in the contrary hemisphere to the observer, 90° must be added to the declination, and the sun's distance from the elevated pole will be 111° 39' 30".

It will be useless, in calculating the horary angle, to take the proportional parts for the seconds; consequently, there may be added to, or subtracted from, the three given quan-

CHAP. IV. CALCULATION OF THE HORARY ANGLE.

tities of the calculation, the number of seconds necessary to cause the logarithms of the trigonometrical lines which correspond to them, to be found directly in the Tables. Attention must also be paid to make these small changes in the given quantities, so that the tens in their sum may be an even number, as follows:—

True alt. of the ☉. $28^{\circ} 58' 10''$			
Latitude	- -	$55^{\circ} 40'$	Comp. cos. - 0.0023285
Polar distance	-	$111^{\circ} 39' 30''$	Comp. sine. 0.0817968
Sum	- - -	$146^{\circ} 33' 20''$	
Sum	- - -	$73^{\circ} 16' 40''$	- - cos. 9.4589882
Sum — altitude	$44^{\circ} 18' 30''$	- -	sine 9.8441785
			Sum 19.3372920
			$\frac{1}{2}$ Sum sine 9.6686460
			Half the horary angle $27^{\circ} 47' 30''$
			Multiplying by - - - - - 8
When the observation was made in the after-noon.			Horary angle - - - - - $3^h 42' 20''$
When the observation was made in the morning, subtracting from 12^h			- - - - - $8^h 17' 40''$
Time by the watch			- - - - - $8\ 10\ 43.9$
The watch is too slow by			- - - - - $0^h\ 6'\ 56''.1$

The time by the watch is less than the time at the place by $6' 56''.1$; the watch is therefore slower than true time, by this quantity. If the time by the watch had been the greater, the difference of the two would have been what it was too fast, or before true time.

77. The time may also be obtained by observing the altitude of a star. The rules already given relative to the observations of the sun's altitude must be followed, both for taking advantage of the most favourable circumstances, and for calculating the horary angle. In this case, the horary angle

of the star will be the difference in right ascension at the instant of the observation, between the celestial meridian of the place and the circle of declination of the star. The right ascension of the star's circle of declination, or the right ascension of the star itself, being known, it will be easy to derive from it the right ascension of the meridian; which is done in the following manner:—When the altitude of the star has been observed to the west of the meridian, add its horary angle to its right ascension reduced into time; the sum will be the right ascension of the meridian. When the star is observed towards the east, subtract its horary angle from its right ascension, and the remainder will be the right ascension of the meridian. Then take the right ascension of the sun from that of the meridian, and there will be obtained the difference of the right ascensions of the sun and the meridian, or the hour at the place. Instead of the right ascension of the sun, the *Connaissance des Temps* contains the distance between the equinox and the sun*, which is the complement to 360° , or 24 hours; to obtain the time at the place, it will therefore be necessary to add this quantity to the right ascension of the meridian: if the sum exceeds 24 hours, the excess will be the time required. The distance from the equinox to the sun, should be calculated for the time at the first meridian, deduced from the estimated hour at the place and its longitude; when the time resulting from this calculation differs more than 5 minutes from the estimated time at the place, the distance from the equinox to the sun may be calculated again, and a second result will be obtained much more accurate than the former. It would be possible to arrive at the greatest degree of accuracy, by

* The sun's right ascension is taken immediately from the second page of the month in the *Nautical Almanac*, and must be added to the right ascension of the meridian, or subtracted from it, as directed in the rule. *Trans.*

making a third calculation of the distance from the equinox to the sun; but the second will always have a sufficient degree of precision.

EXAMPLE.

Being in $21^{\circ} 11'$ of south latitude, and $30^{\circ} 6'$ west longitude, on the 20th of May 1810, at 10^h and $\frac{1}{2}$, several altitudes of *Antares* were observed, the mean of which was $59^{\circ} 22' 30''$. The mean time by the watch was $9^h 48' 55''$; and the elevation of the eye $19\frac{1}{2}$ feet nearly. Required the true time of the observation.

The hour at the first meridian corresponding to the time at the place is $12^h 15'$. The declination of *Antares* is $25^{\circ} 59' 57''$ south; and its distance from the elevated pole $64^{\circ} 0' 3''$. Its right ascension $244^{\circ} 27'$, and in time, $16^h 17' 48''$. The distance from the equinox to the sun is $20^{\circ} 11' 56''.7$.

Apparent altitude of <i>Antares</i>	- - - - -	$59^{\circ} 22' 30''$
Elevation $19\frac{1}{2}$ feet. Depression	- - - - -	$4' 21''$
		<hr/>
		$59^{\circ} 18' 9''$
Refraction	- - - - -	$0' 34''$
		<hr/>
True altitude of <i>Antares</i>	- - - - -	$59^{\circ} 17' 35''$
True alt. of the star	$59^{\circ} 17' 40''$	
Latitude	- - - $21\ 11\ 0$	Com. cos. - 0.0303842
Polar distance	- $64\ 0\ 0$	Com. sin. - 0.0463298
Sum	- - - $144\ 28\ 40$	"
$\frac{1}{2}$ Sum	- - - $72\ 14\ 20$	cos. - 9.4843696
$\frac{1}{2}$ Sum — altitude	$12\ 56\ 40$	sin. - 9.3502600
		<hr/>
	Sum	- - - 18.9113536
	Half-sum. Sin.	9.4556768
	Half horary angle	$16^{\circ} 35' 30''$
	Multiplying by	- - - 8
		<hr/>
	In time	- - - $2^h 12' 44''$

		In time - - - - -	2 ^h 12' 44"
		Right ascen. of the star	16 17 48
The star to the east.	<i>Differ.</i>	} Right ascen. of the merid. }	14 ^h 5' 4"
The star to the west.	<i>Sun</i>		
		Dist. from the equator to ☉.	20 11 57
		Sun. Time at the place	10 ^h 17' 1"
		Time by the watch - -	9 43 55
		Watch behind true time	0 ^h 33' 6"

The time which results from the calculation, differs only 2' from the estimated time at the place; it is therefore not necessary to make a second calculation for the distance from the equinox to the sun.

Calculation of the Altitudes of the heavenly Bodies.

78. This problem is the reverse of the preceding one. In the former, the horary angle is found from an observation of the altitude; in this, the altitude is to be calculated by means of the horary angle: this requires a knowledge of the time at the place. When the altitude of the sun is to be calculated, the horary angle is easily found. It is equal to the true time, if the altitude is to take place after noon; and it is equal to the complement of the true time, to 24 or to 12 hours, when the altitude is to take place in the morning, or before noon. But when it is the altitude of the moon or a star that is required, the horary angle must be calculated in the following manner.

79. First, by means of the longitude, find the time at the first meridian corresponding to the time at the place, and take from the Nautical Almanac, the sun's right ascension. Add this right ascension to the time at the place, and subtract 24 hours from the sum, if necessary, and the result will be the right ascension of the meridian. The difference between the right ascension of the meridian and

the right ascension of the moon or a star, which has been calculated for the instant at which the altitude is required, will be the horary angle of the moon or the star at the time proposed. Then find, in the *Nautical Almanac*, or *Connaissance des Temps*, the declination of the moon or star at the same instant, from which its distance from the elevated pole is obtained. The horary angle, the polar distance, and the latitude, are the three necessary data for the calculation, which may then be performed in the following manner.

80. Write down, first, the horary angle, and take its half; below this half, write the distance of the heavenly body from the elevated pole; and immediately after it the latitude. Then subtract the latitude from the polar distance, and take the difference between the remainder and 90° ; write half this difference below. Take from the Tables, the logarithm cosine of half the horary angle; write below it half the logarithm sine of the polar distance, and half the logarithm cosine of the latitude; lastly, take the arithmetical complement of the logarithm cosine of the half difference from 90° . The sum of these four logarithms will be the logarithm sine of an auxiliary angle; write down the logarithm cosine of this auxiliary angle, and subtract from it the arithmetical complement of the logarithm cosine of the half difference from 90° . The remainder will be the logarithm cosine of the half-sum of 90° plus the altitude; double the arc which corresponds to this logarithm cosine, and, after having subtracted 9 from the tens and hundreds of the degrees, the remainder will be the true altitude required.

81. When it has been impossible to observe the altitudes of the heavenly bodies, the distance of which has been taken, they may be calculated by this method; they serve, as will shortly be shown, to correct this distance for effects of refraction and parallax. An error of a minute in the calcu-

lated altitude cannot have a sensible influence on the true distance; the seconds may therefore be neglected, in making the calculations, and the logarithms may be taken only to five places of decimals. The preceding rules are applied to the calculation of the altitude of a star, and to another of the same, in the following examples.

EXAMPLE I.

On the 19th of June 1793, being in south latitude $9^{\circ} 45' 50''$, and $148^{\circ} 43'$ of east longitude; when a watch indicated $3^h 41' 5''$, it was found by observations of the sun's altitudes, that the watch was too slow by $1^h 21' 34''$. It is required to find the altitude of *Antares*, when the time by the same watch was $6^h 8' 10''$. Between these two observations the vessel had advanced $1'$ towards the north, and $4'$ in longitude towards the east.

Time by the watch	-	-	-	-	$6^h \ 8' \ 10''$
Watch too slow (Add)	-	-	-	-	$1 \ 21 \ 34$
True time	-	-	-	-	$7^h \ 29' \ 45''$

The place of the second observation is $4'$ to the east of the first, or $16''$ of time (Add) } $+ 16$

True time at the place of the required altitude $7^h \ 30' \ 1''$

Estimated time at the first meridian - $21^h \ 35'$

True time at the place of the altitude - $7^h \ 30' \ 1''$

Dist. from the equinox to the \odot . (subtract) $18 \ 6 \ 54$

Right ascension of the meridian - $13^h \ 23' \ 6''$

In degrees - $200 \ 45 \ 38$

Right ascension of *Antares* - $244 \ 11 \ 40$

Horary angle - - - - - } $43^{\circ} \ 25' \ 2''$

Antares east of the meridian - - - - - }

Latitude of the place of the altitudes, S. $9 \ 44 \ 50$

Declination of *Antares* - - - - - $25 \ 57 \ 30$ S.

Distance from the elevated pole - - - $64 \ 3 \ 30$

Half the horary angle	-	21° 43'	cos.	-	9.96803
Polar distance	-	64 3	$\frac{1}{2}$ sine	-	4.97692
Latitude	-	9 45	$\frac{1}{2}$ cos.	-	4.99684
Polar distance—Latitude		54 18			
Difference from 90°	-	35 42			
Half difference from 90°	-	17 51	com. cos.	-	0.02143
Sine auxiliary angle	-				9.96322
Cos. auxiliary angle	-				9.59627
(Cos. auxiliary angle — com. cos. $\frac{1}{2}$ differ.) cos.					9.57484
$\frac{1}{2}$ (90° + altitude)	-				67° 56'
(Double — 90°) TRUE ALTITUDE of the star	-				45° 52'

EXAMPLE II.

The given quantities being the same as in the preceding example, required the altitude of the moon at the same instant.

Estimated time at the first meridian		21 ^h 35 ^m			
Right ascension of the meridian	-	200 46' 38"			
Right ascension of the moon	-	208 7			
Horary angle of the ☾, to the east		20' 22"			
Declination of the moon	-	25 0 S.			
Distance from the elevated pole	-	82 35			
Half the horary angle	-	3° 40'	Cos.	-	9.99911
Polar distance	-	82 35	$\frac{1}{2}$ Sin.	-	4.99817
Latitude	-	9 45	$\frac{1}{2}$ Cos.	-	4.99684
Polar dist. — Latitude	-	72° 50'			
Difference from 90°	-	17 10			
Half diff. from 90°	-	8 35	Com. cos.	-	0.00489
Sin. auxiliary angle	-				9.99901
Cos. auxiliary angle	-				8.82888
(Cos. auxiliary angle — com. cos. $\frac{1}{2}$ differ.) cos.					8.82399
$\frac{1}{2}$ (90° + altitude)	-				86° 10'
(Double — 90°) TRUE ALTITUDE of the moon	-				82° 20'

CHAPTER V.

On regulating Marine Chronometers, and employing them in the Determination of Longitude.

82. The difference of longitude of any two places being equal to the difference of time reckoned at the same instant at both places, if a well regulated watch be taken on board, which will preserve the time at the place from which the vessel sails, it will show the time at the same place at every subsequent instant. Observations of the sun's altitude will also make known the time at the several places of the vessel at these instants; hence it follows, that watches may be equally employed in finding the difference of longitude between the place of departure and each of those where the altitudes are observed, or even the absolute longitude, if it can be ascertained how much the watch is too fast or too slow, with respect to the time at the first meridian. This property of marine chronometers has given them the name of time-keepers. It is conceived to be impossible that a watch should preserve exactly the time at the place of departure; but watch-making has been carried to such perfection, that it may be supposed, without apprehending any considerable error, that the daily variation of a watch is the same quantity. Thus, when this error is known, the watch may be used for ascertaining longitude. The method of finding the quantity which a watch varies daily from the

time at the port sailed from, shall first be shown, and then the manner of calculating the longitude.

On regulating Marine Chronometers.

83. The method of comparing the time by a watch with true time, or that which has been immediately concluded from observations, has already been explained. It has been shown that parts of time having the same denomination, were measured by equal parts of the equator, which the circle of the sun's declination describes during the diurnal revolution of the earth. Thus, 24 hours always answers to 360° , and 1 hour to $15'$. The earth always occupies the same time in making one revolution, and its motion on its axis is uniform; whence, if the sun remained immoveable, or if his motion of right ascension were uniform, it is evident that equal parts of the equator would always be passed over by the circle of the sun's declination in equal times. But the changes in right ascension are subject to the inequalities of the sun's motion in his orbit, and may not be the same for equal intervals of time: from which it follows that the subdivisions of true time, having the same denominations, ought not to be equal to each other. These inequalities arise also from this, that the equal parts of the ecliptic, intercepted between two circles of declination, do not always differ by the same quantity, from the parts of the equator which are intercepted between the same circles: the arc of the ecliptic is greater than that of the equator, when the two circles of declination are near the equinoctial points; it is, on the contrary, smaller when the circles of declination are near the solstices. It results from the combination of these two causes, that at certain times of the year, two consecutive days differ from each other by a quantity sufficiently sensible: and as their increase or

decrease operates progressively, it follows that the hours of true time are not equal to each other: the same for minutes and seconds. At the end of December, the true days differ by half a minute; but for the greatest part of the year the differences are much less, and become nearly insensible for an interval of two or three hours: this is the reason they were not attended to in calculating the hourly angle of a heavenly body, and the time which ought to be given at noon by a watch that has been compared with true time in the morning or evening. But it is not the same when it is required to regulate a marine chronometer intended to give the longitude.

84. The mechanism of watches has been so conceived as to give to the wheels, and consequently to the hands, a motion as uniform as possible; these hands ought, therefore, to describe on the dial-plate equal angles in equal times. The comparison of the time given by a watch with true time, the corresponding intervals of which are unequal, is therefore not proper to give an idea of the regularity of its movements. Astronomers who refer the positions of all the heavenly bodies to those of the fixed stars, compare the motions of clocks and watches to a uniform motion taken immediately in nature, and for this purpose they make use of sidereal time. A sidereal day is the interval of time which elapses between the passage of a star over the meridian and its return to the same meridian. The stars being fixed, and the motion of the earth on its axis uniform, the circles of declination of the stars ought to describe, on the equator, equal arcs in equal times. The hours of sidereal time, as well as their subdivisions, are all equal to each other, and may therefore be used in ascertaining the regularity of the motions of a clock or a watch.

85. Mariners make most of their observations on the sun; and when they observe the other heavenly bodies, they refer

their positions to that of the sun. They are therefore obliged to compare the motions of marine chronometers with another uniform motion, which approaches nearer the real motion, by virtue of which, the circle of the sun's declination passes over the equator. This motion is purely artificial, and does not exist in nature, but has been obtained by a very ingenious hypothesis. It was supposed that a circle of declination, setting off at the same time as the sun from the point where he commences his motion, moved uniformly over the equator, and passed over its whole circumference in the same time as the sun described the ecliptic. This imaginary circle of declination ought to advance on the equator each day, in proceeding from west to east, through a space equal to $59' 8''$; but the quantity which the sun's circle of declination really advances is also known; the position of the imaginary circle of declination, with respect to the real one, is therefore known at any instant: likewise, the times between the passage of this circle over any meridian, and its return to the same meridian, will always be equal to each other; and the equal parts of the equator that are described in consequence of the diurnal motion, always correspond to the equal intervals of time. The time which is derived from the position which the supposed circle of declination ought to have on the equator, is called mean time, to distinguish it from true time, which is immediately derived from the real position of the sun; mean time has the advantage over true time, as it is susceptible of being used for verifying the movements of marine chronometers.

86. The interval reckoned in mean time, is equal to the arc of the equator comprised between the meridian of the place and the circle of declination of mean time; this arc, like that of true time, ought to be reckoned from east to west. The difference between true and mean time, is equal to the angle formed by the real circle of the sun's declination

and that of mean time, or that measured by the arc of the equator comprised between these two circles; that is, equal to the difference between the sun's real right ascension and his mean right ascension.

This difference is what is called the equation of time. The motion of the circle of declination of mean time is sometimes quicker, and sometimes slower, than that of the sun's declination; it will therefore be sometimes before, and at others after this last. When it is before it, the equation of time must be added to the true time which is obtained directly from observation; when the circle of declination of mean time is after that of the sun, the equation of time is to be subtracted from the true time, to obtain the corresponding mean time.

The equation of time is generally given in Ephemerides for every day at noon; but in the *Connaissance des Temps*, instead of the equation of time, there is inserted the time which a clock or watch, regulated according to mean time, ought to give at the instant of the sun's passage over the meridian*. This quantity is denoted by the title of mean time at true noon; and is given for every day at the instant of true noon at the observatory at Paris. It will be easy to calculate it for any other instant, by the rules already given in the first chapter. When the mean is before the true time, the number that is found in the *Connaissance des Temps* is equal to the equation of time; and it is to be added to the hour obtained from the calculation of the horary angle, when mean time is required. But when the mean time is slower than the true, the equation of time is subtractive; but in this case the mean time at true noon is its

* In the *Nautical Almanac*, it is the equation of time that is given in the second page of every month, for every day at noon; and which is to be added to the time obtained from the calculation, or subtracted from it, as there directed, in order to obtain the mean time required. *Trans.*

complement to 12 hours; and, to obtain mean time, it will be equally necessary to add the quantity which is found in the *Connaissance des Temps*, to the hour that results from the calculation of the horary angle; and then 12 hours must be subtracted from their sum. From what has been said, it will be easy to understand the following rules.

87. When the true time corresponding to any instant is known, and the mean time answering to the same instant is required; the proposed time is to be added to the mean time at true noon.

If the mean time be known, the mean time at true noon must be subtracted from it, and the remainder will be the true time corresponding to the same instant.

88. Altitudes of the sun intended for the regulation of marine chronometers, should be taken as near as possible to the instant when he passes the prime vertical: that is, when he attains the altitude given in Table XV. In the case when the sun is not in the same hemisphere as the observer, the observations of altitude may commence when he is at least 7° above the horizon. Then the errors of the estimated latitude, and those of the altitude, will have the least possible influence upon the calculated time. Six altitudes may be taken in succession; and the hour, minute, and second, answering to each observation written down; and the apparent mean altitude corresponding to the mean time of the observations taken. The calculation of the horary angle should be performed according to the rules given in art. 75; and it will give the true time corresponding to the mean time by the watch. The mean time at true noon, taken from the *Connaissance des Temps* for the nearest period at the first meridian, must be added to the true time, and the corresponding mean time will be obtained; or if the *Nautical Almanac* be used, the equation of time must be added or subtracted as it is preceded by the sign + or - : from

which it will be easy to deduce the gain or loss of the watch with respect to mean time, at the instants in which the observations were made.

Suppose that several days after the first observations had been taken, they were repeated; the mean time corresponding to the mean of the second set of observations must be calculated in the same manner; and the gain or loss of the watch may be deduced, with respect to the mean time of this second set of observations.

If the gain or loss of the watch, found from the second series of observations, is the same as that found from the first, it will be a proof that the watch has exactly kept mean time during the interval. But if the gain from the second observations be greater than that from the first, the motion of the watch has been quicker than that of mean time; and the difference of the two quantities gained will be the gain of the watch during the interval. If the gain from the second series of observations had been less than that from the first, the watch would have lost in the interval. A quantity equal to the difference of the two gains, as determined from the calculations of the horary angles. In the case in which the watch may be found to be slower than mean time, an increase in the loss as found from the first, would indicate that the watch has lost between the two epochs at which the observations were made: a diminution in the loss would show, on the contrary, that the watch had gained with respect to mean time. When the gain or loss of the watch in the interval between the observations is known, the gain or loss in 24 hours may be found in the following manner. This last quantity is what is called the diurnal variation of the watch, or more simply its rate. This proportion will give the rate or diurnal variation, viz. as the interval between the observations is to 24 hours, so is the gain or loss in that interval to the diurnal variation;

which will be obtained by multiplying the second and third terms together, and dividing the product by the first term. The following is an example.

It is essential to remark, that in the calculation of the horary angle, the seconds of a degree must be used, and the proportional parts taken, to obtain the logarithms of the trigonometrical lines which enter into the calculation.

EXAMPLE.

On the 29th of March 1793, in the harbour of Tongataboo, in $21^{\circ} 7' 35''$ South latitude, $177^{\circ} 33' 14''$ West longitude, the altitudes of the sun's lower limb were taken in the morning. The mean time was $7^h 34' 28''.82$, and the mean altitude of the sun's centre $19^{\circ} 23' 13''.4$. The corresponding true time is to be calculated by art. 75, and the absolute gain or loss of the watch, with respect to mean time, deduced from it in the following manner:

True time of the observations	-	-	-	$7^h 29' 0''.89$
Mean time at true noon	-	-	-	$0 \quad 4 \quad 54.23$
Mean time of the observations	-	-	-	$7^h 33' 55''.12$
Time by the watch	-	-	-	$7 \quad 34 \quad 28.82$
The 29th of March at $7^h \frac{1}{2}$, the watch was	} $0^h \quad 0' \quad 33''.7$			
before mean time				

In the morning of April 7th, being at the same place, a second series of observations were taken. The mean time by the watch was $7^h 57' 3''.23$, and the apparent altitude of the sun's centre was $23^{\circ} 26' 20''$. The operation is to be performed in the same manner as for the former observation.

True time of the observations	-	-	-	$7^h 53' 31''.32$
Mean time at true noon	-	-	-	$0 \quad 2 \quad 10.98$
Mean time of the observations	-	-	-	$7^h 55' 42''.30$
Hour by the watch	-	-	-	$7 \quad 57 \quad 3.23$
The 7th of April at $7^h 53'$, or the 6th at	} $0^h \quad 1' \quad 20''.93$			
$19^h 53'$, the watch was too fast with re-				
spect to mean time	-	-	-	-

The 4th of April, at 7 ^h 53',	watch too fast by	0 ^h 1' 20 ^s 33 ^{ms}
The 29th of March, at 7 ^h 53',	too fast by	- 0 0 33 ^{ms}
In nine days the watch had gained	-	0' 47" 23 ^{ms}
In 24 hours	-	+ 5 24

89. When the vessel is at anchor, and the horizon is not bounded by land, the altitudes intended for calculating the diurnal variation of the watch may be observed with a sextant or a reflecting circle. The observations should be made near the sun's passage over the prime vertical, or near the instant of his greatest azimuth; and, the latitude of the anchorage may be obtained with a sufficient degree of accuracy. But notwithstanding all these precautions, there is still reason to apprehend an error of 3 or 4 seconds in the time; and even sometimes an error a little greater. The observations should not therefore be limited to a single series of six altitudes, as is generally done at sea. It will be better to observe three or four series; and then it will be probable that the mean gain or loss of the watch derived from all these, will have a precision of 2 or 3'. The gain or loss of the same watch in the interval of the observations may therefore be affected with an error double of these quantities, that is, of 4" or 6", this error will take place when the errors of the first and second days of observation have then greatest values, and act in a contrary sense. In this case, the interval between the observations should exceed 6 days, that the probable error of the diurnal variation may be less than a second. Such an error is considerable; the following means of attenuating it should not be neglected.

90. It has been remarked, that the same observer measures all the altitudes either a little too great or a little too small, the errors arising from this defect of sight, would therefore take place in the same sense in all the altitudes, but those errors, which will influence the time calculated from observations taken in the morning in one direction,

will have an influence in a contrary direction on the time concluded from those taken in the evening. The greatest errors will consequently take place in the gain or loss derived from comparing the result of an observation taken in the morning, with the result of an observation taken in the evening; hence it is necessary to compare together the results from observations in the morning only, and the results from observations taken in the evening with each other. The probable error in the gain or loss calculated in this manner will not be more than about 3"; and at the end of 6 days, we may conclude that the diurnal variation has been obtained to nearly half a second. A greater degree of precision may even be attained, by taking a mean between the diurnal variations which results from observations taken in the morning, and that which results from those taken in the evening. The contrary will take place with respect to the absolute gain or loss of the watch, the last day of the observations, which, as well as the diurnal variation, should be used in calculating the longitude: the mean between the result from the observations of the morning, and that from those of the evening, must be taken. Then the errors which are of such a nature as to act in opposite ways on these two results, will only influence the gain or loss of the watch by half their difference.

91. When the horizon of the sea cannot be seen, the observations must be made on land. The best means undoubtedly is, to take the altitudes with the repeating circle furnished with a level, the description and use of which has been given by M. Biot, at page 273 of the first volume of his *Treatise on Physical Astronomy*, in such a manner as to leave nothing to be desired. But the object of this work is to show the use that may be made of reflecting instruments; and we shall therefore describe a new instrument proper for observing the altitudes of the sun, when the

horizon of the sea is not visible. An artificial horizon is then to be used. The principal piece in this instrument is a round plane glass, set in a brass frame, sustained by three screw feet, the use of which is to place the glass in a horizontal position. The under surface of this glass is unpolished and blacked, so that the image of the sun can only be reflected by the upper surface, which should be carefully polished, and an exact plane: by this means, the errors that might arise from a defect of parallelism in the two surfaces are avoided. The artificial horizon, such as here described, should be placed on a very firm table or on the ground; then an air level is to be laid on the upper surface of the glass, and the feet screws turned to level the instrument. When the bubble rests in the middle of the tube, in all positions of the level, the surface of the glass is in a horizontal plane.

Let it now be supposed, that the direct rays of the sun fall upon the glass; they will be reflected so that the angle of incidence will be equal to the angle of reflection; and, since the surface of the glass is in a horizontal plane, each of these angles will be equal to the sun's altitude. The image that arrives at the eye by the reflected rays will appear to be depressed below the horizontal plane, by a quantity equal to the elevation of the direct image. Thus the angle formed at the eye of the observer, by the rays which proceed, on the one part from the reflected image in the glass, and from the direct image on the other, will be double his altitude. This angle may be measured with a reflecting instrument, by taking the distance from the direct to the reflected image; that is, by making the image reflected by the great mirror of the instrument, and that reflected by the artificial horizon, coincide in the field of the telescope. If the nearest edges of these two images be brought into contact, they will give double the altitude of the sun's lower

limb; and if their most distant edges be brought into contact, double the altitude of the upper limb will be obtained. The nearest and farthest edges should therefore be observed alternately, and then the apparent altitude of the sun's centre will be directly obtained, by dividing the sum of an even number of altitudes by double the number of the observations. The altitudes of the sun near the meridian, and the meridian altitude, may be observed with an artificial horizon; but as the angles measured with this instrument are double of the altitudes, its use is limited. The artificial horizon will not answer when the altitude of the sun exceeds 63° , for reflecting instruments cannot obtain the measure of angles more than 126 degrees.

On finding Longitude by Marine Chronometers.

92. Marine chronometers, as already remarked, preserve such a regularity in their movements, that these may be considered as uniform during a certain lapse of time, without apprehending any material error. It amounts to the same to suppose that the diurnal variation at the place of departure remains always the same during the voyage, which immediately succeeds the epoch at which the observations had been made. When it is wished that the rate of a chronometer or watch should vary as little as possible from this supposition, the greatest care should be taken that it do not experience any sudden jerks, or even any strange motion that might alter the duration of the oscillations of the balance by which its movements are regulated. The first rule therefore which ought to be observed, is never to carry it about one. It has been observed, that a chronometer which had been regulated, while suspended vertically, changed its rate when it was placed in a horizontal position; hence the chronometer should be kept in the same position as it was

when the diurnal variation was observed. The common practice is to place it in a box or case, which should always remain in a horizontal position. It would be advantageous that it should be in a place where the rays of the sun never penetrate, in order to avoid frequent and sudden changes of temperature. It would also be best to place it near the centre of motion of the vessel, that its motion might have the least possible influence on the movements of the balance. In taking altitudes or distances, a good seconds watch may be used, which has been compared with the chronometer before the observations are made; and the comparison will give the time by that watch which ought to correspond to the mean time of the observations. A second comparison should also be made after the observations are finished, to ascertain if the rate of the seconds watch has been altered during their continuance. Whenever all these precautions have been attended to, it may be concluded that the movements of the watches have been as regular as possible, and expected that the longitude will be found within the limits of that precision which the safety of navigation requires.

93. When the absolute gain or loss of a watch with regard to mean time at any place is known, and its diurnal variation, it is very easy to deduce its absolute gain or loss in reference to the same species of time at the same place, for any period subsequent to that at which the watch was regulated. Suppose that a series of observations on the sun's altitude had been made at sea, for obtaining the longitude by a marine chronometer; the absolute gain or loss of the chronometer, with respect to mean time at the place where it was regulated, may be calculated by the following rules.

If the chronometer was before mean time, and it is known to gain a certain number of seconds every day; add to the absolute gain the product of this number of seconds

by the number of days and parts of a day between the two epochs of the observations: if, on the contrary, the diurnal variation is a loss, this product must be subtracted from the absolute gain observed at the place where the chronometer was regulated, and the remainder will be the absolute gain corresponding to the proposed epoch.

In the case in which the chronometer is too slow, there must be added to its loss, the product of its diurnal loss multiplied by the days and fraction of a day elapsed between the two epochs of the observations: on the contrary, the product of the diurnal gain by the number of days and parts must be subtracted from the absolute loss; and we shall have the absolute loss of the chronometer, with respect to mean time, at the place where it was regulated, for the required time.

The absolute gain must then be subtracted from the mean time corresponding to the mean altitude; or else the absolute loss added to the same time; and there will be obtained the mean time that should be reckoned at the place where the chronometer was regulated, at the instant of observing the horary angle. Add to or subtract (art. 87) from this, the equation of time, and the sum or remainder will be the true time corresponding to the same moment. The calculation of the horary angle will give the true time at the vessel; the difference of these two times will be equal to the difference of longitude between the place of the vessel and that where the chronometer was regulated; which may be reduced into degrees by the known rules. The vessel will be to the east of the place, if the time resulting from the calculation of the horary angle is the greater; and on the west of it, when this time is the less. Then add the difference of longitude to that of the place where the chronometer was regulated, or subtract it from it, according as the vessel is on the east or west of that meridian; and the longitude of the vessel will be obtained, reckoned from the first meridian. When the

chronometer has been regulated for the first meridian, the difference between the true time obtained by the chronometer, and that resulting from the calculation of the horary angle, gives the longitude of the vessel directly.

EXAMPLE.

On the 15th of April 1793, being in South latitude $19^{\circ} 51' 20''$, and $167^{\circ} 40'$ East longitude, by account; that is, 8 days after the last observations made at Tongataboo for regulating the marine chronometer (see the ex. art. 88); the altitudes of the sun's lower limb was observed, at about $2^h 46'$ after noon, in order to obtain the longitude by the chronometer. The elevation of the eye above the surface of the sea was $20\frac{1}{2}$ feet. The longitude of the harbour of Tongataboo is $177^{\circ} 33' 14''$ West.

It would be useless to enter into the detail of the calculation of this example; all the given quantities that should be employed will be found in the following specimen, in the order the most convenient and proper for facilitating the operations; this will be sufficient to show the manner in which all other calculations of the same kind should be performed.

April 15th, 1793

Latitude by account, S	-	-	-	19° 51' 20
Longitude by account, E.	-	-	-	167 40 0
Sum of the observed alts. of the ☉.	-	-	-	233 56 40
Mean altitude of the ☉.	-	-	-	38 59 26
Elevation of the eye $20\frac{1}{2}$ feet.	Depression	-	-	— 4 24
Remainder	-	-	-	38 55 2
Semi-diameter of the ☉.	-	-	-	+ 15 57
				<hr/> 39 10 59
Refraction — parallax	-	-	-	— 1 6
True altitude of the ☉'s centre				<hr/> 39 9 53

Watch before mean time at Tongataboo,	}	0 ^h 1' 20".93
the 7th of April, at 7 ^h 53'. (See the Ex. to art. 88)		
Daily advance + 5 ^m 24; in 8 ^d 3 days	-	+ 43 49
Watch before mean time, at Tongataboo,	}	0 2 4.42
15th April		
Time at the first meridian, the 14th April,		14 38 0
Declination of the sun, N.		9° 53' 15"
Distance from the elevated pole		99 53 15

Longitude of the island of Panghaimodoo, in the harbour of Tongataboo, the place where the chronometer was regulated,

		177° 33' 14"
In time	-	11 ^h 50' 13"
Times by the chronometer	{	0 23 19
		0 23 57
		0 24 42
		0 25 21
		0 26 58
		0 27 42
		151 59
Mean time	-	0 25 19 83
Time by the chronometer, at the mo-	}	3 38 8 17
ment of comparison. Add		
Sum	-	4 3 28
Time by the seconds watch Subtract	-	0 15 0
Time by the marine chronometer	-	3 48 28
Before mean time at Tongataboo Subtract	-	0 2 4.42
Mean time at Tongataboo	-	3 46 23 58
Mean time at true noon. Subtract	-	11 59 56 47
True time at Tongataboo	-	3 46 27 11

True alt. of the ☉	39° 9' 59"		
Latitude - - -	19° 51' 20"	Com. cos. -	0.0206172
Polar distance - -	99° 53' 10"	Com. sin. - -	0.0064972
Sum - - -	158° 54' 20"		
Half sum - - -	79° 27' 10"	- - cos. -	9.2625599
Half sum—alt. of ☉	40° 17' 20"	- - sin. -	9.8106638
Sum - - -			19.1063381
Half sum - - -		- - sin. -	9.5531690
Half horary angle	22° 56' 10"		
Multiplying by - -	- - -	- - -	8
In the morning, take the } True time at			2 ^h 47' 29" 20'''
comp. to 12 hours - } the vessel			
True time at Tongataboo			3 46 27 7
When the time at the } The vessel is now			
vessel is the greater, } to the West of			0 58 57 47
it is to the East - } Tongataboo			
In degrees -			12 44 27
Longitude of Tongataboo W.			177 23 14
Subtract from 360°			
the longitude ne- } Longitude of the			192 17 41
ver exceeds 180 } vessel West - }			
Longitude of the vessel, East			167 42 19

91 It ought to be remarked, that in order to obtain the absolute gain or loss of the chronometer for every day, with respect to mean time, at the place where it was regulated, the diurnal variation of the chronometer must be successively either added to or subtracted from, the gain or loss found from the observations. The quantity which should be added or subtracted daily is therefore the sum of all the diurnal variations of the preceding days. From the moment that the movement of the chronometer experiences a change, the diurnal variation employed is affected with an error, which has a daily influence, equal to its whole value, on the longi-

tude derived from the time kept by the chronometer. At the end of a certain time, the errors of longitude are equal to the sum of all the errors in longitude observed during the preceding days. It follows from this, that marine chronometers can only give with precision the differences of longitude of the places where the observations have been made at epochs very near to each other. For this reason, they are employed with the greatest success in the construction of marine charts; in which case, they show the relative positions in longitude of all the places inserted in these charts. But when they are used for the common purposes of navigation; that is, for calculating the distance of the port to which the vessel is sailing, it would be imprudent to rely wholly upon them; and it is necessary to compare the longitudes obtained by the chronometer with those deduced from observations on the distances of the moon from the sun and the stars: these last ought always to be within the limits of a known precision, and are very proper for ascertaining whether chronometers preserve the same regularity in their movements, and whether the longitudes obtained by them can be depended upon, without exposing the safety of the vessel.

95. The method of obtaining longitude by marine chronometers, is perhaps, that which has contributed the most to the progress of hydrography and geography. To be convinced of this, it is only necessary to glance at the astronomical observations published in a series of relations of long voyages, both French and English, that have been made since the first voyage of Captain Cook: it will then be seen what advantage has been derived from them. But it cannot be concealed that those chronometers, of such generally acknowledged utility, may suddenly experience derangements, and without our being able to assign the cause, the consequences of which may prove fatal, if the other means which nautical astronomy furnishes for determining

the position of the vessel, be neglected. It is therefore impossible, and it would even be dangerous to endeavour to estimate the errors with which the longitudes from chronometers may be affected at the end of a certain time. The regularity of most of the chronometers now in use only serves to confirm their general utility: there ought, however, to be no hesitation in saying, that we cannot compare a watch with itself. Though all probabilities are in favour of chronometers that have been proved, we dare not yet assert that a chronometer, the rate of which has always been regular, will preserve that regularity of motion, which the greater or less humidity of the atmosphere, or different degrees of extreme temperature, may cause it to lose. And, therefore, the necessity of verifying the longitudes obtained by means of chronometers, by observations of the distances of the moon from the sun and the stars, cannot be too much insisted upon.

Marine chronometers whose rates have been best ascertained, have generally given the longitude to about half a degree, at the end of a voyage of three months. The chronometer, No. 14, of M. *Louis Berthoud*, which was used during the voyage of Rear-Admiral *D'Entrecasteaux*, has always given the longitude of the vessel to about a quarter of a degree, even at the termination of a voyage of more than three months. But this astonishing precision, which ought in reality to be attributed in a great measure to the regularity of its movements, may also have arisen from some of the errors in longitude having been of such a nature as to compensate others. In general, good marine chronometers, like those that have been mentioned, preserve a very regular rate during a period of about two years, after being taken from the hands of the watch-maker; but at the end of that time the oil begins to thicken, and wants renewing; then the movement changes successively by a small quantity, and generally tends towards acceleration.

Means of correcting the Longitudes obtained by Marine Chronometers.

96. When marine chronometers have been used for directing the course of a vessel and bringing it to a coast, the observations that may be made during the stay of the ship at that place, cannot be of any other utility than that of ascertaining the diurnal variation of the chronometer, which should be employed in finding the longitude during the following passage. But if the geographical position of some of the places at which she has touched has been determined, then the diurnal variation observed during a succeeding stay in port, may serve, in certain cases, to correct the longitudes of these places, and greatly to increase their accuracy. These corrections become altogether indispensable when the diurnal variation has changed considerably in the interval between the observations that have been made for regulating the chronometer. The method of calculating these corrections shall now be explained.

97. Suppose it were known from astronomical observations, that the diurnal variation of a marine chronometer was not the same at any place as it was at the port from which the ship sailed. Calculate, first, the difference of longitude which there ought to be between the port of departure and that arrived at, with the diurnal variation observed immediately before the commencement of the voyage; then take half the sum of the two diurnal variations, and calculate the same difference of longitude with this mean variation. The result of the second calculation will be the corrected difference of longitude; and the quantity which it is greater or less than the former will be the correction that ought to be applied to the first difference of longi-

tude: this difference should be used in finding all the corrections of the other longitudes observed during the same voyage. It should be observed that, if this correction place the port arrived at to the east or west of the positions assigned it in the calculation made with the diurnal variation of the port of departure, all the other corrections ought to be employed in the same sense.

Search, in Table XI, opposite the number which expresses that of the days elapsed since the chronometer was first regulated, for another number, entitled, *Multiple of the Second Difference*; then, by means of logarithms, divide the correction of the longitude at the place arrived at by this number, and it will give the second difference of the corrections of all the longitudes observed during the voyage. The correction of other longitudes will be found by multiplying this second difference by the multiple corresponding to the number of days elapsed from the time the chronometer was regulated, to the time when the longitude, for which the correction is to be calculated, was observed. These rules shall be illustrated by an example.

EXAMPLE.

It has been found in the Ex. art. 88, that the diurnal gain of the chronometer, No. 14, at Tongataboo, was $+5^{\circ}21'$; the 6th of April 1793, at $19^{\text{h}}43^{\text{m}}31^{\text{s}}.44$, the last day of the observations, the chronometer was before mean time at Tongataboo, $0^{\text{h}}1^{\text{m}}20^{\text{s}}.93$. Having sailed from the last place to the harbour of Ballada, and made a fresh series of observations for ascertaining the diurnal variation of the same chronometer; it was found $+8^{\circ}56'$. The 22d of April, the first day of the observations at Ballada, the chronometer was before mean time at this port $1^{\text{h}}24^{\text{m}}23^{\text{s}}.71$.

Diurnal variation found at Tongataboo - 13° 34'
 Diurnal variation of Ballada - 8° 56'

Sum - 13° 30'

Half sum. Mean diurnal variation + 6° 9'

Difference in longitude between the harbour of Tongataboo and that of Ballada, by the first diurnal variation; + 5° 24'

Difference in longitude by the mean variation 20 17 55

The difference of longitude ought to be diminished, and the harbour of Ballada to be more to the east by 40 6 39

Required the correction of the longitude observed on the 17th of April, at 7° 34'.

Correction of the longitude of Ballada, after 16 days, 6° 39', or 399° } log. 2.60097

Multiple from Table XI, corresponding to 16 days - - 136 } Comp. log. 7.86646

Constant log. 0.46743

From the 6th of April to the 17th, 11 days } Multiple 66 log. 1.81954

Sum - 2.28697

Correction of longitude on the 17th of April - 3° 14'

The correction of the longitude on the 17th, ought to cause the situation of the place of observation to be more to the east, because that Ballada should also be to the east of the position calculated from the diurnal variation found at Tongataboo.

The correction of longitude may be calculated for other days of the same voyage, by adding to the constant logarithm, the logarithm of the multiple from Table XI, which answers to the number of days elapsed from the 6th of

April to the time when the longitude to be corrected was observed.

98. This correction of longitudes observed at the end of a long voyage is indispensable, during which the diurnal variation has experienced changes. The corrections of the longitudes near the commencement of the voyage will always be very small, and consequently less necessary; but those observed at the middle of a long voyage must be very uncertain, and the positions fixed by them but little susceptible of correction, except from the results obtained from distances. Suppose that after a voyage of three months, it was ascertained that the diurnal variation of the chronometer had changed several seconds; then the corrected longitudes of the first and last month, may be considered as approaching near the true longitudes, but those of the second month must always be regarded as uncertain.

CHAPTER VI.

On finding the Longitude by the Distances of the Moon from the Sun and the Stars.

99. The method of the distances of the moon from the sun and the stars is generally allowed to be the best of all those that can be employed for finding the longitude at sea. It has already been said, that it ought to be used for verifying the longitudes obtained by the use of marine chronometers, and that there is not any other means of establishing the regularity of their movements: it may therefore be regarded as that which has given us the solution of the problem of longitudes, with which all the learned astronomers of Europe were so long occupied. The accuracy of the results obtained by the method of distances, depends upon the precision with which the position that the moon ought to occupy in the heavens at any instant can be ascertained. The slow progress which this method at first made should be attributed to the complicated nature of the theory of the lunar motions, and the difficulties which astronomers had always to encounter when they wished to calculate her inequalities. *Johann Mayer*, by the assistance of this theory and observations, constructed tables which have served to predict the moon's place with a degree of accuracy sufficient for the safety of navigation. Since their publication, the distances of the moon from the sun and some of the princi-

tables have been inserted in all the Ephemerides; and navigators, having been made acquainted with the utility of observations of these distances, began to practise them. But, notwithstanding the great care and pains that were taken to perfect these tables, their precision still left something to be desired. In short, *M. Laplace*, in submitting the lunar motions to the calculations of analysis, discovered irregularities in them which, till then, had escaped all investigation, and obtained the means of giving to the method of distances the greatest degree of precision of which it is susceptible. With the assistance of *Delambre's* solar tables, and the tables of the moon calculated by *M. Bérgh*, from the theory of *Laplace*, both of which have been published by the Bureau des Longitudes, it is possible to predict the distances, and to obtain the longitudes, with a degree of precision which we should not have dared to flatter ourselves with being able to attain, when this method was first brought into practice. The perfection which artists have given to sextants, and the invention of the reflecting circle, have also added great advantages; in the actual state of things, navigators can no longer dispense with employing observations which may make known their position on the globe within some leagues, and afford them the power of obtaining from marine chronometers whatever assistance they are capable of affording.

100. The object of employing this method is to ascertain the true distance of the moon and the sun or a star, at any given instant; for the purpose of deducing from it the time which, at that instant, is reckoned at the first meridian; the time at the place which corresponds to the same instant is obtained from the altitude of the sun; these times being thus determined, their difference reduced into degrees, is the longitude required.

101. It has been shown that the altitudes of the heavenly

bodies appear greater than they ought to be from the effects of celestial refraction; the altitudes of the sun and moon appear less on account of their parallax. From the union of these two causes, it follows, that the observed distances are not equal to the true ones; they must therefore be corrected for the effects of refraction and parallax, when it is wished to obtain the true distance, from which the time at the first meridian may be directly concluded. It has been stated, art. 29, that the quantity by which the apparent altitudes of the heavenly bodies are too great from the effects of refraction, and in art. 33, that the quantity by which they appear too little on account of parallax, depend upon the apparent altitudes of these bodies; thus, to know the absolute values of these quantities, the altitudes of the two bodies must be measured at the same moment as their distance is observed, or else the method of obtaining these altitudes from calculation must be found. It is this which shall first be explained. We shall then treat of calculating the true distance; but the object of this treatise being to perform all the calculations of nautical astronomy, with the sole assistance of the Nautical Almanac, or the *Connaissance des Temps*, and a table of logarithms to seven places, we shall content ourselves with giving the method which is generally known by the name of *Borda's*: it is the shortest that can be employed, when tables of common logarithms only are used.

On the Methods of obtaining the Altitudes of the heavenly Bodies, the Distances of which have been observed.

102. When neither a marine chronometer nor a seconds watch is employed, the observation of distances requires three observers: while one of them measures the distance, the other two should take the altitudes; by this means,

the distance and the two corresponding altitudes are obtained by three simultaneous observations. But the distance is that which it is of the greatest importance to obtain with precision, because the errors by which it may be affected will have a greater influence on the result than the errors in the altitudes; each of the observers who takes the altitudes must therefore bring the body he is observing to the horizon, and take care to follow its movements with the repelling screw of the instrument, so that one of its edges may always be in contact with that circle. At the instant that he who observes the distance has brought the limb of the sun or a star to coincide with the limb of the moon, he informs his two co-operators, and they reckon on their instruments the two simultaneous altitudes. The two altitudes and the distance are written down separately when this last is taken with a sextant. Four observations must be made in this manner, but whenever it can be done, six should be taken. When the distance is observed with a reflecting circle, the arc passed over by the index is read off only at the end of the last observation, and it will give directly the sum of the observed distances. The sum of the altitudes of each of the bodies and that of the distances being divided by the number of observations, will give the mean altitudes and the mean corresponding distance.

EXAMPLE.

The 16th of June 1793, at $1\frac{1}{2}$ hour after noon, being in South latitude $10^{\circ} 16' 40''$, and East longitude 149 by account, six distances of the nearest limbs of the sun and moon were observed, and at the same instants, six altitudes of the lower limb of the sun, and six of the upper limb of the moon, were taken.

Altitudes of the ☉.		Altitudes of the ☾.	
48° 49'		26° 56'	
48 28		27 27	
48 18		27 51	
48 6		28 8	
47 57		28 22	
47 47		28 37	
<hr/>		<hr/>	
Sum - - -	289° 25'	Sum - - -	167° 21'
Sixth - - -	48 14 10"	Obs. alt. ☾	27° 53' 30"
Rect. of the inst.	+ 2 0		
Obs. altitude ☉.	48° 16' 10"		
Sum of the distances ☉ ☾ - - -		500° 40' 40"	
Observed distance ☉ ☾ - - -		83° 26' 46"	

103. The difficulty of exactly following the motion of the heavenly bodies with the repelling screw of the instrument, renders the altitudes taken in this manner less susceptible of precision, than in those observations where the observer employs the altitude of a celestial object only when he is certain of having made a good observation. The accuracy of the altitudes cannot be answered for at least within 2', and sometimes the errors amount to 3'. These errors can never have a great influence upon the true distance; but as the time at the place of observation must be calculated with the altitude of the sun, they may have a sensible effect upon the longitude. This is the reason that the sun's altitude should always be taken by an observer well experienced in this kind of observations, and with a well rectified instrument.

104. When a marine chronometer; or simply a seconds watch is possessed, the following method will always be preferable. Take an account of the hour, minute and second, at which each observation of the distance is made; then a mean distance corresponding to the mean time may be obtained. A few instants before these observations are

to be made, take one or more altitudes of the heavenly bodies of which the distance is to be observed, and also an account of the time answering to each of these altitudes. Immediately after observing the distances, take the altitudes of the same two bodies again; the difference of the altitudes observed before and after the distance will give the movement in altitude of each body in the interval of the observations, which is equal to the difference of the times corresponding to these altitudes. Then take the difference between the time of the first observation of the altitude and the mean time corresponding to the mean distance, and it will give a second interval; next calculate, by proportion, the movement in altitude which corresponds to it. Add this last to the first observed altitude when the altitude is increasing, but subtract it when it is decreasing, and the altitude corresponding to the mean distance will be obtained. These rules shall be illustrated by an example.

EXAMPLE.

On the 17th of June 1793, at $4^h 32'$ in the evening, being in South latitude $9^\circ 57'$, and $148^\circ 50'$ of East longitude, the following observations of the distance between the sun and the moon were taken, and of the altitudes of these two bodies, with a seconds watch, the elevation of the eye being $20\frac{1}{2}$ feet.

Time of the Distances.

$1^h 48' 55''$	}	Sum of the distances $\odot \odot .$ }	$571^\circ 2' 0''$
49 55			
51 2			
52 34			
53 53			
54 51		Mean distance -	$95^\circ 10' 20''$
Sum - $311^\circ 10''$			
Mean time $1^h 51' 51''.6$			

	Times.		Altitudes ☉.
1st observation	1 ^h 49' 25"	-	32° 21' 30"
2nd observation	1 54 22	-	31 22
1st interval	0 ^h 4' 57"	Difference	0° 59' 30"
Time of the first observation		-	1 ^h 49' 25"
Time of observing the distance		-	1 51 51
2nd interval	-	-	0 ^h 2' 26"
1st inter. 4' 57": 2d inter. 2' 26" :: 1st chan. in alt. 59' 30" : x.			
1st change in altitude	- 59' 30"	- : log.	- 3.55267
1st interval	- - - - 4 57	Com. log.	7.52724
2nd interval	- - - - 2 26	- log.	2.16435
		log. x =	3.24426
x, or 2nd change in altitude	-	-	0° 29' 15"
1st altitude ☉	-	-	32 21 30
The ☉ descends. Difference.	Altitude ☉.		31° 52' 15"

	Times.		Altitudes ☾.
1st observation	1 ^h 51' 2"	-	40° 45'
2nd observation	1 52 34	-	41 5
	0 ^h 1' 32"	Difference	- 0° 20'
Time of the first observation		-	1 ^h 51' 2"
Time of the distance	-	-	1 51 51
2nd interval	-	-	0 ^h 0' 49"
1st inter. 1' 32": 2d inter. 0' 49" :: 1st chan. in alt. 0° 20' : x.			
1st change in altitude	- 0° 20'	- log.	- 3.07918
1st interval	- - - - 1 32	Com. log.	- 8.03621
2nd interval	- - - - 0 49	- log.	- 1.69020
		log. x =	2.80559
x, or the second change in altitude	-	-	0° 10' 39"
1st altitude of the moon	-	-	40 45 0
The ☾ ascends. Sum.	Altitude of ☾	-	40° 55' 39"

105. The observations may be made in this manner by a single observer; but it would be advantageous if he who measures the distances had an assistant to take the altitudes, and especially those of the sun. These last have the inconvenience of greatly fatiguing the sight, when the sun is not very elevated; then his reflection often renders the horizon so bright, that his light must be weakened by means of a coloured glass. The altitudes may be taken 7' or 8' before and after the observation of the distance; but it must be remarked, that the altitudes corresponding to the distance will be susceptible of much greater accuracy when they are taken nearer to the instant at which that distance is observed. It is also necessary that the mean time corresponding to the mean distance, should be between the times corresponding to the two observed altitudes. Whenever all these circumstances have been attended to, the altitudes calculated by proportional parts will have a precision nearly equal to those which have been directly obtained from observation.

106. When the visual horizon is limited by land in the direction of one of the heavenly bodies of which the distance has been taken, and a seconds watch was used, its gain or loss, with regard to mean time, must be ascertained by observing the sun's altitude when he answers to a point of the horizon where the sea appears clear. Then the altitude of the heavenly body may be calculated by the rules given in arts. 79 and 80.

107. The difficulty experienced in observing the altitudes of the stars, and even those of the moon during the night, has been mentioned. Errors of 5' or 6', of which they are susceptible, will not have a great influence upon the true distance of the moon from a star; thus, if preferred, the altitudes for correcting the distance may be observed. But, as an error of 5' or 6' may, in some cases, occasion an error

in the lunar angle of 30" of time, and even sometimes more, the time at the place should never be calculated with the altitude of a star. To supply its place, the gain or loss of the watch by which the time corresponding to the distances should be calculated from an observation of the sun's altitude, made either on the evening which precedes, or the morning that follows the time at which the distance is taken; and then, by means of the way made in longitude, the time at the place where the distances were observed should be found. In this case, the observations of the altitudes of the two bodies may be dispensed with; for they may be obtained with much greater accuracy from calculation than by observation. This method was recommended by *Borda* in his treatise on the reflecting circle; and it is that which ought to be practised. Articles 79 and 80, contain circumstantial details relative to the operations which should be performed for calculating the altitudes of the heavenly bodies.

Calculation of the true Distance, and of the Time at the first Meridian.

108. When the altitudes corresponding to the mean distance have been obtained by the methods already explained, the true distance and the time at the first meridian must be calculated by the following rules. An example shall first be given for the case in which the altitudes have been procured directly from observation; then the method that should be followed when the true altitudes of the heavenly bodies, corresponding to the distance, have been obtained by calculation, shall be explained in a second example.

109. First, calculate the time at the first meridian corresponding to the instant of the observations, by means of the estimated or true time at the place, and the longitude by account; then take from the *Nautical Almanac*, the semi-

diameters of the sun and moon at that instant. Find, in Table II, the augmentation of the moon's semi-diameter answering to her altitude, and it will give her apparent semi-diameter. Then find her equatorial parallax for the moment of the observation, and Table III will show, by means of the latitude, the quantity which this parallax ought to be diminished in order to obtain the parallax at the place of observation. These given quantities will serve for ascertaining the apparent distance between the centres of the sun and moon, or the apparent distance of a star from the centre of the moon, as well as the apparent and true altitudes of the centres of these two bodies.

110. When distances of the sun and moon are taken, the observation always gives the distance of their nearest limbs; then their semi-diameters must be added to the observed distance. If the distance between the moon and a star be taken, it gives the distance between the star and the enlightened limb of the moon, which is sometimes the nearest and sometimes the most distant; it must therefore be observed, in making the observation, which limb has been used. When the nearest limb has been observed, the apparent semi-diameter of the moon must be added to the observed distance, according to the preceding rule; but if the distance between the star and the most distant limb of the moon was observed, the moon's apparent semi-diameter must be subtracted from the observed distance. The distance thus found is called the *apparent distance*.

111. Then, correct the observed altitudes for the depression of the horizon, and the semi-diameter of either the sun or the moon; and the results will be the apparent altitudes of each of these bodies. Next find the refractions and parallaxes which answer to these altitudes, and when corrected for these, the true altitudes will be obtained. It is unnecessary to enter into greater detail relative to these correc-

tions, since the rules which should be followed have been explained in the second chapter. Those who are not familiar with these operations, may have recourse to what has there been said on the subject. The refractions of Table V, and those of Table VIII, ought always to be corrected according to the elevation of the mercury in the barometer and thermometer, whenever the altitude of either of the two bodies is less than 40° .

112. When the true altitude of the moon's centre has been obtained by calculation, search first in Table VIII, with this altitude instead of the apparent altitude, for an approximative number, which will sometimes differ from that which ought to express the true parallax of the altitude less refraction, by nearly a minute. With this number calculate a first apparent altitude, and then search in the same table the number that corresponds to it; this will be the parallax in altitude less refraction, which is to be subtracted from the true altitude resulting from the calculation, in order to obtain the apparent altitude of the moon's centre.

113. The apparent distance of the two bodies, their apparent altitudes, and their true altitudes are the five data with which the true distance is to be calculated. The following are the necessary rules.

Write, in the following order; first, the apparent distance of the two heavenly bodies, then the apparent altitude of the sun or the star, and lastly, the apparent altitude of the moon; add these three quantities together, and take half their sum. The apparent distance and the half sum being thus known, subtract the less of these quantities from the greater. Below this remainder, write the true altitude of the sun or the star, and afterwards that of the moon; add these two altitudes together, and take half their sum. When this preparation for the calculation has been made, look successively in the logarithm tables, for the arithmetical

complements of the logarithm cosines of the apparent altitudes; find also in the same manner, the logarithm cosines of the half sum of these altitudes, and of the apparent distance, as well as the logarithm cosine of their half difference, and write these two logarithms below the two arithmetical complements before found: then write, also below the last, the logarithm cosines of the true altitudes. Add together the two complements and the four logarithms, and take half the sum thus obtained; from this half sum subtract the logarithm cosine of the half sum of the true altitudes, and the remainder will be the logarithm sine of an auxiliary angle. Place the logarithm cosine of this auxiliary angle below the logarithm cosine of half the sum of the true altitudes; then the sum of these last two logarithms, will be the logarithm sine of half the true distance. Double of the corresponding arc will be the distance corrected for the effects of refraction and parallax, or the true distance with which the time at the first meridian ought to be calculated.

When the true distance has been calculated by this method, it may happen that the sum of the apparent distance and the apparent altitudes may be greater than 180° ; then it will not be necessary to continue the calculation, and the apparent distance may be corrected, by first taking the difference of the correction of the moon's altitude and that of the altitude of the sun or star, and then subtracting this difference from the apparent distance of the two bodies.

114. Search, in the Nautical Almanac, for the two distances between which the distance resulting from the calculation is found; write these below each other; then take their difference, which will be the change in the distance answering to three hours. Also take the difference between the calculated distance and the first in the tables; and having the change which answers to 3 hours, the interval of time answering to this last difference may be found by proportion.

This second interval should be calculated by logarithms. It must always be added to the time of the first distance in the tables, and the sum will be the required time at the first meridian.

All the operations which are to be performed, either in procuring the apparent distance and altitudes, or for obtaining the true altitudes; or, lastly, for calculating the true distance from which the time at the first meridian and the longitude are found, shall now be explained: the example in art. 102 may be resumed, in which the altitudes and distances have been obtained by simultaneous observations.

EXAMPLE.

On the 16th of June 1793, at about one hour and a half after noon, being in South latitude $10^{\circ} 16' 40''$, and 149° of East longitude, by account, six observations of the distance between the sun and moon were taken, and six simultaneous altitudes of each of these two bodies. The mean distance of their nearest edges was found to be $83^{\circ} 26' 46''$; the mean altitude of the sun's lower limb was $48^{\circ} 16' 10''$; and that of the moon's upper limb $27^{\circ} 53' 30''$.

It is found, by means of the estimated time at the place of observation and the longitude, by account, that the estimated time at the first meridian which corresponds to the observation of the distance, is the 15th of June at $15^h 34'$. The semi-diameter of the sun, taken from the Nautical Almanac, was at that instant $15' 46''$. The semi-diameter of the moon was $14' 54''$; the small Table II shows that, at $27^{\circ} 53'$ or 28° of altitude, that there must be added $7''$ to have the apparent diameter which will then be $15' 1''$: these last quantities should be employed in obtaining the apparent distance and apparent altitudes of the centres of the sun and moon. The equatorial parallax is $54' 41''$, but at 10° of

altitude it must be diminished by $2'$; hence there must be used in the calculation only $54' 40''$.

When these first elements are known, the apparent distance must be calculated. The calculations of all the quantities which are to be obtained in order to find the true distance, and from it the longitude, shall now be successively given; but, to render the proceeding still clearer, all these quantities have been collected into a table which is subjoined to the end of the calculation of the time at the place of observation; and will serve as a guide to those who wish to exercise themselves in calculating the longitude from observed distances.

Observed distance between the limbs of the \odot & \odot	$83^{\circ} 26' 46''$
Semi-diameter of the \odot	$+ 15 46$
Semi-diameter of the \odot	$+ 15 1$
Apparent distance of the centres of the \odot & \odot	$83^{\circ} 57' 33''$

The observed altitudes of the two heavenly bodies are to be corrected for the depression of the horizon and their semi-diameters, and the apparent altitudes of their centres will be obtained; then these altitudes must be corrected for the effects of refraction and parallax, by means of the numbers found in Tables V and VIII, which will give the true altitudes. It is essential to attend to the variations which these last numbers experience relatively to the height of the mercury in the barometer and thermometer, as will be shown.

Observed altitude of the \odot	$48^{\circ} 16' 10''$
Elevation of the eye - 20 $\frac{1}{2}$ feet. Depression	$- 4 24$
	$48 11 46$
Semi-diameter of the \odot	$+ 15 46$
Apparent altitude of the \odot	$48 27 32$
Refraction — Parallax	$0 45$
Thermometer + 78.98	$- 2$
Barometer = 29.988 in.	0
True altitude of the \odot	$48^{\circ} 26' 49''$

Observed altitude of the α	-	-	27° 30'
Elevation of the eye 20 ft.	-	Depression	- 4 34
			<hr/>
			27° 49'
Semi-diameter of the α	-	-	- 15 1
			<hr/>
			27° 34'
Parallax — Refraction	-	- 46' 38"	
Thermometer 78° 98	-	- + 5	
Barometer 29.99 in.	-	- 0	
			<hr/>
True altitude of the α	-	-	28° 20' 48"

In calculating the distance, the proportional parts must be taken, in order to have the logarithms corresponding to the seconds of a degree. This however may be, in a great measure avoided, if from the apparent distance there be subtracted such a number of seconds as will make the remainder contain only even tens of seconds. For example, in this case, $83^{\circ} 57' 30''$ may be used instead of $83^{\circ} 57' 38''$; but the $3''$ that have been subtracted are to be written above the distance with the sign $+$, which indicates that they ought to be added to the true distance obtained by the calculation. Subtract from the apparent altitudes, in the same manner, the number of seconds necessary to make them contain only tens of seconds, or else add this number to complete them. These small changes should always be made in such a manner that the tens of seconds of the sum of the distance and of the apparent altitudes may be an even number; then the half sum, and the difference of that half sum and the distance, as well as the apparent altitudes, will contain only tens of seconds; we shall therefore be able to take two arithmetical complements and two logarithms, without being obliged to calculate the proportional parts.

It is important not to neglect, in writing down the true altitudes, to add to, or subtract from them, the same number of seconds that has previously been added to, or subtracted from the apparent altitudes, in order that the difference of

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the true and apparent altitudes of each body may remain always the same. It is on account of this difference that the value of the refraction of the apparent distance, or the difference of this and the true distance, in a great measure depends. In this example, we have subtracted 2" from the sun's apparent altitude, they must therefore be likewise taken from his true altitude, and then $48^{\circ} 26' 47''$ is to be written instead of $48^{\circ} 26' 49''$. There has also been 5" subtracted from the apparent altitude of the moon, and therefore, in the calculation of her true altitude, only $28^{\circ} 20' 43''$ must be used instead of $28^{\circ} 20' 48''$.

Appar. Dist. \odot \odot	85 20 55		
Appar. Alt. \odot	48 27 30	com. cos.	0.1783787
Appar. alt. \odot	27 34 0	com. cos.	0.0523345
Sum	159 59 0"		
Half sum	79 59 30"	cos.	9.2400283
Dist.	3 58 0	cos.	9.9989584
True alt. \odot	48 26 47	cos.	9.8217234
True alt. \odot	28 20 43	cos.	9.9445339
Sum	70 47 30"	Sum	39.2359565
$\frac{1}{2}$ Sum	19 61 7 1/2"		9.7238069 sine auxiliary angle.
Half sum	38 23 45"	cos.	9.8941715
Auxiliary angle		cos.	9.5205723
Sum		sin.	9.8227496
Half the distance	43 40' 26"		
Double Distance	83 20' 52"		
Add the seconds neglected	+ 3"		
TRUE DISTANCE	85 20' 55"	1st inter. 3 ^h	log. 4.0334
Distances in the \int at 15 ^h	83 2 9	1st diff. 0 ^h 18' 46"	log. 2.05154
Table at 15 ^h	84 24 32	2d diff. 12' 24"	log. 6.30592
Time of the first tabular distance	13 0' 0"	Sum	3.39088
2d interval	0 34' 0"	2d interval	0 ^h 41' 0"
Time at the first meridian	15 44' 0"		

115. The distance which results from the calculation is $85^{\circ} 20' 52''$; the 3", which were neglected before beginning

the calculation, must be added to it, and the true distance will be $82^{\circ} 20' 55''$. The two distances in the Canon of the Terns, between which this calculated distance is found, are $83^{\circ} 2' 9''$, and $83^{\circ} 24' 35''$. The first of these places is at 15^h , and the second at 18^h . The interval by which they are separated, and which is called the first interval, is 3^h . Write these distances, and the hours which correspond to them, under the true distance, as above; then take the difference of the true distance and the first tabular distance, which will give a first difference; this is to be placed on the right of the distance to which it corresponds. Also take the difference of the two tabular distances, and it will be the second difference, which is the change in distance in 3^h , or in the first interval. These quantities are to be used in calculating the second interval, or that which answers to the difference between the calculated distance and the first tabular distance, which has been called the first difference. Make this proportion, the second difference is to the first interval of 3^h , as the first difference of $18' 40''$ is to the second interval, which must always be added to the time of this first distance to obtain the hour at the first meridian. The fourth term of this proportion should be calculated by logarithms; thus, in order to have the logarithm of the second interval, add the constant logarithm of 3^h , that of the first difference, and the arithmetical complement of the second difference together.

Calculation of the Time at the Place of Observation.

116. The method which should be followed in finding the time at the place depends upon the manner in which the altitudes of the two heavenly bodies have been obtained. When they have been taken, as well as the distance, by simultaneous observations, and the time of these observations

would ~~not~~ be reckoned by the watch, the horary angle of the sun must be calculated by means of his observed altitudes, by the rules in art. 75; then the time at the place may be deduced from it. The difference which exists between this time and the time at the first meridian, that has been calculated from the true distance, reduced into degrees, will be the longitude of the vessel. When the time at the place is greater than that at the first meridian, this longitude is east; but when it is less, the vessel is to the west of the first meridian, and its longitude is west.

117. Now to find what time at the vessel corresponds with the time at the first meridian, which has been previously calculated. Take first, in the *Connaissance des Temps*, or the *Nautical Almanac*, the sun's declination which answers to the calculated time at the first meridian; in this case it is $23^{\circ} 22' 47''$ North, but the latitude is $10^{\circ} 16' 40''$ South: the distance of the elevated pole will therefore be $113^{\circ} 22' 47''$. Proceed with these quantities and the true altitude of the sun to calculate the horary angle. It should be remarked, that the altitude increased or diminished by a certain number of seconds, must not be employed in the calculation, but that which is immediately deduced from the observed altitude. Thus, in the present example, the altitude of the sun which should be employed in calculating the time, is $48^{\circ} 26' 49''$, instead of $48^{\circ} 26' 47''$.

True altitude \odot .	$48^{\circ} 26' 49''$			
Latitude, South	$10^{\circ} 16' 40''$	-	comp. cos.	0.0070251
Polar distance \odot	$113^{\circ} 22' 47''$	-	comp. sin.	0.0372070
Sum	$172^{\circ} 6' 16''$			
Half sum	$86^{\circ} 3' 8''$		cos.	8.8378864
$\frac{1}{2}$ Sum — altitude	$37^{\circ} 36' 19''$		sin.	9.7854251
Sum		-		18.6676086

Half sun	12 ^h 00 ^m 18 ^s
Half of the horary angle	12 ^h 47 ^m 18 ^s
Multiplying by	
(Add 24 ^h). Time of the place	1 ^h 39 ^m 38 ^s 24 ^h
Time at the first merid.	15 ^h 41 ^m 0 ^s 0 ^h
Difference	9 ^m 58 ^s 38 ^h 24 ^m
LONGITUDE, East	149° 39' 36"

The time at the place of observation, in this case, appears to be less than the time at the first meridian, but it is really greater. In fact, a day more is reckoned on board the vessel, and 24 hours must be added to the time deduced from the calculation, which is the 16th of June at 1^h 39' 38" 24^h, while at the first meridian it was only the 15th of June, and, at the instant of the observation, the hour was 15^h 41'.

118. This method of obtaining the time at the place of observation should be employed only when the time of the observations cannot be estimated by a good watch. Whenever a marine chronometer or a good seconds watch is used, whether the observations be simultaneous, or the altitudes corresponding to the distance be calculated by proportion, an account must always be taken of the time at which each observation of the distance was made. Some time before observing the distance, or a little after the observations have been made, the sun's altitudes may be observed, which will give the time at the place where these altitudes were taken. The apparent distance, with the altitudes observed at the same place must be corrected, as in the preceding example; but then the difference between the time at the first meridian, concluded from the true distance, and the time at the place where the horary angle was observed, is to be taken: by this means, the longitude of the place will be obtained. This second method possesses a great advantage, when a

marine chronometer is used, which will also give the time at the place where the horary angle is observed; for it procures two results which can be directly compared with each other without any previous reduction being made. In the same case where the time of the observations can only be reckoned on a common seconds watch, the observation of a horary angle, taken before and after that of the distances, will have the advantage of greatly shortening the calculation. In fact, if a single series of observations of the distance be not thought sufficient, the same horary angle will suffice for calculating the longitude from all the series that have been observed.

119. It has already been remarked, that the time at the place of observation should not be calculated with the altitudes of the stars, but obtained from the altitude of the sun, taken either in the evening which precedes, or the morning which follows the observation of the distance between the moon and a star; it has also been recommended to calculate the true altitudes of the two bodies with the hour at the place where the horary angle was observed, referred to that where the distance was observed by means of the way made in longitude in the interval between the two observations. If the time used in the calculations of the altitudes be compared with the time at the first meridian, concluded from the calculation of the true distance, the longitude of the place where the distances were observed will be obtained; but it would be better, as before, to take the difference of the time at the first meridian, and that of the place where the horary angle was observed, in order to obtain a longitude that may be directly compared with that obtained by a marine chronometer.

120. The circumstances in which the distances of the moon from the stars can be taken, are much more frequent than those in which her distance from the sun can be mea-

sured. This kind of observation must not therefore be neglected, as it is often the only one from which the position of the vessel can be concluded. With a seconds watch, the observation of the distances between the moon and the stars is as easy as that between the sun and moon. It cannot be too much recommended to navigators, not to suffer themselves to be terrified by the length of the calculations, which, in this case, are indeed increased by that of the altitudes; what they may find difficult and tedious at the beginning will soon disappear: for, by exercising themselves during a short time will render the calculations familiar. Besides, it is useless to aim at an imaginary precision, of which the observation is not susceptible, and it will be sufficient to calculate the altitudes to a minute, and to take the logarithms only to five places of decimals. The calculation of the true distance of a star from the moon is the same as that of the distance between the sun and moon; and if attention be paid to all the abbreviations which have been indicated, the calculation of two or three series of observations may be made in a very short time. The following example may be used as an exercise. The rules that ought to be followed will be found in arts. 79 and 80, and in this chapter. We shall, therefore, give only a simple enunciation of the question, and the result of the calculation: but we have united in the same table, as has likewise been done with respect to the former example, all the data with the quantities necessary for the calculation; in which, they are arranged in the order most proper for facilitating the operations.

EXAMPLE II.

On the 19th of June, 1793, being in South latitude $9^{\circ} 45' 50''$, and East longitude $148^{\circ} 43'$. When the time by the watch was $3^h 41' 5''$. (See the Examples in

art. 81.), it was found from altitudes of the sun, that it was $1^h 21' 34''$ behind true time. At $6^h 8' 10''$ by the same watch, it was found from a series of six observed distances between the moon and *Antares*, that the distance of this star from the farthest limb of the moon was $39^\circ 12' 18''$. It is required to find the longitude of the place where the horary angle was observed.

The calculated altitudes are the same as those in the example above referred to, where the operations necessary for finding them are explained.

Time at the place of obser. of the horary angle	$31^h 29' 45''$
Time at the first meridian	$21 32 21$
Difference	$9^h 57' 24''$
LONGITUDE, East	$145^\circ 21' 0''$

121. Whenever two or three series of distances can be observed, the longitude may be obtained to within about $15'$ or $20'$ of the truth. This error can never have a greater influence than from 5 to 6 $\frac{1}{2}$ leagues upon the position of the vessel.

122. The true distance which is found by the preceding method, is obtained on the hypothesis that the earth is spherical. There are found, in almost all treatises on the calculations of nautical astronomy, the means of correcting it in order to find that which would have resulted from the calculation, if the earth had been considered as a spheroid flattened at its poles. It has been thought proper to omit these corrections in this place, because they can never be of sufficient consequence to merit attention; in the most favourable circumstances, they can never influence the calculated longitude more than $3'$ or $4'$ of a degree. But when the altitude must be calculated, the illipticity of the earth may be taken into the account without increasing the calculation; it would simply be necessary, instead of calculating

these altitudes with the latitude of the place, to employ that latitude diminished by a quantity which is found in collections of astronomical tables, and which is called the angle at the vertical. All mention of these corrections has been hitherto avoided, * in order that the calculation of longitude might not be rendered unnecessarily complicated; on the contrary, we have been desirous of finding methods of increasing its simplification.

123. The distances may be observed at land, and, in this case, they will give the longitude of the place for which the marine chronometer has been regulated; but as the probable error from each observation is 15' or 20', a great number of results should be obtained for determining the longitude of the same point, in order to diminish as much as possible the errors with which they may be affected. The probability of the accuracy of longitudes thus determined may still be increased, by deducing them in the following manner. In the first place, it should be recollected, that it has been said the same observer always measures the angles either a little too great or a little too small, either from the nature of his sight, or from the manner in which he is accustomed to make the limbs of the objects coincide in the field of the telescope. It follows from this, that the distances observed by the same person are all either too great or too little. The errors by which they may be affected from this cause are subject to variation, but those which act in one sense, on the time at the first meridian, concluded from observations made when the distances in the

* There will be found in the notes subjoined to this treatise, and at the end of the explanation of the construction of Tables XII and XIII, the use that may be made of these tables for correcting the observed altitudes, and obtaining the proper quantities for computing the true distance, on the hypothesis that the earth is an oblate spheroid.

tables increase, will act in a contrary or opposite sense when the distances in the tables diminish. The distances increase when the sun or the star is on the west of the moon, and then the distances are called *west distances*; they diminish when the sun or the star is on the east of the moon, and, in that case, they are named *east distances*. It is therefore necessary to take an arithmetical mean between the mean longitude concluded solely from west distances, and that from east distances only, in order to obtain a final longitude, that may, in a great measure, be free from errors arising from the sight of the observer. It is probable that the errors in longitude obtained by thus combining the results, will not amount to more than $10'$ of a degree.

124. The distinction which is here made between the longitudes obtained from west distances, and those found solely from east distances, can only be of much utility when the results are derived from distances between the sun and moon. Many causes, the explanation of which would be too long in this place, render the distances between the moon and the stars subject to irregularities, the different influences of which it is impossible to ascertain from the circumstances that accompany the observations; hence the longitude of the port arrived at should always be determined from the mean longitude of all the observations, which can be taken without making the distinction between the longitudes obtained from east and those from west distances. The errors of longitudes obtained from distances between the moon and the stars, ought never to exceed $15'$ of a degree; they will therefore be susceptible of an accuracy a little less than that derived from distances between the sun and the moon; this is the reason why they should be employed in ascertaining a geographical position, only when a sufficient number of the latter observations cannot be obtained.

125. Longitudes observed at sea are generally concluded

from distances taken in different places, which, at first sight, do not appear capable of giving the position of the vessel with any great accuracy; it will, nevertheless, be easy to give them a precision, whenever a good marine chronometer is used, equal, perhaps, to that of the longitudes observed during a period of anchorage. In fact, these chronometers afford the means of measuring, with accuracy, the differences of longitude of all the places where the distances have been observed; it will therefore be possible to refer the results of longitudes observed at different places to the same place, the longitude of which will have a much greater precision than if it had been determined from the small number of observations which it was possible to make at that place. Whenever the following rules can be complied with, the longitude will only be affected with the error arising from that of the distances; and the influence of the errors of different longitudes, taken even a long time after the chronometer has been regulated, may be considered as nothing.

126. Refer the longitudes from distances, which have been taken on consecutive days, or those but little distant from each other, to the place of which the longitude has been determined by the marine chronometer, in the morning or evening of the day which is equally distant in time from the extreme observations of the distance.

If the longitudes obtained from distances between the sun and moon are required, refer all the longitudes from east distances to the same point. In the same manner, those obtained from west distances are to be referred to another point; then the longitude of a third point is to be calculated by referring the longitudes of the two intermediary days, one of which was the result from east, and the other from west distances, to the place where the longitude has been found by the chronometer, in the morning or evening of the day equally distant in time from the two intermedi-

any days above mentioned. The longitude of this third point will have all the accuracy of which the method of distances observed at sea is susceptible; it will even be nearly as accurate as the longitude obtained from observations made at the place itself.

127. When the longitudes obtained from distances between the moon and the stars are required, the mean longitudes which have been deduced from two or more series of observations, may be referred to a single point in the same manner, but without any distinction into those from east and west distances: the only attention which is necessary is, that the interval between the intermediary days, answering to each series of observations, may not exceed 20 or 30 days. By this means, so great a number of observations may be made to concur in determining the longitude of a single place, that it will be obtained with a precision, perhaps, equal to that of the longitude derived from distances between the sun and the moon.

CHAPTER VII.

On finding the Declination of the Magnetic Needle, by Observations of the Sun's Azimuth or Amplitude, and by the Astronomical Bearing of a terrestrial Object.

128. THE declination of the magnetic needle is the angle which the direction of this needle makes with the north and south line. If the bearing of a terrestrial object situated exactly north and south, be taken, the observation will give directly the declination of the needle; but as all the points of the compass make, with the true points of the horizon, angles equal to its declination, it will be sufficient to take any object, the true bearing of which is known: then the difference of this bearing, and that which has been observed, is the declination required. The question is, therefore, reduced to that of finding, by any means, the true bearing of an object so situated that its bearing can be taken with the compass. The sun is the only object which can be conveniently observed with the compass at sea. The bearing of the sun is an arc of the horizon comprised between the vertical circle and the meridian of the place, and ought therefore to be equal to the angle formed by these two circles, or to the azimuth of the sun. This azimuth must therefore be found from calculation for the instant that his bearing is taken. Nautical astronomy also teaches the means of observing and calculating the true bearings of terrestrial objects: hence, near the

short, observations of these objects may be employed for finding the declination of the magnetic needle. These last bearings are called *Astronomical bearings*; and as they are those which are susceptible of the greatest degree of precision, and also contribute much to the perfection of hydrographical or marine charts, it is proposed to treat of them here at some length.

Calculation of the Sun's Azimuth and Amplitude.

129. It has already been remarked, that the altitude of the sun varies at every instant of his course, and that the time at any place may be found by an observation of this altitude: the sun's azimuth, corresponding to the same instant of observation, may also be obtained by calculation. Hence, when it is wished to ascertain the declination of the magnetic needle, it is only necessary to observe the sun's azimuth with a compass at the same instant that his altitude is taken. The difference of the observed and calculated azimuths will be the required declination.

130. The circumstances under which the observation of the sun's altitude gives his azimuth with the greatest accuracy, are nearly the same as those in which that altitude ought to be observed for ascertaining the time at the place where the observations are made. Now, as the calculated azimuth is almost always susceptible of much greater accuracy than the azimuth observed with the compass, it will not be necessary in the present case, to pay any regard to the rules that have been given relative to the circumstances which should accompany an observation of the solar angle. It will always be most advantageous to make the observation when the sun is very near the horizon; then his azimuth may be observed with a compass, much more easily than when he has attained a certain altitude. The errors with which the calculated azimuth

may, in this case, be affected from the uncertainty of refraction and the latitude of the place, will be very small in comparison with those of which the observed azimuth is itself susceptible. Whenever, however, remaining at the horizon with a reflecting instrument, the direct image of the sun cannot be seen in the field of the telescope, we may commence the observation: the altitude will then be a little more than four degrees. The bearings of the sun become susceptible of great errors when he has attained 15° of altitude; the azimuth should not, therefore, be observed when his altitude exceeds 15°. We might, in strictness, practice this kind of observation, as often as the sun can be seen through the sights placed on the lid of the compass; but when it is desired to obtain all the accuracy of which it is susceptible, the observation must terminate when the altitude is equal to 15°.

131. While two observers are occupied in taking the bearing of the sun with a compass, a third observer should take the altitude of the sun with a sextant or a reflecting circle; bringing the sun's image to the horizon, and, following its movements with the repelling screw of the instrument, always preserving its lower limb in contact with the horizon. At the moment when the two observers who take the bearing are certain of having made a good observation, they inform him who takes the altitude, and he reckons the arc marked by the index on the limb of his instrument: which will be the altitude corresponding to the observed azimuth. Another observation may be made, and the mean altitude concluded, answers to the arithmetical mean between the two observed azimuths. If the altitude be taken with a reflecting circle, the arc passed over by the index should be reckoned only at the end of the second observation; and the mean altitude corresponding to the mean azimuth, may be concluded in the usual manner. It

would be advantageous to observe in this manner several series, each consisting of two observations; the arithmetical mean of the declinations deduced from each of these series, will be susceptible of considerable accuracy. It is not necessary to reckon the time at which each of these observations was made on a seconds watch; the estimated time will be sufficient, which may differ 15' or 20' from the true time at the place of observation without inconvenience.

132. The following is the method of calculating the azimuth. Calculate the time at the first meridian corresponding to the instant of the observation, by means of the estimated time at the place, and the longitude by account. Search in the Nautical Almanac the declination of the sun for the time of observation, from which his distance from the elevated pole may be concluded. This polar distance, the true altitude, which is to be deduced from the observed altitude by the rules in Chapter II, and the latitude of the vessel, are the three data necessary for the calculation, which is to be performed as follows.

Write below each other in the following order, the distance of the sun from the elevated pole, his true altitude, and the latitude. Add these three quantities together, and take half their sum. Then below this half sum write the difference between it and the polar distance; that is, subtract the less of these quantities from the greater. Take, in the tables, first, the arithmetical complements of the logarithm cosines of the true altitude of the sun and the latitude; then write below these complements the two logarithm cosines of the half sum, and the difference between this half sum and the polar distance. Add these four logarithms together, and half their sum will be the logarithm cosine of half the azimuthal angle; double of the corresponding arc will be the sun's azimuth, which is always reckoned to commence towards the elevated pole: hence, if

the elevated pole is in the northern part of the meridian, the azimuth will be reckoned from the north; but if the elevated pole is towards the south, the azimuth will be reckoned from the south. The azimuth observed with the compass must consequently commence at the same part as that obtained by calculation, that the declination of the magnetic needle may be deduced by comparing them together.

The calculation of the azimuth may be made without regarding the seconds of a degree; and the logarithms need only be taken to five places of decimals.

133. It has already been observed, that the declination of the needle is equal to the difference between the azimuth observed with the compass and that derived from calculation; but in order to know on which side of the meridian it should take place, it will be necessary to attend to the following remarks: suppose, for a moment, that we were turned towards the sun, and looking in the direction of his bearing; then it would be very easy to know whether the azimuth resulting from the calculation, answered, on the card of the compass, to the left or *larboard* side of the azimuth observed with the compass; or whether it corresponded to the right or *starboard* side. But the direction of the magnetic needle ought to be situated, with respect to the north and south line, exactly in the same manner as the calculated azimuth is situated with respect to that which has been observed with the compass; hence, whenever the calculated azimuth answers on the card of the compass to the *larboard* of that observed with the needle, it follows, that the direction of the needle ought to be to the *larboard* side of the north pole: in this case, the needle declines towards the west, and its declination takes the name of north-west. If the calculated azimuth place the sun on the *starboard* side of the observed azimuth, the needle declines towards the east, and the de-

130. CALCULATION OF THE SUN'S AZIMUTH, &c. CHAP. VII.
clination take the denomination of north-east. Mariners commonly call the declination of the needle the *Variation of the Compass*, and say that the variation is north-east or north-west.

134. When the needle declines two points of the compass towards the north-west, or to the larboard side of the north, the true direction of the north point of the compass is the north-north-west; and when it declines two points towards the north-east, or starboard side, the true direction of the same point of the compass is north-north-east. The corrected point is therefore always situated, with respect to the observed point, in the same manner as the north of the compass is with regard to the north of the world. This consideration induces us to believe, that there would be an advantage in applying both these denominations to the declination of the magnetic needle; we should say, for example, declination north-west or larboard, and declination north-east or starboard. This double appellation would afford a very simple general rule for correcting the course of a vessel and the bearings observed with the compass. It would be sufficient to employ the declination of the needle in such a manner that the corrected bearing may be on the larboard or the starboard of the observed bearing, according to the denomination which that declination ought to have. The denominations of north-east and north-west are more naturally derived from principles, and are essential to those who occupy themselves with the theory of magnetism; the other denominations would be a great convenience in practice, and might prevent many mistakes that take place, only because men, even the most experienced, are subject to be deceived relative to the true sense in which the bearings should be corrected. Mariners are, doubtless, guided by an analogy of this kind, when they say that the lee-way is on the larboard or starboard, and that

the variation is on the same or the opposite side. It is not attempted to introduce a new term, but only proposed to render a denomination general, which has been used in a particular case.

EXAMPLE

On the 2nd of March 1792, at about six in the morning, being in South latitude $34^{\circ} 48'$, and East longitude $35^{\circ} 49'$ the altitude of the sun's lower limb, was observed to be $6^{\circ} 15'$. At the same instant the sun's azimuth, observed with the compass, was $57^{\circ} 17'$; that is, the centre of the sun was taken at $57^{\circ} 17'$ from the south towards the east; the elevation of the eye was $20\frac{1}{2}$ feet above the surface of the sea. Required the sun's true azimuth, and the declination of the needle.

The time at the first meridian corresponding to the instant of the observation is the 1st of March, at $15^h 37'$; the declination of the sun for that time was $6^{\circ} 57'$ South. But the latitude is of the same denomination as this declination, consequently, the distance of the elevated pole is $83^{\circ} 3'$. The true altitude of the sun's centre is $6^{\circ} 19'$. The given quantities may be disposed, and the calculation performed in the following manner.

Distance of the elevated pole	$83^{\circ} 3'$		
True altitude of the \odot .	$6 19$	comp. cos.	9.99264
Latitude	$34 48$	comp. cos.	9.98558
Sum	$124^{\circ} 10'$		
Half sum	$62 5$	cos.	9.67042
Polar distance — $\frac{1}{2}$ Sum	$20 58$	cos.	9.97025
Sum			19.72889
Half sum		cos.	9.86444
Half azimuth	$42^{\circ} 57'$		

<i>Double.</i> Azimuth from the South to the East.	$85^{\circ} 54'$
The sun's bearing was from the South	$- 57^{\circ} 17' \text{ E.}$
DECLINATION of the magnetic needle	$- 28^{\circ} 37' \text{ N.W.}$ or larboard.

In this example, the south pole is that which is elevated above the horizon, consequently, the calculated azimuth of the sun is reckoned from that pole. The azimuth observed with the compass must also be reckoned from the same pole, and it is S. $57^{\circ} 17'$ East. The difference of this observed azimuth, and that which results from the calculation, is the required declination of the needle, and is found to be $28^{\circ} 37'$. Now, in order to know in what direction this declination ought to take place, it may be remarked, that the calculated azimuth being greater than the azimuth observed with the compass, it ought to answer on the card of the compass to the left or larboard of the observed azimuth; it follows then, that the declination of the needle is north-west; and if we adopt the double denomination which has been proposed, it will be north-west or larboard.

135. The instant at which the sun's bearing can be the most easily taken with the compass, is that of his rising or setting, because he is then found very nearly in the plane of the compass card. Mariners make more use of this observation, than of the preceding one, because the calculation is shorter, and it is not necessary to observe the sun's altitude, which is nothing when his centre is in the horizon but the result is not susceptible, as will be seen, of so much precision as it is possible to attain by the other method. There are inserted in almost all collections of tables on nautical astronomy, tables of a double entry, by the assistance of which it is easy to find, with the latitude of the place and the declination of the sun, his amplitude at the moment of his

rising or setting. This arc is only a part of the horizon comprised between the sun and the true east or west point; it is the complement of the sun's azimuth, or, in certain cases, it is equal to the quantity which this azimuth exceeds 90° . The difference of the amplitude found in the tables, and that observed with the compass, is equal to the declination of the needle. It may be known by means similar to those which have been given for the azimuth, whether the needle declines towards the north-east or north-west.

The table of amplitudes, in order to be useful, ought to have a certain extent. The limits to which we have been obliged to confine the collection of tables at the end of this treatise, has obliged us to suppress this; but its place may be supplied by a very short calculation, which will give the sun's amplitude by the addition of two logarithms of five figures each.

136. Before making the calculation, find the time at the first meridian corresponding to the moment of the rising or setting of the sun; and take, from the Nautical Almanac, the declination of the sun at that instant. Then add the logarithm sine of the declination to the arithmetical complement of the logarithm cosine of the latitude, and the sum will be the logarithm sine of the sun's amplitude. When the sun is found on the north of the equator, his rising and setting will be north of the east and west line; and when he is on the south of the equator, he rises and sets on the south side of the same line; the amplitude is therefore always of the same denomination as the declination.

137. The amplitude found in the tables, and that obtained by the preceding calculation, supposes the bearing of the sun to be taken with the compass, at the instant when his centre was really in the horizon; but, on account of refraction, the centre of the sun ought then to appear to have an elevation of $33'$, it will therefore be necessary to

observe the bearing only when the sun's lower limb has an altitude nearly equal to his semi-diameter. It is the difficulty of seizing this instant that renders the declination of the needle concluded from observations of the amplitude, less susceptible of precision than those which result from observations of the azimuth. Nevertheless, when the sun is taken a short time before his lower limb is detached from the horizon, and again when the altitude of this limb does not appear greater than his whole diameter, then the error of the calculated amplitude will never be much more than half a degree, provided the latitude do not surpass 60° . But, on the other hand, the observed amplitude may be affected with an error of half a degree: also, when circumstances are not very favourable, that is, when the sea is but slightly agitated, the declination of the magnetic needle may be obtained within nearly a degree: this accuracy is sufficient for the purposes of navigation; but if it be wished to obtain it with greater precision, observations of the sun's azimuth only must be employed.

EXAMPLE.

On the 11th of June 1792, at about $6^h 50'$ in the morning, being in South latitude $27^\circ 10'$, and East longitude $164^\circ 22'$; the easterly amplitude of the sun was observed with the compass, and found to be $37^\circ 27'$ towards the North. Required the declination of the magnetic needle.

The time at the first meridian, at the moment of the sun's rising at the place of observation was the 10th of June at $7^h 53'$, consequently, his declination was $23^\circ 7'$ North.

Declination of the \odot North	$23^\circ 7'$	sin.	9.59396
Latitude	$27^\circ 10'$	comp. cos.	0.05077
		sum. sin.	9.64473
Amplitude of the \odot .		E.	$26^\circ 11' N.$

Amplitude of the ☉. - E.	26° 11' N.
The sun was taken to the E.	37° 27' N.
DECLINATION of the magnetic needle.	11° 16' N.E.
	or starboard.

In this example the calculated bearing answers on the card of the compass, to the right or starboard of the bearing taken with the compass; the declination of the needle is therefore North-East or starboard.

Of Astronomical Bearings.

138. Having given the method of calculating the azimuth or bearing of the sun from an observation of his altitude; if, at the same time that his altitude is taken, the distance between the sun and a terrestrial object be measured, and the altitude of that object be observed, nautical astronomy furnishes the means of calculating, from these data, the difference between the bearings of the sun and that object at the moment of observation. The bearing of the sun being known, and the difference between this bearing and that of the object, the bearing of this last is easily determined. It is these bearings that are called astronomical bearings, because they are immediately derived from observations of the heavenly bodies. They are the most proper, as will be shewn, for ascertaining the declination of the magnetic needle; they ought also to be employed in preference to bearings taken with the compass, in the construction of hydrographical or marine charts.

139. The observation of astronomical bearings requires the concurrence of two observers; while one takes the altitude of the sun, the second measures the distance of the object from his nearest limb. Two observations of this distance and altitude may be taken, and the mean of each

deduced. The altitude of the object should always be very small, and may not vary by a sensible quantity in a short interval of time, it may therefore be measured a little before or after the observations of the altitude and distance of the sun. As it is not necessary to take the bearings of terrestrial objects to a small number of seconds, the rules that have been recommended to be observed in taking the distance of the moon from the sun or the stars need not be attended to here; hence the exact simultaneous observations of the sun's altitude, and his distance from the object will not always be absolutely necessary.

The hour, minute and second at which the observation was made need not be taken, and the quantities taken from the Nautical Almanac may be calculated with the time at the first meridian, deduced from the estimated time at the place of observation, which may be 15 or 20 minutes from the exact time without inconvenience.

When it is intended to deduce the declination of the magnetic needle from the astronomical bearing of a terrestrial object, two other observers must take the bearing of the same object with the compass, at the instant its distance from the sun is observed. This method of observing the declination of the needle requires the assistance of four observers, that is, one person more than when it is obtained from the azimuth of the sun. The method of amplitudes requires only two observers, and this is one of the reasons which renders it more convenient in practice.

140. The calculation of astronomical bearings, from what has been said above, consists of two parts; 1st. the calculation of the sun's azimuth; 2nd, the calculation of the difference of the azimuths of the sun and the object. Hence the accuracy of the result depends upon that with which the sun's azimuth is obtained from his altitude; and also the precision with which the calculated difference of the

azimuths is determined. It has been shown that the motion in altitude is very slow near the meridian; consequently, altitudes taken near the meridian are not proper for ascertaining the corresponding azimuth. In general, the azimuth of the sun should never be calculated with altitudes taken within an hour and a half of noon. The altitudes taken at any other time of the day, give the azimuth within about $2'$ of the truth. There are circumstances in which the astronomical bearings may be obtained from a distance observed between half past ten in the morning and half past one in the afternoon, but then it is necessary to calculate the sun's azimuth with the horary angle instead of his altitude. The method of performing this calculation will be given in the subsequent pages.

141. All circumstances are not equally favourable for observing the difference between the azimuth of the sun and that of a terrestrial object; observations may even be made, the results from which would be very defective; this renders it essential to consult the following precepts, before the observations are made, and if care be taken to conform to them, the azimuth will be obtained, in all cases, within $2'$ or $3'$ of the truth.

1st. Never observe the astronomical bearing when the altitude of the sun exceeds 60° .

2nd. Choose an object when it is nearly 90° distant from the point where the vertical circle of the sun cuts the horizon.

3rd. When an object cannot be observed about 90° from the vertical circle of the sun; choose another so situated with respect to the sun, that the angle of inclination of the instrument with which the distance is measured, may not be more than 45° .

An error of 10° or 12° in the estimate, either of the difference between the azimuths of the sun and object, or in the

inclination of the instrument with which the distance is measured, will not have a great influence upon the result.

142. The following are the rules which ought to be observed in procuring the quantities necessary for the calculation. From the estimated time and longitude, the time at the first meridian corresponding to the moment of observation must be deduced; then take from the Nautical Almanac the sun's declination at that instant, by which the distance of the elevated pole is to be found. Correct the observed altitude of the sun for the depression of the horizon and semi-diameter, and his apparent altitude will be obtained, which must be diminished by refraction, to have the true altitude, with which the calculation of the azimuth is to be performed, according to the rules in art. 139. Add the semi-diameter of the sun to the observed distance, when his nearest limb was brought into contact with the object; but subtract it when his furthest limb was used. In these two cases, the apparent distance between the centre of the sun and the object will be obtained; the depression of the horizon must also be subtracted from the altitude of the object, and the remainder will be its apparent altitude, which, with the apparent altitude and distance of the sun, are to be used in finding the azimuths, according to the rules given in the following article.

143. Write, in the following order, the apparent distance, and the apparent altitudes of the sun and object; add these three quantities together, and take half their sum; also take the difference of that half sum and the apparent distance. Then take the arithmetical complement of the logarithm cosines of the apparent altitudes of the sun and object. Write below these two complements, the logarithm cosines of the half sum, and the difference of the half sum, and the apparent distance. Half the sum of these four logarithms

will be the logarithm cosine of the half difference of the azimuths of the sun and object. Double of the corresponding angle will be the difference of the azimuths required.

Suppose, for a moment, that we face the elevated pole, and remark whether the vertical circle of the sun is to the right or left of that pole; and also whether the object of which the distance had been taken is to the right or the left of this vertical circle. When the vertical circle of the sun is on the left of the elevated pole, and the object on the left of that circle, add the difference of the azimuths to the azimuth of the sun. The same addition must also be made, under the same circumstances on the right of the elevated pole; but when the sun is on the right of the elevated pole, and the object on the left of his vertical circle, and reciprocally, the difference between the sun's azimuth, and that which results from the calculation, must be taken. The azimuth of the object calculated according to these rules, will always be reckoned to commence at the elevated pole, and in the same direction as the sun's azimuth: this rule is general when the sum of the results of the two calculations is taken; but in the case in which they have been subtracted from each other, and the difference of the azimuths is greater than the azimuth of the sun, then the azimuth of the object ought to be reckoned in a contrary direction to that of the sun; that is, the one will be towards the east, and the other towards the west.

EXAMPLE.

On the 10th of July 1792, at 7 in the morning, being in south latitude $7^{\circ} 31'$, and east longitude $153^{\circ} 10'$, the altitude of the sun's lower limb was observed to be $10^{\circ} 30'$; at the same time, the distance between the summit of a distant mountain and his nearest limb was taken, and found to be equal to $95^{\circ} 16'$. This mountain was situated on the left

of the vertical circle of the sun, and the observed altitude of its most elevated part was, at the instant of the observation, $3^{\circ} 20'$; the elevation of the observer's eye was $20\frac{1}{4}$ feet. Required the bearing of the mountain.

Latitude, South	-	-	-	-	$7^{\circ} 31'$
East Longitude	-	-	-	-	$153 10$
In time	-	-	-	-	$10^h 37'$
Estimated time at the place of observation	-	-	-	-	$19 0$
Time at the first meridian	-	-	-	-	$8^h 23'$
Declination of the \odot .	-	-	-	-	$22^{\circ} 14' N.$
Distance of the elevated pole	-	-	-	-	$112 14$
Observed altitude of the \odot .	-	-	-	-	$10^{\circ} 30'$
Elevation of the eye $20\frac{1}{4}$ feet.	Depression	-	-	-	$- 4$
					$10^{\circ} 26''$
Semi-diameter of the \odot .	-	-	-	-	$+ 16$
Apparent altitude of the \odot .	-	-	-	-	$10^{\circ} 42'$
Refraction	-	-	-	-	$- 5$
True altitude of the \odot .	-	-	-	-	$10^{\circ} 37'$
Distance of the nearest limb of the \odot .	-	-	-	-	$95^{\circ} 16'$
Semi-diameter of the \odot .	-	-	-	-	$+ 16$
Distance of the centre of the \odot .	-	-	-	-	$95^{\circ} 32'$
Altitude of the mountain	-	-	-	-	$3^{\circ} 20'$
Elevation of the eye $20\frac{1}{4}$ feet.	Depression	-	-	-	$- 4$
Apparent altitude of the mountain	-	-	-	-	$3^{\circ} 16'$

Calculation of the Azimuth of the ☉.

Polar distance of the ☉.	112° 14'		
True altitude of the ☉.	16° 37'	comp. cos.	0.00750
Latitude - - -	7° 31'	comp. cos.	0.00375
Sum - - -	130° 22'		
Half sum - - -	65° 11'	- cos.	9.62296
Polar distance — $\frac{1}{2}$ Sum	47° 3'	- cos.	9.83358
	Sum - - -		19.46759
	Half Sum - -	cos.	9.73379
	Half azimuthal angle		57° 12'
<i>Double.</i> The sun is from the South -			114° 24'E.

Calculation of the difference of the azimuths.

Appar. distance of the ☉.	95° 32'		
Appar. altitude of the ☉.	10° 42'	comp. cos.	0.00762
Appar. alt. of the mount.	3° 16'	comp. cos.	0.00071
Sum - - -	109° 30'		
Half sum	54° 45'	- cos.	9.76129
Appar. dist. — Half sum	40° 47'	- cos.	9.87920
	Sum - - -		19.64882
	Half sum	cos.	9.82441
	Half diff. of the azimuths		48° 8'
The mount. on the left of the vertical circle of ☉. difference of azimuths.		}	96° 16'
The ☉. to the left of the elevated pole, remains to the south.			114° 24'E.
<i>Add.</i> THE MOUNTAIN was to the S.			210° 40'E.
<i>Subtract</i> 180° - - - or to the N.			30° 40'W

The latitude in this example is south, consequently the

south pole is the elevated pole; and all the bearings that are immediately derived from calculation, ought to be reckoned from that pole. The azimuth of the sun is $114^{\circ} 24'$, and because the observation was made in the morning, his bearing is south $114^{\circ} 24'$ east; the vertical circle of the sun was therefore on the left of the elevated pole. But at the time of the observation, the mountain was also on the left of the vertical circle; hence the azimuth of the sun must be added to the difference of the azimuths; this sum will be the bearing of the mountain, reckoned from the south pole towards the east, the same way as the azimuth of the sun is counted. In the present case, the sum of the two quantities is equal to $210^{\circ} 24'$, and it is greater than 180° , which shews that the mountain is beyond the north pole, and to the left of that pole; consequently 180° must be subtracted from it, and the remainder will be the bearing of the mountain, or north $30^{\circ} 24'$ west, as above.

144. It has been said that the azimuth of the sun might be obtained to nearly $2'$; the difference between his azimuth and that of the terrestrial object may be equally ascertained to $2'$ or $3'$. Consequently, if we conform to the rules that have been given relative to the circumstances under which the observation should be made, we may be certain that the astronomical bearings resulting from the calculation will not be affected with an error of more than 4 or 5 minutes.

145. When the sun passes the meridian at a less altitude than 60° , it is possible, as already remarked, to obtain the astronomical bearing of a terrestrial object, by an observation made within an hour and a half of noon. These bearings may even be observed very near the passage of the sun over the meridian. In this case, the time corresponding to the observed distance between the sun and the object must be reckoned on a seconds watch, the gain or loss of which, with respect to true time, had been ascertained near the time at

which this distance was taken. The gain or loss of the watch will serve to find the time which ought to be reckoned at the place of observation of the horary angle, at the moment in which the observation of the astronomical bearing was made. By means of the way made in longitude, this time must be referred to the place where the bearing is observed, and the true time corresponding to the distance between the sun and the object will be ascertained. If this distance has been taken before the passage of the sun over the meridian, by taking its complement to 24 or 12 hours, we shall have the horary angle of the sun; but when the observation is made after noon, the horary angle will be equal to the time at the place of observation. By means of the longitude, the time at the first meridian may be calculated; this time will then serve to find the sun's declination, by which the distance of the elevated pole may be obtained. The polar distance of the sun, the complement of the latitude, and the horary angle of the sun, are the three data, with which the azimuth must be calculated, the following are the rules to be observed.

146. Write down the polar distance of the sun, and below it the complement of the latitude; take the sum and difference of these two quantities. Write, in succession, the half sum and the half difference, and below them write the horary angle, and take its half. Add together the arithmetical complement of the logarithm sine of the half sum, the logarithm sine of the half difference, and the logarithm cotangent of half the horary angle; the sum of these three will be the logarithm tangent of an arc which is called the first angle. Write down on the right hand of the former logarithms, the arithmetical complement of the logarithm cosine of the half sum, the logarithm cosine of the half difference, and the logarithm cotangent of half the horary

angle. Add these three logarithms together, and the sum will be the logarithm tangent of a second angle, the arc corresponding to which must be taken from the tables.

It is to be remarked that these two calculations have a common logarithm, and that there is only to look in the tables for five logarithms. The calculations will be much abridged if the seconds of all the given quantities are suppressed, and the logarithms taken only to five places of decimals. These given quantities may be placed as in the following example; then immediately after taking the arithmetical complement of the logarithm sine of the half sum, that of its cosine, which it by its side, may be taken; in the same manner the logarithm sine and cosine of the half difference may be taken, at one opening of the tables.

When the sun passes the meridian towards the depressed pole, add the 1st and 2nd angles, that have been found by the calculation, together, their sum will be the sun's azimuth, which will be reckoned from the elevated pole; that is, in this case, from the side opposite the passage of the sun over the meridian: it will, therefore, be greater than 90° , and often near 180° .

When the sun passes the meridian towards the elevated pole, take the difference of the two angles found by the calculation; this difference will be the sun's azimuth, which will be reckoned from the elevated pole, that is, from the side on which the sun passes the meridian, and in this case it will always be less than 90° .

The difference of the azimuths of the sun and the observed object, is to be calculated by the rules in art. 142, and the azimuth of the object, or its bearing, may be obtained in the same manner as when the sun's azimuth was calculated from his altitude.

These rules are illustrated by the following example.

EXAMPLE.

On the 17th of June 1792, being in south latitude $22^{\circ} 53'$, and east longitude $164^{\circ} 43'$, when the time by the watch was $2^h 25' 31''$, the distance of the nearest limb of the sun from the most elevated summit of the isle of Pines, situated at the south-east extremity of New Caledonia, was found to be $85^{\circ} 51'$. This island was on the right of the vertical circle of the sun. The altitude of his lower limb at the same instant was $43^{\circ} 41'$; that of the object $5^{\circ} 10'$; and the eye of the observer was elevated 20 feet above the surface of the sea.

It was known, from observations of the sun's altitude made in the morning, that the watch was $2^h 2' 27''$ before true time; the place where the bearing was observed was $2'$ of a degree, or $8'$ of time, to the west of that where the horary angle had been ascertained.

Time by the watch	-	-	-	-	$2^h 25' 31''$
Before true time. Subtract.	-	-	-	-	$2 \quad 2 \quad 27$
Time at the place of the horary angle	-	-	-	-	$0^h 23' 4''$
The place of the bearing to the W.	-	-	-	-	$- \quad 8$
True time of the bearing, or horary angle	-	-	-	-	$0^h 22' 56''$
Horary angle in degrees	-	-	-	-	$5^{\circ} 44'$
Latitude S.	-	-	-	-	$22^{\circ} 53'$
Complement of latitude	-	-	-	-	$67 \quad 7$
Longitude East	-	-	-	-	$164^{\circ} 43'$
Longitude in time	-	-	-	-	$\{ 10^h 59'$
Time of observing the bearing	-	-	-	-	$\{ 0 \quad 23$
Time at the first meridian	-	-	-	-	$13^h 24'$
Declination of the ☉. N.	-	-	-	-	$23^{\circ} 25'$
Distance of the ☉. from the elevated pole	-	-	-	-	$113 \quad 25$

Observed altitude of the ☉. - - - $43^{\circ} 11'$

Elevation of the eye $20\frac{1}{2}$ feet. Depression. - - - 4

$43^{\circ} 7'$

Semi-diameter of the ☉. - - - $+ 16$

Apparent altitude of the ☉. - - - $43^{\circ} 23'$

Distance of the nearest limb of the ☉. - - - $85^{\circ} 51'$

Semi-diameter of the ☉. - - - $+ 16$

Distance of the centre of the ☉. - - - $86^{\circ} 7'$

Observed altitude of the mountain. - - - $5^{\circ} 10'$

Elevation of the eye $20\frac{1}{2}$ feet. Depression. - - - 4

Apparent altitude of the mountain. - - - $5^{\circ} 6'$

Calculation of the Sun's Azimuth.

Dist of ☉ from elev. pole $125^{\circ} 25'$

Comp. of the latitude $7^{\circ} 7'$

Sum - - - $180^{\circ} 32'$

Difference - - - $46 18$

Half sum - - - $90^{\circ} 16'$ comp. sin. 0.00000 comp. cos. 2.31216

Half difference - - - $23 9$ - - - sin. 9.59455 - - - cos. 9.96355

Horary angle - - - $5 44$

Half horary angle - - - $2 52$ cotang. 1.30038 cotang. 1.30038

tang. 0.89493 tang. 3.59608

1st angle $82^{\circ} 44'$ 2nd angle $89^{\circ} 59'$

1st angle $82 44$

The sun passes the meridian towards the depressed pole. $172^{\circ} 43'$

The sun remains to the S. - - - $172^{\circ} 43' W$

Calculation of the difference of the Azimuths.

Appar. distance of the \odot .	86° 7'		
Appar. altitude of the \odot .	43° 23'	comp. cos.	0.13860
Appar. alt. of the mount.	5° 6'	comp. cos.	0.00172
Sum	134° 36'		
Half sum	67° 18'	cos.	9.58648
Appar. dist. — Half sum	18° 49'	cos.	9.97615
Sum			19.70295
Half sum		cos.	9.85147
Half diff. of the azimuths	44° 44'		

The Mount ON THE RIGHT of the vertical circle of \odot . difference of azimuths. } 89 28

The \odot ON THE RIGHT of the elevated pole, remains from the south. } 172 43 W

Add — THE MOUNTAIN is *from* the S. 262 11' W
 Subtract 180 — — — or from the N. 82 11 E.

The observation was made after noon, consequently the sun was on the right of the south pole, which, in the present case, was the elevated pole; but the mountain was also on the right of the vertical circle of the sun, the difference of the azimuths must therefore be added to the azimuth of the sun. The sum $262^{\circ} 11'$ is an arc reckoned from the south pole towards the west, or in the same direction as the sun's azimuth. This arc being greater than 180° , terminates beyond the north. Therefore 180° must be subtracted from the sum that has been found; then the true bearing of the mountain is north $82^{\circ} 11'$ east, as shewn above.

147. Astronomical bearings observed near noon are not, in general, susceptible of such accuracy as those which result from observations made when the sun is but a little elevated

above the horizon; but they are always preferable to bearings observed with the compass, provided the rules given in art. 141, relative to the circumstances under which the observations ought to be made, are attended to. When the elevation of the sun does not exceed 40° , the error with which they may be affected will never be more than $6'$ or $8'$; and if the sun's altitude approach to 60° , the error will not surpass $12'$ or $14'$. It may now be useful to remark that the errors will, almost always, be much less than the quantities here assigned them.

148. When astronomical bearings are to be employed in the construction of hydrographical or marine charts, an object must be chosen on the shore which is best defined, and most advantageously situated with respect to the vertical circle of the sun and the bearing observed. Now, when this observation is made, several observers should take the angular distances between the object fixed upon, and all the other objects that are to be placed on the chart, with reflecting instruments. It will be easy to conclude, from all these angles, the bearing of each particular object. The errors in the angular distances, measured with octants or common sextants, will never be more than $1'$ or $2'$. Bearings observed in this manner, will therefore have nearly the precision of the astronomical bearings from which they have been derived; and the charts constructed from these bearings will consequently possess very great accuracy.

149. Circumstances do not always permit astronomical bearings to be observed; then we are obliged to take the bearings with the compass, but in this case the following method should be adopted: it possesses the advantage of remedying a part of the imperfections of which bearings taken with the compass is susceptible. It may be supposed, from what has been said, that the declination of the magnetic needle has been determined as accurately as possible

by the method explained in this chapter. Choose a very distinct object, and sufficiently distant that its bearing may not be sensibly changed during the short time occupied by the observation; those objects that are seen very nearly, either before or behind the vessel, ought to be preferred. Observe, first, the bearing of the chosen object with the compass; then derange its sights, and take a second bearing. Three or four observations may be taken in the same way, and there will be obtained from the mean of all these, a final bearing, which will be much more exact than if a single observation only had been made. While this bearing is taken, other observers should measure, with reflecting instruments, as in the preceding case, the angular distances between the object fixed upon, and all the others which are to be inserted in the chart; these angles will give the bearing of each of the objects in particular. The angular distances may be considered as very exact; since the errors of all the bearings will be nearly the same, and consequently will have little influence on the relative positions which were derived from these bearings.

NOTES.

THE object of the preceding treatise was not to show the manner of making astronomical observations at sea; but to explain at some length the methods of calculating them. It was thought requisite to add to the rules that have been prescribed, some elucidations proper for facilitating their application. It is with this view that we have endeavoured to explain, by such simple reasonings as might be understood by all classes of readers, the different rules that are derived from the elementary principles of the sphere; but it was indispensable to refer those demonstrations which involve the more complicated theory of spherical triangles, to the end of the work; and this is the place for fulfilling the engagement which has been made. It shall be shown how the formulæ are to be found, according to which the different calculations of the various examples that have been given are performed; then the principles of the construction of the new tables, for referring an altitude taken in any place to another place a little distant from the former, and situated under the same meridian, shall be developed. It will be seen, and perhaps not without interest, that these tables may also be used for correcting the observed altitudes of the sun and moon, in order to obtain the reduced distance of these two bodies, upon the hypothesis that the earth is a spheroid flattened at the poles. We shall give, lastly, a demonstration of a very simple method, which has been mentioned in the 2nd chapter, for calculating the inclination of the visual ray, when it meets the shore

by which the horizon is bounded. This will explain the reasons which have caused it to be used.

Let $z h n o$ figs. 1, 2, be the meridian, z the zenith, and $h o$ the horizon. If p be the elevated pole and $e q$ the equator, the arc $p o$ will be equal to the latitude, and $z p$ will be its complement to 90° . Suppose the sun to be at the point s of the parallel to the equator $a v$, the arc $p s$ will be the distance of the sun from the elevated pole, and the arc $s i$ will be his altitude; consequently $s z$, which is his zenith distance, will be the complement of his altitude. The triangle $z p s$ is formed by the polar distance $s p$, and by the sides $z p$ and $z s$, which are the complements of the latitude and altitude. It ought to be observed that the angle $z p s$, formed by the circle of declination $s p$ and the meridian, is the horary angle of the sun s ; the angle $p z s$ formed by the vertical circle and the meridian, is the azimuth; lastly, the angle $z s p$, formed by the vertical circle $z s$ and the circle of declination $p s$, is the angle of variation. Whether it be required to find the horary angle or azimuth of a heavenly body by an observation of its altitude, or to calculate the altitude from the horary angle, or the latitude with the angle of variation, it is necessary to resolve the triangle $z p s$.

Call the altitude $s i$ of a heavenly body h , its distance from the elevated pole p , the latitude $p o$ call L , and let A denote the horary angle $z p s$, a the azimuth $p z s$, and v the angle of variation $z s p$, which will give denominations to all the parts of the triangle $z p s$; and these are employed in the following calculations.

Trigonometry teaches the method of finding a great number of different formulæ, any of which would be proper for calculating one of these six quantities when three of the others are given. Of these known methods, those have been chosen, which are generally regarded as the most simple; and new ones have been introduced, only when they appeared to be still more proper than the old ones, either for simplifying the calculations, or rendering the operations more uniform.

NOTE I.

Calculation of the horary angle.*

We have generally, in the triangle zps ,

$$\cos zps = \frac{\cos zs - \cos pz \cdot \cos ps}{\sin pz \cdot \sin ps};$$

but $zps = h$, $zs = 90^\circ - \delta$, $pz = 90^\circ - L$ and $ps = D$; we shall therefore have the following equation:

$$\cos h = \frac{\sin \delta - \sin L \cdot \cos D}{\cos L \cdot \sin D}.$$

According to the rules of trigonometry,

$$\cos h = 1 - 2 \sin^2 \frac{1}{2} h,$$

and

$$\sin L \cdot \cos D = \sin (L + D) - \cos L \cdot \sin D.$$

Substituting in the preceding equation their values for the $\cos h$, and the $\sin L \cdot \cos D$, we shall have,

$$2 \sin^2 \frac{1}{2} h = \frac{\sin (L + D) - \sin L}{\cos L \cdot \sin D}.$$

But,

$$\sin (L + D) - \sin L = 2 \cos \frac{1}{2} (L + D + L) \cdot \sin \frac{1}{2} (L + D - L);$$

from which it follows that

$$\sin^2 \frac{1}{2} h = \frac{\cos \frac{1}{2} (L + D + L) \cdot \sin \frac{1}{2} (L + D - L)}{\cos L \cdot \sin D};$$

otherwise

$$\sin \frac{1}{2} h = \left(\frac{\cos \left(\frac{L + D + L}{2} \right) \sin \left(\frac{L + D - L}{2} \right)}{\cos L \cdot \sin D} \right)^{\frac{1}{2}}.$$

The rules in art. 75 are derived from this formula, which is

* See art. 75.

Borda's. The same number of logarithms are required as by the other methods; but the preparation for the calculation is rather more simple.

NOTE II.

Calculation of the Altitude.*

THE EQUATION of the preceding problem,

$$\cos h = \frac{\sin H \cos L \cos D}{\cos L \sin D},$$

gives us

$$\sin H = \sin L \cos D + \cos h \cos L \sin D;$$

but $\cos h = 2 \cos^2 \frac{1}{2} h - 1$, and by substituting this value in the equation we have

$$\sin H = \sin L \cos D + 2 \cos^2 \frac{1}{2} h \cos L \sin D - \cos L \sin D;$$

or else

$$\sin H = 2 \cos^2 \frac{1}{2} h \cos L \sin D - \sin (D - L).$$

But on the other hand,

$$\sin H = 1 - 2 \cos^2 \frac{1}{2} (90^\circ + H),$$

and

$$\sin (D - L) = 2 \cos^2 \frac{1}{2} (90^\circ - D - L) - 1.$$

Substituting these values in the preceding equation, we have

$$\cos^2 \frac{1}{2} (90^\circ + H) = \cos^2 \frac{1}{2} (90^\circ - D - L) - \cos^2 \frac{1}{2} h \cos L \sin D;$$

from which we obtain

$$\sin M = \frac{\cos \frac{1}{2} h (\cos L \sin D)}{\cos \frac{1}{2} (90^\circ - D - L)}$$

and

$$\cos \frac{1}{2} (90^\circ + H) = \cos \frac{1}{2} (90^\circ - D - L) \cos M.$$

* See art 80.

Borda has given, in his *Treatise on the Reflecting Circle*, the following formula for resolving the same problem.

$$\tan g M = \frac{\sin \frac{1}{2} h. (\cos L. \sin D)^{\frac{1}{2}}}{\sin \frac{1}{2} (90^\circ - L + D)},$$

and

$$\sin \frac{1}{2} (90^\circ - h) = \frac{\sin \frac{1}{2} (90^\circ - L + D)}{\cos M}.$$

It ought however, to be observed that the $\sin \frac{1}{2} (90^\circ - h) = \cos \frac{1}{2} (90^\circ + h)$; but the altitude is more easily found from $\frac{1}{2} (90^\circ + h)$ than from $\frac{1}{2} (90^\circ - h)$, and this is one of the advantages of the formula which has been adopted. Then, according to the present arrangement of the calculation, we likewise obtain the $\cos \frac{1}{2} (90^\circ - D - L) \cdot \cos M$ with greater facility than

$$\frac{\sin \frac{1}{2} (90^\circ - L + D)}{\cos M};$$

the method which we have adopted, therefore, simplifies the calculation a little.

NOTE III.

Calculation of Latitude from two Altitudes of the Sun taken out of the Meridian, and the interval of time between the observations.*

Let s , fig. 1, and 2, be the place of the sun where the observation of the less altitude is taken, s' his place at the instant of the greater altitude. Suppose the two places of the sun to be joined by the arc of a great circle ss' ; preserving the denominations that have been adopted, and denoting the greater altitude $s'i$ by n' , and, the interval of time between the observations, reduced into degrees by t . The angle sps' , formed by the two circles of declination, ps and ps' , is equal to t , and the arc of the great circle ss' is the distance of the two places of the sun.

This being supposed, in the triangle sps' , which may be considered as isosceles for abridging the operations, calculate the

* See art. 67.

distance ss' , and the first angle at the sun $ps's'$ formed by this distance, and the circle of declination corresponding to the less altitude. In the triangle zss' , there are known the three sides, one of which is the distance of the two places of the sun, and the other two are the complements of the two observed altitudes; we can therefore calculate the second angle at the sun zss' , formed by the circle of the distance and the sun's vertical circle at the time of observing the less altitude. The difference of the two angles at the sun, $ps's' - zss'$, fig. 1, or their sum $ps's' + zss'$, fig. 2, is the angle of variation zsp of the triangle zps , which serves to calculate the side zp , or the latitude.

Distance of the sun's places, and the first angle at the sun. If we suppose the two declinations to be the same, the sides ps and ps' will be equal, and the triangle $ps's'$ will be isosceles; then, by the common rules of trigonometry, we have

$$\sin \frac{1}{2} ps's' = \sin \frac{1}{2} t \cdot \sin D,$$

and

$$\text{tang } ps's' = \frac{\cot \frac{1}{2} t}{\cos D}.$$

These are the two equations from which the distance ss' and the first angle at the sun $ps's'$ are calculated.

Second angle at the sun. In the triangle zss' , we have the equation

$$\cos zss' = \frac{\cos zs' - \cos zs \cdot \cos ss'}{\sin zs \cdot \sin ss'}.$$

If we employ the denominations that have been adopted, this will become

$$\cos zss' = \frac{\sin H' - \sin H \cdot \cos ss'}{\cos H \cdot \sin ss'};$$

but

$$\cos zss' = 1 - 2 \sin^2 \frac{1}{2} zss',$$

and

$$\sin H \cdot \cos ss' = \sin (H + ss') - \cos H \cdot \sin ss'.$$

Substituting these values of the $\cos zss'$, and $\sin H \cdot \cos ss'$

in the preceding equation, and making the necessary reductions, we have

$$2 \sin^2 \frac{1}{2} Z S S' = \frac{\sin (H + S S') - \sin H'}{\cos H \sin S S'}$$

We have also

$$\sin (H + S S') - \sin H' = 2 \cos \frac{1}{2} (H + S S' + H') \sin \frac{1}{2} (H + S S' - H'),$$

therefore

$$2 \sin^2 \frac{1}{2} Z S S' = \frac{2 \cos \frac{1}{2} (H + S S' + H') \sin \frac{1}{2} (H + S S' - H')}{\cos H \sin S S'};$$

or else

$$\sin \frac{1}{2} Z S S' = \left(\frac{\cos \left(\frac{H + S S' + H'}{2} \right) \sin \left(\frac{H + S S' - H'}{2} \right)}{\cos H \sin S S'} \right)^{\frac{1}{2}}$$

This formula, by which the second angle at the sun is found, is analogous to that by which the horary angle is calculated. The given quantities should be disposed in the same manner, and the preparation for the calculation is as simple.

Latitude. The triangle Z S P gives us

$$\cos P S Z = \frac{\cos Z P - \cos P S \cos Z S}{\sin P S \sin S Z'}$$

$$\cos V = \frac{\sin L - \cos D \sin H}{\sin D \cos H},$$

from which we obtain

$$\sin L = \cos V \sin D \cos H + \cos D \sin H.$$

It is known that $\cos V = 2 \cos^2 \frac{1}{2} V - 1$, and by substituting this value we have

$$\sin L = 2 \cos^2 \frac{1}{2} V \sin D \cos H - \sin (D - H):$$

but

$$\sin L = 1 - 2 \cos^2 \frac{1}{2} (90^\circ + D)$$

and

$$\sin (D - H) = 2 \cos^2 \frac{1}{2} (90^\circ - D - H) - 1.$$

Hence by the substitution of these two values of $\sin L$, and $\sin (D - H)$, we shall have

$$\cos^2 \frac{1}{2} (90^\circ + L) = \cos^2 \frac{1}{2} (90^\circ - D - H) - \cos^2 \frac{1}{2} \sin D \cos H;$$

from which we derive

$$\sin M = \frac{\cos \frac{1}{2} (\sin D \cos H)}{\cos \frac{1}{2} (90^\circ - D - H)},$$

and

$$\cos \frac{1}{2} (90^\circ + L) = \cos \frac{1}{2} (90^\circ - D - H) \cos M.$$

This formula is analogous to that for calculating the altitude, and consequently possesses the same advantages.

NOTE IV.

Calculation of the true distance between the Moon and the Sun or a Star.*

Let z be the zenith, fig. 3, zH the moon's vertical circle, and zo that of the sun. If L be the apparent place of the moon, and L' the true place; also if s be the apparent place of the sun and s' the true place, SL will be the apparent distance, and $S'L'$ the true distance. Call H the apparent altitude HL of the moon, and H' the true altitude; B the apparent altitude of the sun, and B' the true altitude; Δ the apparent distance, and x the true distance required.

This being supposed, in the triangle LZS , formed by the apparent distance of the two heavenly bodies, and the apparent zenith distance of each body, we have

$$\cos Z = \frac{\cos \Delta - \sin H \sin B}{\cos H \cos B}.$$

In the triangle $L'ZS'$ composed of the true altitudes and the true distance, we have

* See art 113.

$$\cos Z = \frac{\cos x - \sin H' \sin B'}{\cos H' \cos B'}$$

therefore

$$\frac{\cos H' \sin B'}{\cos H' \cos B'} = \frac{\cos x - \sin H' \sin B'}{\cos H' \cos B'}$$

but

$$\sin H' \sin B' = \cos H' \cos B' - \cos (H' + B'),$$

and

$$\sin H' \sin B' = \cos H' \cos B' - \cos (H' + B').$$

Substituting these values of $\sin H' \sin B'$, and of $\sin H' \sin B'$, we shall have

$$\frac{\cos \Delta + \cos (H' + B)}{\cos H' \cos B} = \frac{\cos x + \cos (H' + B)}{\cos H' \cos B},$$

and

$$\cos x = \frac{\cos H' \cos B'}{\cos H' \cos B} \left\{ \cos \Delta + \cos (H' + B) \right\} - \cos (H' + B').$$

We know that

$$\cos \Delta + \cos (H' + B) = 2 \cos \frac{1}{2} (H' + B + \Delta) \cos \frac{1}{2} (H' + B - \Delta),$$

$$\cos (H' + B) = 2 \cos^2 \frac{1}{2} (H' + B) - 1,$$

$$\cos x = 1 - 2 \sin^2 \frac{1}{2} x.$$

By substituting these values in the equation, we shall have

$$\sin^2 \frac{1}{2} x = \cos^2 \frac{1}{2} (H' + B') \frac{\cos \frac{1}{2} (H' + B + \Delta) \cos \frac{1}{2} (H' + B - \Delta) \cos H' \cos B'}{\cos H' \cos B}.$$

Now making

$$\sin M = \frac{\left(\frac{\cos \frac{1}{2} (H' + B + \Delta) \cos \frac{1}{2} (H' + B - \Delta) \cos H' \cos B'}{\cos H' \cos B} \right)^{\frac{1}{2}}}{\cos \frac{1}{2} (H' + B')};$$

we shall have, lastly,

$$\sin \frac{1}{2} x = \cos \frac{1}{2} (H' + B') \cos M.$$

This formula is known by the name of *Lorda's*; and is generally used when the tables of common logarithms only are employed.

NOTE V.

*Calculation of the Sun's Azimuth and Amplitude.**

The triangle P S P, fig. 1, and 2, gives us the equation.

$$\cos P Z S = \frac{\cos P S \cos P Z \cos S Z}{\sin P Z \sin S Z},$$

or

$$\cos A = \frac{\cos D + \sin L \sin H}{\cos L \cos H};$$

but

$$\cos A = 2 \cos^2 \frac{1}{2} A - 1,$$

and

$$\sin L \sin H = \cos L \cos H - \cos (L + H).$$

Substituting these values of the $\cos A$ and the $\sin L \sin H$, in the preceding equation, we have

$$2 \cos^2 \frac{1}{2} A = \frac{\cos D + \cos (L + H)}{\cos L \cos H}.$$

According to the known rules,

$$\cos D + \cos (L + H) = 2 \cos \frac{1}{2} (D + L + H) \cos \frac{1}{2} (L + H - D),$$

we have therefore

$$\cos^2 \frac{1}{2} A = \frac{\cos \frac{1}{2} (D + L + H) \cos \frac{1}{2} (L + H - D)}{\cos L \cos H},$$

or

$$\cos \frac{1}{2} A = \left(\frac{\cos \left(\frac{D + L + H}{2} \right) \cos \left(\frac{D + L + H - D}{2} \right)}{\cos L \cos H} \right)^{\frac{1}{2}}$$

This formula is extracted from *Borda's Treatise on the reflecting circle*. It appears from inspection of figs. 1 and 2, that the angle P Z S is always formed by the vertical circle of the sun, and that part of the meridian adjacent to the elevated pole.

* See art 132.

Thus the calculated azimuth A ought, in all cases, to be reckoned from the pole which is above the horizon, as has been observed in art. 132.

Let the equation be resumed,

$$\cos A = \frac{\cos D - \sin L \sin H}{\cos L \cos H}.$$

Suppose that the sun is in the horizon, then H becomes equal to nothing, $\cos H = 1$, and we have,

$$\cos A = \frac{\cos D}{\cos L}.$$

If the declination d be employed instead of the polar distance, we have $\sin d = \cos D$. On the other hand $90^\circ - A$ or $A - 90^\circ$ is the amplitude of the heavenly body, we shall therefore have

$$\text{sine amplitude} = \frac{\sin d}{\cos L}.$$

This is the formula that has been employed in calculating the amplitude of the sun. (See art. 136.)

The rules which have been given in art. 146, for calculating the sun's azimuth by means of the horary angle, and derived from Napier's two well known analogies, which serve to calculate one of the angles of a spherical triangle, when two sides and their contained angle are given. In effect, in figs. 1, and 2, in the triangle PZS , knowing the angle $ZPS = h$, the side $PZ = 90^\circ - L$ and the side $PS = D$, we have

$$\text{tang } \frac{1}{2}(A \sim v) = \cot \frac{1}{2} h \frac{\sin \frac{1}{2}(90^\circ - L \sim D)}{\sin \frac{1}{2}(90^\circ - L + D)}$$

and

$$\text{tang } \frac{1}{2}(A + v) = \cot \frac{1}{2} h \frac{\cos \frac{1}{2}(90^\circ - L \sim D)}{\cos \frac{1}{2}(90^\circ - L + D)}.$$

When the sun passes the meridian towards the depressed pole, fig. 1, A is greater than v , and we shall have

$$A = \frac{1}{2}(A - v) + \frac{1}{2}(A + v),$$

that is, the azimuth is equal to the sum of the first and second angles.

If the sun pass the meridian towards the elevated pole, fig. 2, Λ is, on the contrary, less than v , and we shall have

$$\Lambda = \frac{1}{2} (\Lambda + v) - \frac{1}{2} (v - \Lambda),$$

that is, the azimuth is equal to the difference of the first and second angles. It ought to be understood, by inspection of figs. 1 and 2, that the angle Λ is always reckoned, as in the preceding calculation, to commence at the elevated pole.

NOTE VI.

*Calculation of the difference between the Azimuth of the Sun and the Azimuth of a terrestrial object.**

Let s be the sun's apparent place, m the summit of the mountain of which the distance sm from the sun has been observed. Let the apparent distance sm be denoted by Δ ; the apparent altitude sh of the sun by h ; and the apparent altitude of the object m , by o ; also let z be the difference of the azimuths, or the angle szm . In the triangle zsm , we have

$$\cos szm = \frac{\cos sm - \cos zs \cos zm}{\sin zs \sin zm},$$

or otherwise,

$$\cos z = \frac{\cos \Delta - \sin h \sin o}{\cos h \cos o};$$

but

$$\cos z = 2 \cos^2 \frac{1}{2} z - 1,$$

and

$$\sin h \sin o = \cos h \cos o - \cos (h + o).$$

Substituting these values, we have

$$2 \cos^2 \frac{1}{2} z = \frac{\cos \Delta - \cos (h + o)}{\cos h \cos o}.$$

* See art. 142.

According to the rules of trigonometry,

$$\cos \Delta - \cos (H + O) = 2 \cos \frac{1}{2}(\Delta + H + O) \cdot \cos \frac{1}{2}(H + O - \Delta),$$

we shall therefore have

$$\cos^2 \frac{1}{2} Z = \frac{\cos \frac{1}{2}(\Delta + H + O) \cdot \cos \frac{1}{2}(H + O - \Delta)}{\cos H \cdot \cos O},$$

and finally

$$\cos \frac{1}{2} Z = \left(\frac{\cos \left(\frac{\Delta + H + O}{2} \right) \cos \left(\frac{\Delta + H + O - \Delta}{2} \right)}{\cos H \cdot \cos O} \right)^{\frac{1}{2}}.$$

This last formula is *Borda's*; it is analogous to that which serves for calculating the sun's azimuth by means of his altitude. The quantities that are required to obtain the astronomical bearing of any object from it, may therefore generally be found by two calculations very nearly similar.

NOTE VII.

Principles of the construction of the Tables for finding the correction of the less of two altitudes, taken out of the meridian, in order to find the latitude.

Let A be the azimuth, L the latitude, and H the altitude of the sun. If δH be the change in altitude answering to a small change in latitude δL , we have, by the known rules, $\delta H = \mp \delta L \cdot \cos A$; the sign *minus* is to be used when the sun passes the meridian towards the depressed pole, and the sign *plus* when he passes it towards the elevated pole. In fact, in the first case, fig. 1, the angle A is greater than 90° and its cosine is negative; in the second case, fig. 2, the angle A is less than 90° , and its cosine is positive. Now, if the effect which the change of latitude ought to produce upon the meridian altitude of the sun be considered, it will be seen that the errors in altitude, when the sun is above the meridian, take place in the same sense as those of the meridian altitudes; hence, if the azimuth A be reckoned to commence at the side where the sun passes the meridian, we

shall always have $\delta H = \text{diff. merid. alt.} \times \cos A$. When the change in latitude increases the meridian altitude, the value of δH must be added to the observed altitude, and subtracted in the contrary case. When it is greater than 90° , the cosine of A becomes negative; then δH should be employed with a different sign from that of the variation of the meridian altitude. This last distinction of case may be made to disappear, by employing for the $\cos A$ its value $1 - \text{vers } A$, and we shall have $\delta H = \text{diff. merid. alt.} - \text{diff. merid. alt.} \times \text{vers } A$. When A is less than 90° , the versed sine of A will be less than unity, and the correction will be positive. When it is greater than 90° , the vers. A will be greater than unity, and the correction will be negative. Tables XII and XIII are intended to calculate the versed sine of A , supposing this angle, as already observed, to be reckoned from the side on which the sun passes the meridian.

Instead of the polar distance which has hitherto been used, the declination of the sun may be employed, and let $d =$ this element. The triangle PZS , fig. 1, gives

$$\cos A = \frac{\sin D - \sin L \sin H}{\cos L \cos H},$$

$$\text{vers } A = \frac{\cos^2(L \sim H) - \sin d}{\cos L \cos H}.$$

When the declination is of a different denomination from the latitude, the $\sin d$ changes its sign; we shall therefore have generally

$$\text{vers } A = \frac{\cos^2(L \sim H) \pm \sin d}{\cos L \cos H},$$

The upper sign takes place when the declination is of the same denomination as the latitude, and the lower sign when the denominations are different.

The left hand page of Table XII, contains the first term $\cos(L \sim H)$; this table must be entered with the latitude L , and the altitude H . The right-hand page contains, under the term

argument, the value of the denominator $\cos L \cos H$; and it is with the argument and the declination that we find, in table XIII, the value of the second term $\frac{\sin d}{\cos L \cos H}$.

The angle A which is obtained from the preceding formula, by calculation, is to be reckoned from the elevated pole, but according to what has been said, the azimuth should be reckoned, in all cases, from the side on which the sun passes the meridian, which is sometimes that of the elevated, and sometimes that of the depressed pole; this is done in the following manner.

For the sake of abridgment, make

$$\frac{\cos (L - H)}{\cos L \cos H} = P \quad \text{and} \quad \frac{\sin d}{\cos L \cos H} = S,$$

we shall then have $\text{vers } A = P + S$.

If the declination and the latitude be of the same name, and the declination greater than the latitude, the sun passes the meridian towards the elevated pole: then we have

$$\text{vers } A = P + S.$$

When P and S have been found by the rules in art. 40, the second term must be subtracted from the first

In the case in which the latitude is greater than the declination, the sun passes the meridian towards the depressed pole, and the versed sine of the azimuth, reckoned from this pole, is $2 - \text{vers } A$; we have therefore

$$2 - \text{vers } A = 2 - P + S = (2 + S) - P.$$

The first term must be subtracted from the second increased by 2 units; as before specified.

When the declination has a denomination different from the latitude, the sun passes the meridian towards the depressed pole, and we shall have

$$2 - \text{vers } A = 2 - P - S = (2 - S) - P.$$

In the second part of Table XIII, in which the declination

and latitude are of different kinds, we have to write $2 - s$ instead of s : by this means, the first term must be subtracted from the second, but it is not necessary to increase the second term by two units.

What we have called the multiplier of the way made in latitude is, therefore, the versed-sine of the sun's azimuth reckoned from that side on which his passage over the meridian takes place: the arc which corresponds to this versed-sine is found in Table XIV.

Tables XII and XIII may also be used for correcting the altitudes observed at the same time as the distance of the moon from the sun or a star is taken, in the case in which the reduced distance of the two bodies is required on the hypothesis that the earth is an oblate spheroid. According to what has been said in art. 122, it is sufficient to calculate the altitudes with the latitude of the place where the distance was observed, diminished by the angle at the vertical. Hence, if the altitudes have been obtained directly from observation, there must be added to, or subtracted from them, the quantities by which they ought to be increased or diminished, on account of the decrease in latitude, which will be equal to the angle at the vertical. In order to render the use of Tables XII and XIII uniform, it will be necessary to consider whether a diminution in the latitude of the place where the distance is observed, tends to augment or diminish the meridian altitude of the heavenly body, the altitude of which is to be corrected. Then, the operations, relative to the angle at the vertical, are performed in the same manner as recommended with respect to the change of latitude which takes place between the observations of the altitudes taken out of the meridian, in order to ascertain the latitude. There will, in this case, be obtained, the corrected altitudes, by which the true distance of the bodies is found, on the hypothesis of the earth being a spheroid flattened at the poles.

NOTE VIII.

Means of calculating the inclination of the visual ray meeting the Shore by which the Horizon is bounded.

Let $A O$, fig. 5, be a portion of the earth's surface, and A the point of the shore which bounds the horizon where the vertical circle of a heavenly body meets it. If from the point a , which is elevated by a quantity equal to $B O$, the altitude of that body be observed, and its reflected image be brought to coincide with the point A , the inclination of the visual ray BA , which it is required to find, will be equal to the angle LBA , which this visual ray makes with the horizontal line LB . When the distance AB , or arc AO , is unknown, the angle LBA must be obtained as follows.—While one person observes the altitude of the body at the point B , another is to observe the altitude of the same body from b , at a much greater elevation than B ; the difference of the two heights bo and BO , as well as that of the two observed angles will serve to find the angle LBA .

Let i denote the angle LBA , or the correction of the observed altitude at B , and i' the angle Lba , or the correction of the altitude observed at b . Let h be the elevation of the eye, BO , and h' the elevation bo . Draw AD perpendicular to the radius CO , which is produced to b . This being supposed, the right angled triangles bAD , BAD , by the rules of trigonometry, give the two following equations:

$$\cot A b D = \tan i' = \frac{b D}{A D},$$

$$\cot A B D = \tan i = \frac{B D}{A D};$$

from which we obtain

$$\tan i' - \tan i = \frac{b D - B D}{A D}.$$

The angles i' and i never exceed a very small number of minutes; we shall therefore have, without sensible error*,

$$\sin i' (i' - i) = \frac{h' - h}{AD},$$

and

$$AD = \frac{h' - h}{\sin i' (i' - i)},$$

but

$$\tan i = \frac{BD}{AD} = \frac{BO + DO}{AD} = \frac{h + DO}{AD}.$$

On the other hand, $\tan i = \sin i' \times i$, very nearly, we shall therefore have

$$i = \frac{h}{\sin i' \cdot AD} + \frac{DO}{\sin i' \cdot AD}.$$

It ought to be remarked that DO is the versed-sine of the arc AO , of which the sine AD is known; we must therefore substitute for DO its value in a function of AD .

Supposing the radius equal to unity, we have, by the known rules,

$$\text{vers } AO = \frac{AO^2}{2} - \frac{AO^4}{24},$$

and

$$AO = \sin AO + \frac{\sin^3 AO}{24}.$$

Substituting the value of AO , given by the second equation, in the expression for the versed-sine of AO , we shall have, by neglecting the terms above the fourth powers,

$$\text{vers } AO = \frac{\sin^2 AO}{2} + \frac{3 \sin^4 AO}{24}.$$

Let a be the radius of the earth, and substitute DO in the

* This supposition is so much the more admissible as it is only required to find the angle i to within some seconds of the truth

preceding equation instead of the vers ΛO , and ΛD instead of the $\sin \Lambda O$; and we shall have

$$DO = \frac{\Lambda D^2}{2a} + \frac{3\Lambda D^4}{8a^3}$$

But we already have the equation

$$1 = \frac{h}{\sin 1'' \cdot \Lambda D} + \frac{DO}{\sin 1'' \cdot \Lambda D};$$

substituting here the value of DO , and it will give

$$1 = \frac{h}{\sin 1'' \cdot \Lambda D} + \frac{\Lambda D}{2a \cdot \sin 1''} + \frac{\Lambda D^3}{8a^3 \cdot \sin 1''};$$

but

$$\Lambda D = \frac{h' - h}{\sin 1'' (1' - 1)},$$

the preceding equation therefore becomes

$$1 = \frac{h(1' - 1)}{h' - h} + \frac{h' - h}{2a \cdot \sin^2 1'' \cdot (1' - 1)} + \frac{(h' - h)^3}{8a^3 \cdot \sin^4 1'' \cdot (1' - 1)^3}.$$

The value of the third term of this equation will always be insensible, and we may, in all cases, confine ourselves to the calculation of the first two; we shall therefore have

$$1 = \frac{h(1' - 1)}{h' - h} + \frac{h' - h}{2a \cdot \sin^2 1'' \cdot (1' - 1)}.$$

The first term may be calculated by a simple rule of proportion. As to the second, it would be easy to construct a small table of two arguments; and with the difference of the two heights of the eye, and the difference of the observed angles, it might be found, without being obliged to take proportional parts. The second term might also include the advantage of enabling us to correct the inclination of the visual ray BA for the effect of terrestrial refraction; for we have

$$\sin \Lambda O = \sin 1'' \cdot \Lambda O; \text{ from which } \Lambda O = \frac{\sin \Lambda O}{\sin 1''}, \text{ or}$$

$$\Lambda O = \frac{\Lambda D}{\sin 1''}. \text{ If it be wished to have } \Lambda O \text{ in parts of the circum-}$$

ference, AD must be used in parts of the radius; then we have

$$AO = \frac{AD}{a \sin 1''}, \text{ or else } AO = \frac{h' - h}{a \sin^2 1'' (i' - i)}; \text{ from which it fol-}$$

lows that the second term of the value of the depression i , is always equal to half the terrestrial arc comprised between the observer and the point of the shore to which the reflected image of the sun is brought: hence we might subtract from this half, the necessary quantity to have the refraction corresponding to the whole arc.

Resume the equation from which the value of i has been obtained.

$$i = \frac{h (i' - i)}{h' - h} + \frac{1}{2 a \sin^2 1''} \times \frac{(h' - h)}{(i' - i)}.$$

By adopting the differential method, and regarding i and $(i' - i)$ as variable quantities, we shall have

$$\delta i = \delta (i' - i) \frac{h}{(h' - h)} - \delta (i' - i) \frac{(h' - h)}{2 a \sin^2 1'' (i' - i)^2}.$$

The error of the first term will be less as $h' - h$ is greater, or as the point h is more elevated. The contrary will take place with respect to the second term: but as this term has a contrary sign to the first, the value of the error of i will be, in all cases, diminished by that with which this term is affected. It may therefore be concluded, in the first place, that the second term must never be neglected; and, in the second, that one of the altitudes should be observed in a very elevated situation. One of the observers should therefore be placed upon the deck of the vessel, and the other at the top of one of its masts; then the quantity $\frac{h}{h' - h}$ may be equal to $\frac{1}{5}$ or $\frac{1}{6}$, and the error of the angle i will always be less than $\frac{1}{5} \delta (i' - i)$ or $\frac{1}{6} \delta (i' - i)$.

The altitude observed at the lower station may be obtained within about $1'$; but the observer placed at the top mast, where the motion of the vessel is more sensible, frequently cannot observe the altitude within less than $3'$ or $4'$. It may therefore be supposed

that the error of $r' - 1$ is $\frac{4}{5}$; then that of the angle i , or of the depression which would be obtained from the calculation, would be $\frac{4}{5}$ or $\frac{4}{6}$ of a minute, which is equal to $48''$ or $40''$. Whenever the distance from land exceeds a league, there will not be much advantage in using this method. And it ought to be perceived that it must never be employed in determining geographical positions.

The circumstances which it appears at first ought to be most advantageous, are those in which the vessel is at least two miles from the shore; now ($r' - 1$) will then become greater, the error increases in proportion, and may even be more than a minute. On the other hand, at a small distance from land, the undulations of the shore become sensible, and the two observers would be exposed to the inconvenience of referring their images of the sun to two different points, which might both be out of the vertical circle of the sun. It is difficult to value the errors with which $r' - 1$, and the altitude itself, might be affected, in this case; we only know that they might be very considerable, and render the results very defective. It was chiefly this last cause which induced us not to give, in the preceding treatise, the method of correcting altitudes observed near land, by simultaneous observations made at very different elevations. Besides, it appears that the difficulty of taking observations in places so elevated as the top-masts of a vessel, has deterred navigators; and they make very little use of this method: they prefer removing from the shore, as has been recommended, when they wish to obtain altitudes upon which they can depend.

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APPENDIX.

CONTAINING a series of practical Examples adapted to the various Rules given in the preceding treatise; and designed to assist the young Mariner in obtaining a knowledge of this important part of practical Navigation, by furnishing him with a copious collection of exercises on the subject of Nautical Astronomy.

PRACTICAL EXAMPLES TO

CHAPTER I.

Conversion of Longitude into Time. Arts. 11 and 12.

Example 1.

Required the time answering to $97^{\circ} 55' 39''$ of longitude.

$97^{\circ} 55' 39''$

Multiply by $\begin{array}{r} - \\ - \\ - \\ - \\ 4 \end{array}$

Product $\begin{array}{r} - \\ - \\ - \\ - \\ 6^{\text{h}} 30' 42'' 36''' \end{array}$

Then by dividing the thirds by 6, gives $36''' \div 6 = 6''$, the decimal of a second; and therefore $6^{\text{h}} 30' 42''.6$ is the time required.

Example 2.

What is the time corresponding to $141^{\circ} 13' 51''$ of longitude.

Ans. $9^{\text{h}} 24' 55''.4$

Example 3.

Reduce $76^{\circ} 43' 27''$ of longitude into time.

Ans. $5^h 6' 53.8$.

Example 4.

What are the hours, minutes, and seconds, corresponding to $187^{\circ} 54'$ of longitude?

Ans. $9^h 10' 36''$.

Conversion of Time into Longitude. Arts. 13 and 14.

Example 5.

What is the longitude corresponding to $7^h 54' 32.8$ of time?

Multiplying the tenths of a second by 6, to obtain the thirds, gives $.8 \times 6 = 48''$, then

Dividing by 4), $54' 32'' 48''$

Quotient - $13^{\circ} 38' 12''$

$15 \times 7 = 105$

Longitude required $118^{\circ} 38' 12''$

Example 6.

Find the longitude answering to $6^h 44' 10''$ of time.

Ans. $101^{\circ} 2' 30''$.

Example 7.

What longitude corresponds to $2^h 3' 17.8$?

Ans. $30^{\circ} 49' 27''$.

Example 8.

If the time elapsed be $57' 43.3$, what is the corresponding longitude?

Ans. $14^{\circ} 25' 49.5''$.

Declination of the Sun. Art. 16.

Let τ denote the time between the epoch in the Nautical Almanac preceding and that following the time for which the quantity is to be calculated; and t , the time between the first epoch and the given time. Also let q express the quantity in the Almanac, answering to the first epoch, and q the change corres-

ponding to the time T ; then $T:t::q:\frac{q+t}{T}$ = the change answering to the time t ; and consequently the quantity required will be equal to

$$\text{noon} \quad q \pm \frac{q+t}{T}$$

where the sign $+$ is generally used when the quantity q is increasing, and the sign $-$ when it is decreasing. This simple formula may easily be remembered, and will render it unnecessary to refer to any written rule.

Example 9.

What was the sun's declination on the 12th of January, 1814, at $10^h 20'$ in the morning, civil time, in west longitude $63^\circ 42'$?

First, $63^\circ 42' = 4^h 14' 48''$, the time that the sun passes the meridian of $63^\circ 42'$ of west longitude after it has passed that of Greenwich; therefore when it is $10^h 20'$ in the morning at the former place, it is $10^h 20' + 4^h 14' 48'' = 14^h 34' 48''$, or $2^h 34' 48''$ after noon, civil time, at the latter. But as the astronomical day commences at noon, the required declination is for the 12th of January at $2^h 34' 48''$, astronomical time.

Sun's declination 12th Jan. 1814, at noon.	-	$21^\circ 42' 16''$
Ditto - - 13th - (Subtract.)	-	<u>$21 \quad 32 \quad 21$</u>
Decrease of declination in 24 hours	- -	<u>$0^\circ 9' 55''$</u>

By proportional parts, and taking only tenths of a second	{	In 2	-	-	0' 49"·6
		30'	-	-	0 12·4
		3	-	-	0 1·2
		1	-	-	0 0·4
		48'	-	-	0 0·3
		<hr/>			
		2 ^h 34' 48"		1' 3·9	
					or 1' 4" nearly.

The same result may also be obtained by the following proportion.

As $24^h : 2^h 34' 48'' :: 9' 55'' : 1' 3'' 98$, or $1' 4''$ nearly.

Now as the declination is *decreasing*, this must be subtracted from the sun's declination on the 12th at noon; hence

The declin. of sun, 12th Jan. at noon	cond	-	$21^\circ 42' 16''$
Decrease in $2^h 34' 48''$		tract.)	$\frac{1 \quad 4}{1}$
DECLINATION required			$21^\circ 41' 12''$

By the preceding formula

$$\frac{2^h 34' 48'' \times 9' 55''}{24} = \frac{2^h 58' \times 9' 92''}{24} = (45' \times 2' 48'') = 1' 3'' 98,$$

As it is troublesome to multiply the second and third terms of this proportion together, on account of the different denominations they contain, the operation will be facilitated by reducing the lower denominations in the second term to decimals of an hour, and those in the third to decimals of the highest denomination it contains; and then the answer, or fourth term, will be of the same name as the third. This reduction is very easily made by dividing each lower denomination by 6 and annexing the quotient as decimals to the next higher.

Thus $34' + \frac{48}{6} = 34' 8$, and $2^h 34' 8 = 2^h + \frac{34' 8}{6} = 2^h 58$,

and $9' 55'' = 9' + \frac{55}{6} = 9' 92$ very nearly.

Therefore $24^h : 2^h 58 : : 9' 92 : 1' 3'' 98$, as before.

As some mariners may prefer the method of obtaining the fourth term of the proportion by the use of logarithms, with which they are so familiar, especially when it is thought necessary to retain three or four decimal places in either of the factors, it is proper to observe, that this may be expeditiously done by the following simple rule; viz.

Add the logarithms of the second and third terms to the constant logarithm -2.6197888 , when the quantities in the Nautical Almanac are calculated for every 24 hours, but to -2.9208188 , when they are calculated for every 12 hours. Thus, in the above example,

Constant Logarithm	-	-2.6197888
Logarithm of $2^h 58$	-	0.4116197
Ditto of $9' 92$	-	0.0965117
Nat. Numb. $1' 0664$	$= 1' 3'' 98$	0.0279902

or $1' 4''$ nearly, and $21^{\circ} 42' 16'' - 1' 4'' = 21^{\circ} 41' 12''$ S. the declination required.

NOTE. The tables usually given for correcting the declinations of the sun and moon, found in the Nautical Almanac for noon or midnight, for any other time of the day, and for finding the time of the moon's passage over any other meridian than that of Greenwich, generally require the proportional parts to be calculated; in order to obtain a near approximation. Thus, in the preceding example the use of Table VI, in the *Requisite Tables*, requires a double entry of the table, two subtractions, with one calculation for the proportional parts, and would have required two if the minutes in the time had not consisted of tens only. It is therefore frequently much better to find the whole correction at once by calculation; which may generally be done with great facility by regarding the formula $\frac{q \times t}{T}$ as a fraction,

and cancelling both its terms by their common factors, as above. Besides, this method possesses the advantage of accuracy arising from calculating the correction from the actual variation for the day on which it is required; for the daily change in the sun's declination on the corresponding days in the several quarters is not the same. Taking a promiscuous example, the four corresponding days in the table above referred to, are February 23rd, May 18th, August 24th, and October 18th. The variations in the declination of the sun between noon on each of these days and the following noon, as given in the Nautical Almanac for 1814, are $22' 3''$, $18' 10''$, $20' 34''$ and $11' 51''$ respectively. The mean of these four is $16' 54''\frac{1}{2}$, which differs from each of them respectively by $5' 8''\frac{1}{2}$, $3' 44''\frac{1}{2}$, $3' 39''\frac{1}{2}$ and $5' 3''\frac{1}{2}$.

In the above example, the correction resulting from the use of the table is $1' 0''\cdot92$, but the accurate correction from the calculation is $1' 3''\cdot98$; the difference of which is therefore $3''\cdot06$. Hence, whenever accuracy is required, the correction should *always* be calculated from the actual variation on the day for which it is required.

Example 10.

On the 10th of March, 1814, being in longitude $54^{\circ} 37'$ East; what was the declination of the sun at the time of his passing the meridian? Ans. $4^{\circ} 17' 3''$ S.

Example 11.

Being in East longitude $121^{\circ} 35' 6''$, by account, at $8^h 57' 35''$ A.M. on the 26th of October, 1814, civil time; required the sun's declination at that moment. Ans. $12^{\circ} 10' 34''$ S.

Example 12.

Required the declination of the sun at $4^h 50$ minutes P.M. on the 15th of May, 1815, civil time, in $76^{\circ} 43' 27''$ West longitude. Ans. $18^{\circ} 49' 24''$ N.

Declination of the Moon. Art. 16.

Example 13.

Required the moon's declination on the 20th of March, 1815, at 3 P.M. civil time, in longitude $134^{\circ} 38' 4''$ East.

First, $134^{\circ} 38' 4'' = 8^h 58' 33'' 6$ of time, which must be subtracted from the time in the question, as the place is east of the first meridian; and therefore, the hour reckoned from the commencement of the civil day is $3^h + 12^h - 8^h 58' 33'' 6 = 6^h 1' 26'' 4$ A.M. But as the astronomical day begins 12 hours after the civil, the 20th of March has not yet commenced, and the astronomical time is the 19th of March at $18^h 1' 26'' 4$: hence,
Moon's declination at midnight, 19th March, N. - $23^{\circ} 2'$
Ditto - - at noon, 20th - N. - $22^{\circ} 35'$
Decrease of declination in 12 hours. - Difference - $0^{\circ} 27'$

Then, as $12^h : 6^h 1' 26'' 4 :: 27' : 13' 33'' 1$, the change of declination corresponding to $6^h 1' 26'' 4$; consequently,

At midnight	-	-	$23^{\circ} 2'$
Decrease	-	-	$13' 33'' 1$
DECLINATION required	-		$22^{\circ} 48' 26'' 9$
Taking the nearest second			$22^{\circ} 48' 27''$

Example 14.

Required the moon's declination at the time of her rising at the Royal Observatory, Greenwich, on the 30th of October, 1814, which is 6^h 11' A. M. civil time. Ans. 14° 32' 51" N.

Example 15.

Required the moon's declination on the 17th of May, 1814, at 2^h 57½ P. M. civil time, in 137° 54' of West longitude?

Ans. 7° 52' 40" N.

Example 16.

Required the moon's declination on the 28th of June, 1814, at the time of her setting at Greenwich, which is 1^h 29' A. M. civil time?

Ans. 11° 40' 5" S.

*Right Ascension of the Sun. Art. 16.**Example 17.*

Required the right ascension of the sun on the 22nd of February, 1814, at 11^h 44' P. M. civil time, in longitude 55° 25' 12" West of the meridian of Greenwich.

First, 55° 25' 12" of longitude converted into time gives 3^h 41' 40"·8, which must be added to the time at the place of observation to obtain the hour at the first meridian, because the longitude is *west*; hence, the astronomical time, reduced to the first meridian, is the 22nd, at 3^h 25' 40"·8.

Sun's right ascen. in time 22nd, at noon	-	22 ^h 20' 56"·1
Ditto - - - 23rd, - - -	-	22 24 44·6
Increase in 24 hours - - - - -	-	0 ^h 3' 48"·5

Then, as 24^h : 3^h 25' 40"·8 : : 3' 48"·5 : 32"·6, taking only one decimal place in the fourth term; consequently, as the right ascension is increasing,

150 RIGHT ASCENSION OF THE MOON.

Right ascension at noon, 22nd Feb. $22^h 20' 56''.1$
 Increase in $8^h 25' 40''.8$ - (Add) 32.6
 RIGHT ASCENSION, in time, required - $22^h 21' 28''.7$
 or, $22^h 21' 29''$ nearly.

Example 18.

What was the right ascension of the sun on the 3rd of May, 1814, at $3^h 36' 20''$ P.M. civil time, in longitude $120^\circ 54' 7''$ East from Greenwich? Ans. $2^h 39'$.

Example 19.

Required the sun's right ascension at the time of his rising at Greenwich, on the 26th of November, 1814, which is at $7^h 31'$ civil time. Ans. $16^h 5' 48''.1$.

Example 20.

Required the right ascension of the sun at the moment of his passage over the meridian of Port Royal, in Jamaica, situated in West longitude $76^\circ 50' 30''$, on the 12th of August, 1815. Ans. $9^h 25' 54''.8$.

Right Ascension of the Moon. Art. 16.

Example 21.

What was the right ascension of the moon on the 26th of April, 1814, at $9^h 50'$ P.M. civil time, in $39^\circ 13'$ East longitude from Greenwich?

First, $39^\circ 13' = 2^h 36' 52''$, which must be subtracted from $9^h 50'$, which gives $7^h 13' 8''$ for the time reduced to the first meridian, or that for which the right ascension is required. Then,

Moon's right ascension, 26th April, at midnight	-	$128^\circ 89'$
Ditto - - - 26th, - at noon	-	$121 16$
Increase in right ascension in 12 hours	-	$7^\circ 23'$

SEMI-DIAMETER OF THE SUN.

By proportional parts, and taking only tenths of a minute.	In 6 hours	3° 41' 5"
	1	36' 8"
	13	6' 1"
	2	1' 2"
	1	6"
		1
	7 ^h 15'	4° 20' 3"

Hence, right ascension at noon $121^{\circ} 16'$
 Increase in $7^h 13' 8''$ - Add. $4^{\circ} 26' 3''$
 RIGHT ASCENSION required $125^{\circ} 42' 3''$

Example 22.

Required the moon's right ascension on the 21st of July, 1814, at 9^h 57' A.M. civil time, at a place situated $35^{\circ} 20'$ West of Greenwich. Ans. $177^{\circ} 55'$.

Example 23.

Required the right ascension of the moon on the 15th of January, 1815, at midnight, in $56^{\circ} 38'$ of East longitude. Ans. $354^{\circ} 49'$.

Example 24.

Required the moon's right ascension at the time of her rising at Greenwich, on the 30th of December, 1814, which is 8^h 36' P.M. Ans. $156^{\circ} 27' 27''$.

Semi-diameter of the Sun. Art. 16

Example 25.

What was the semi-diameter of the sun on the 31st of January, 1814?

Semi-diameter, Jan. 25th	-	$16' 16' 2''$
Ditto Feb. 1st,	-	$16' 15' 3''$
Decrease in 7 days	-	$0' 0' 9''$

Therefore $16' 16'' \cdot 2 - \frac{9'' \times 6}{7} = 16' 16'' \cdot 2 - 8'' = 16' 15'' \cdot 4$, the SEMI-DIAMETER required.

Example 26.

What was the sun's semi-diameter on the 29th of July, 1814, at 4^h A.M. ? Ans. $15' 47'' \cdot 1$.

Example 27.

Required the sun's semi-diameter on the 17th of March, 1815. Ans. $16' 5'' \cdot 4$.

Example 28.

Required the semi-diameter of the sun on the 9th of November, 1815, at noon, in 180° of longitude ? Ans. $16' 11'' \cdot 1$.

*Semi-diameter of the Moon. Art. 16.**Example 29.*

What was the moon's semi-diameter on the 16th of April, 1814, at 1^h 45', P.M. civil time, in East longitude $43^\circ 21'$, supposing her altitude to be 26° ?

$43^\circ 21' = 2^h 53' 24''$; therefore the time in the question reduced to the first meridian is $10^h 51' 36''$ A.M. 16th April, civil time, or $22^h 51' 36''$, 15th April, astronomical time. Therefore,

Moon's semi-diam. 15th midnight	-	$15' 21''$
Ditto - - 16th noon	-	$15' 28''$
Increase in 12 hours	-	$0' 7''$
Then, as $12^h : 10^h 51' 36'' :: 7'' :$	-	$0' 6'' \cdot 3$
Semi-diameter 15th midnight	-	$15' 21''$
Hor. Semi-diam. - - -	-	$15' 27'' \cdot 3$
Augmentation in TABLE II.	-	$7''$
SEMI-DIAMETER required	-	$15' 34'' \cdot 3$

Example 30.

Required the horizontal semi-diameter of the moon when she

passed the meridian of the Royal Observatory, at Greenwich, on the 20th of July, 1814. Ans. $16' 7''$.

Example 31.

Find the moon's horizontal semi-diameter on the 25th of January, 1815, at 2^h 41' P.M. civil time, in longitude $85^{\circ} 56'$ West. Ans. $16' 45''$.

Example 32.

On the 8th of May, 1815, at 6^h 38' A.M. by a chronometer regulated to civil time at the first meridian, suppose the moon's altitude found by observation to be $49^{\circ} 30'$; required her semi-diameter. Ans. $15' 51''$.

*Moon's horizontal Parallax. Art. 16.**Example 33.*

Required the moon's horizontal parallax on the 20th of May, 1814, at 5^h 45' A.M. civil time, in latitude 55° N. and longitude $64^{\circ} 34'$ West.

Given time	- - - -	5 ^h 45'
Longitude West, in time	- Add.	<u>4 17 36"</u>
Time reduced to the first merid.		10 ^h 2' 36"
Moon's horizontal parallax, 19th at midnight	-	60' 49'
Ditto - - - 20th at noon	-	<u>60 55</u>
Increase in 12 hours	- - - -	0' 6'

Then $60' 49'' + \frac{10^h 2' 36'' \times 6''}{12^h} = 60' 49'' + 5'' = 60' 54''$, which

is the horizontal parallax at the equator at the given time; and which reduced, by TABLE III, to that at the latitude of the question, gives $60' 46''$ for the horizontal parallax required.

Example 34.

Required the moon's horizontal parallax at the time of her setting at Greenwich on the 13th of December, 1814; or at 5^h 23' P.M. civil time. Ans. $53' 59''$.

Example 35.

The horizontal parallax of the moon is required for the 12th of March, 1815, at 7^h 20', P.M. civil time, in latitude 54° 20', and longitude 135° 38' East. Ans. 55' 51".

Example 36.

Find the moon's horizontal parallax on the 28th of August, 1815, at 3^h 50' A.M. civil time, in latitude 35° 10', and longitude 72° 43' West. Ans. 57' 40".

Moon's passage over the Meridian. Art. 16.

Example 37.

What time did the moon pass the meridian of 30° 45' West longitude on the 25th of January, 1814?

Moon passed the first meridian 26th at	-	-	3 ^h 54' P.M.
Ditto	-	-	25th - - 3 7
			Difference 0 ^h 47'

Therefore $360^\circ : 30^\circ 45' :: 47' : 4' 3''$, the required variation.
 Moon passes the first merid. Jan. 25th at - 3^h 7' 0" P.M.
 Longitude West, in time - - Add. 2 3 0
 Variation answering to 30° 45' - Add. 4 3
 TIME required - - - - - 5^h 14' 3"

Example 38.

Required the time at which the moon passed the meridian of Canton, in longitude 113° 2' 45" E. on the 20th of September, 1814. Ans. 10^h 21' 45" A.M.

Example 39.

Required the culminating of the moon at Kingston, Jamaica, in longitude 76° 50' 30" W. on the 28th of April, 1815. Ans. 11^h 54' 27" P.M.

Example 40.

At what time of the day will the moon pass the meridian of

Constantinople, East longitude $28^{\circ} 55' 15''$, on the 12th of December, 1815.

Ans. $7^h 44' 35''$ P.M.

Right ascension of the Stars. Art. 18.

Example 41.

What was the right ascension of *Arcturus* on the 31st of May, 1814?

Right ascen. of *Arcturus*, at the beginning of 1815, $14^h 7' 13'' \cdot 38$

Variation for 7 months, or $\frac{2 \cdot 728 \times 7}{12}$ (Subtract.) $1 \cdot 59$

Right ascension required, in sidereal time $- 14^h 7' 11'' \cdot 79$

Example 42.

Required the right ascension of *Sirius* on the 15th of November, 1815.

Ans. $2^h 37' 2'' \cdot 26$.

Example 43.

The right ascension of *Regulus* is required on the 1st of March, 1816.

Ans. $9^h 58' 34'' \cdot 3$.

Example 44.

Required the right ascension of *Aldebaran* on the 30th of December, 1818.

Ans. $2^h 25' 32'' \cdot 54$.

Declination of the Stars. Art. 18.

Example 45.

Required the declination of α *Pegasi* on the 14th of February, 1816.

Declination at the beginning of 1815 $- 14^{\circ} 12' 54'' \cdot 19$ N.

Annual variation, $19'' \cdot 43$, for one year $- + 19 \cdot 43$

Variation for 1 month and 14 days $- + 2 \cdot 43$

Declination required $- - - - 14^{\circ} 13' 16'' \cdot 05$ N.

Or $14^{\circ} 13' 16''$, omitting the decimals.

Example 46.

What was the declination of *Fomalhaut* on the 25th of June, 1814.?

Ans. $30^{\circ} 35' 34'' 4S$.

Example 47.

Required the declination of *Pollux* on the 10th of August, 1816.

Ans. $28^{\circ} 27' 38'' 43N$.

Example 48.

Find the declination of *Aquilæ* on the 20th of November, 1818.

Ans. $8^{\circ} 24' 0'' 05N$.

CHAPTER II.

*Depression of the Horizon. Art. 20.**Example 49.*

Required the depression of the horizon, the observer's eye being elevated 25 feet 9 inches above the surface of the sea.

Depression for 25 feet, Table I.	-	-	-	4' 54"
Proportional parts for 6 inches	-	-	-	3
Ditto - for 3 inches	-	-	-	1.5
Depression required, for 25 feet 9 inches	-	-	-	4' 58".5

Example 50.

Required the depression of the horizon, when the eye of the observer is elevated 16 feet 8 inches above the level of the sea.

Ans. 4'.

Example 51.

The eye of an observer is elevated 22 feet 6 inches above the level of the sea, what is the depression of his visual horizon?

Ans. 4' 39".

*Augmentation of the Moon's Semi-diameter. Art. 28.**Example 52.*

Required the moon's semi-diameter at the moment her centre passed the meridian of Greenwich, on the 30th of March, 1814, supposing her altitude to have been at that instant $52^{\circ} 30'$.

The moon passed the meridian of Greenwich on the day proposed at 8^h 3' P.M. and her horizontal semi-diameter at that time, calculated according to Art. 16, of Example 29, is 16".

$$\text{Augmentation, Table II. } \left\{ \begin{array}{l} \text{Altitude } 45^\circ \quad - \quad - \quad - \quad 11''.5 \\ \text{Ditto } 55 \quad - \quad - \quad - \quad 13''.5 \\ \hline 10'' \quad \text{Difference} \quad 2''.0 \end{array} \right.$$

$$\text{Also } 52^\circ 30' - 45^\circ = 7^\circ 30'.$$

Consequently $11''.5 + \frac{5 \times 2''}{10} = 11''.5 + 1''.5 = 13''$ the required augmentation answering to $52^\circ 30'$ of altitude. Therefore

$$\begin{array}{rcl} \text{Horizontal semi-diameter} & & 16'' \\ \text{Augmentation} & - & - & + 0 \quad 13'' \\ \hline \text{Semi-diam. required} & - & - & 16'' \quad 13'' \end{array}$$

Example 53.

Having observed the moon's altitude on the 5th of November, 1814, to be $61^\circ 10' 30''$, and finding her horizontal semi-diameter at that time to be $14' 50''$; what is her augmented semi-diameter on account of altitude? Ans. $15' 2'' 28''$.

Example 54.

Required the moon's augmented semi-diameter when her altitude was observed to be $35^\circ 24' 45''$, and her horizontal semi-diameter was known to be $15' 20''$ at the time of observation.

Ans. $15' 29''$.

Example 55.

Suppose that at a certain place the moon's horizontal semi-diameter had been found by calculation to be $15' 53''$, and her altitude observed to be $56^\circ 28'$; it is required to ascertain her augmented semi-diameter. Ans. $16' 6'' 4$.

Refraction—Parallax of the Sun. Arts. 31 and 32.

Example 56.

On the 14th of May, 1814, the altitude of the sun's lower limb was observed to be $24^\circ 55'$, when the barometer stood at

30.324 inches, and Fahrenheit's thermometer at $60^{\circ}8$; what was the refraction less parallax at the time?

The semi-diameter of the sun at the time of observation, according to the preceding rules, was $15' 51''$, and therefore the altitude of his centre was $25^{\circ} 10' 51''$. The refraction less parallax, in Table V, for 25° is $1' 55''$, and the difference for the succeeding degree is $6''$, the proportional part answering to $10' 51''$ is therefore very nearly $1''$, which is subtractive; hence

Sun's apparent alt. $25^{\circ} 10' 51''$.	Refraction, Table V.	$1' 54''$
Thermometer $+ 60^{\circ} 8'$ -	Table VI. Subtract. -	1
Sun's apparent alt. $25^{\circ} 10' 51''$ }		
		<hr/>
Barometer - 30 ⁱⁿ .324 -	Table VII. Add. -	2
Sun's apparent alt. $25^{\circ} 10' 51''$ }		
		<hr/>
Corrected REFRACTION - - - - -		$1' 55''$

Example 57.

The altitude of the sun's upper limb was observed to be $21^{\circ} 48'$, on the 7th of November, 1814; required refraction less parallax, the height of the observer's eye being $5\frac{1}{2}$ feet.

Ans. $2' 16''6$.

Example 58.

The apparent altitude of the sun's centre being $15^{\circ} 27' 30''$, the barometer 29.75 inches, and Fahrenheit's thermometer $63^{\circ}72$; required the refraction less parallax. Ans. $3' 13''6$.

Example 59.

The observed altitude of the sun's lower limb on the 30th of April, 1815, being $26^{\circ} 10'$, the barometer 29.86 inches, and Fahrenheit's $56^{\circ}47$; required the corrected refraction at the time of observation, the height of the observer's eye being 25 feet.

Ans. $1' 47''5$.

Parallax less Refraction of the Moon. Arts. 35, 36, and 37.

Example 60.

Suppose the altitude of the moon's upper limb was observed on the 15th of April, 1814, at 6^h A.M. civil time, to be $46^{\circ} 39'$; the latitude of the place of observation being $48^{\circ} 10'$ North, and the longitude $6^{\circ} 30'$ East; the barometer at the time was 29.32 inches, and Fahrenheit's thermometer $64^{\circ} 34'$. Required the corrected parallax less refraction of the moon at that moment; the height of the eye being 16.4 feet above the surface of the sea?

First, $6^{\circ} 30' = 26'$ of time; therefore the time of observation reduced to the first meridian is 6^h 26' A.M. of the 15th, civil time, or 18^h 26' of the 14th of April, astronomical time.

Moon's observed altitude	-	-	-	-	$46^{\circ} 39'$
Moon's semi-diameter	-	-	-	Subtract.	$15 \ 12$
					<hr/> $46^{\circ} 23' 48''$
Depression of the horizon	-	-	-	-	$3 \ 58$
Apparent altitude of the moon	-	-	-	-	<hr/> $46^{\circ} 19' 50''$
Equatorial and horiz. Parallax, April 15th, at 6 ^h 26' A.M.					$55' 43''$
Diminution of the equat. paral. on account of latitude				} Table III.	6
Horizontal parallax for lat. $48^{\circ} 10'$	-	-	-	-	<hr/> $55' 37''$
Paral.—refrac. for $46^{\circ} 19' 50''$					$37' 4''$
For $37''$ of horizon. paral.	-	-	+ 26	} Table VIII.	$37' 30''$
Height of the barometer 29.32 inches					
Apparent altitude	-		$46^{\circ} 19' 50''$	} Table VII.	$+ 1$
Height of the thermometer $64^{\circ} 34'$					
Apparent altitude	-		$46^{\circ} 19' 50''$	} Table VI.	$+ 1$
Parallax—Refraction required	-	-	-	-	<hr/> $37' 32''$

Example 61.

Required the parallax—Refraction of the moon when she

passed the meridian of Greenwich on the 27th of September, 1814; supposing the apparent altitude of her centre at that moment to be $53^{\circ} 10'$, the barometer at 29.42 inches, and Fahrenheit's thermometer $49^{\circ} \cdot 16$. Ans. $32^{\circ} 55''$.

Example 62.

The apparent altitude of the moon's centre having been found to be $43^{\circ} 15' 30''$, and Fahrenheit's thermometer observed to stand at $72^{\circ} \cdot 65$, the barometer at 29.8 inches, and the horizontal parallax to be $57' 30''$: required the corrected parallax less refraction. Ans. $40' 32''$.

Example 63.

Suppose the apparent altitude of the moon's centre, after having been corrected for the dip of the horizon and semi-diameter, to be $15^{\circ} 58'$ at $10^h 15'$ P.M. civil Greenwich time, on the 10th of October, 1815; the height of the barometer being 29.6 inches, and that of the thermometer $68^{\circ} \cdot 45$. Required the parallax—refraction of the moon at that time. Ans. $49' 17''$.

True Altitude of the Sun. Art. 31.

Example 64.

On the 12th of August, 1814, the altitude of the sun's lower limb was observed to be $26^{\circ} 35'$; the height of the thermometer was $71^{\circ} \cdot 6$, that of the barometer 29.12 inches, and the height of the observer's eye $21\frac{1}{3}$ feet; required the sun's true altitude.

Sun's observed altitude	-	-	-	-	$26^{\circ} 35'$
Depression of the horizon answering to $21\frac{1}{3}$	-	-	-	-	$4 32''$
					<hr/>
					$26^{\circ} 30' 28''$
Semi-diameter of the sun, August 12th	-				$+ 15 49 \cdot 5$
					<hr/>
					$26^{\circ} 45' 17 \cdot 5$
Refraction—Parallax to alt. $26^{\circ} 46' 17 \cdot 5$, corrected for the temperature and pressure of the atmosphere	-	-	-	-	} $- 1 38 \cdot 3$
TRUE ALTITUDE of the sun	-	-	-	-	<hr/>
					$26^{\circ} 44' 39 \cdot 2$

Example 65.

The observed altitude of the sun's lower limb being $21^{\circ} 32'$, the height of the eye 28 feet above the level of the sea, and the sun's semi-diameter $15' 58''$. The true central altitude is required.

Ans. $21^{\circ} 40' 31''$.

Example 66.

At $10^h 30'$ A.M. March 21st, 1814, being in East longitude $60^{\circ} 21'$, by account, and having observed the heights of the barometer and thermometer to be 30.12 inches and $63^{\circ} 72'$ respectively; it is required to find the true altitude of the sun's centre, the observed altitude of his lower limb being $30^{\circ} 40' 15''$, and the height of the eye, above the surface of the sea, 18 feet.

Ans. $30^{\circ} 50' 43''$.

Example 67.

Required the true altitude of the sun's centre on the 26th of April, 1815, at $9^h 10'$ P.M. civil time, in longitude $43^{\circ} 20'$ West; supposing the height of the barometer to be 30 inches, that of the thermometer $55^{\circ} 12'$; the height of the observer's eye being $30\frac{1}{2}$ feet, and the observed altitude of the sun's upper limb $37^{\circ} 55''$.

Ans. $37^{\circ} 52' 33''$.

True Altitude of the Moon. Arts. 35, 36, and 37.

Example 68.

On the 31st of May, 1814, at midnight, in longitude $96^{\circ} 25'$ East, by account, suppose the observed altitude of the moon's lower limb was $32^{\circ} 21'$, the height of the eye 23 feet above the level of the sea; and also that the corrected parallax—refraction was found to be $37' 32''$: required the moon's true altitude at that time.

The time proposed reduced to the first meridian, or Greenwich time, is May. 31st, at $5^h 34' 20''$ P.M. at which time the semi-diameter of the moon was $14' 58''$.

Then the moon's observed altitude	-	-	-	32° 21'
Depression of the horizon	-	-	-	— 4 42'
				<hr/> 32 16 18
Moon's semi-diameter	-	-	-	+ 14 58
				<hr/> 32 31 16
Apparent altitude of the moon's centre	-	-		
Corrected Parallax — Refraction	-	Add.	-	37 32
True altitude required	-	-	-	33° 8' 48"

Example 69.

Suppose that on the 29th of March, 1814, the altitude of the moon's upper limb was observed to be $24^{\circ} 35'$, at 36 minutes past 8 at night, civil time; that the height of the observer's eye was $5\frac{1}{2}$ yards, and the longitude $112^{\circ} 55' W.$ by account; the height of the barometer being 30.3 inches, and that of the thermometer $52^{\circ}.5$. Required the true central altitude of the moon at the moment of the observation. Ans. $25^{\circ} 7' .1''$.

Example 70.

Let it be supposed that on the 31st of January, 1815, in longitude $69^{\circ} 24' E.$ by account, the altitude of the moon's upper limb is found to be $44^{\circ} 12'$ at 10 minutes past 9, P.M. civil time, when the barometer was 30.4 inches, Fahrenheit's thermometer $3\frac{1}{2}$ degrees below the freezing point, and the height of the eye 16 feet above the surface of the water; it is required to determine the true altitude of the moon's centre.

Ans. $44^{\circ} 32' 25''$.

Example 71.

Suppose that the altitude of the moon's lower limb is observed to be $38^{\circ} 14'\frac{1}{2}$, on the 28th of May, 1815, at $10^h 29'$ P.M. on board a vessel in longitude $101^{\circ} 50' E.$ according to her reckoning, at the same time that it was ascertained the barometer stood at 28.93 inches, Fahrenheit's thermometer at $79^{\circ}\frac{1}{4}$, and the elevation of the eye above the surface of the sea was 18 feet. Required the true altitude of the moon's centre at the moment of taking her observed altitude. Ans. $39^{\circ} 6' 31''$.

*True Altitude of the Stars.**Example 72.*

Suppose the observed altitude of *Arcturus* to be $53^{\circ} 24'$, the height of the eye $25\frac{1}{2}$ feet above the surface of the water, the barometer 30.3 inches, and the thermometer $60^{\circ}.17$; what is its true altitude?

Observed altitude of the star	-	-	-	53° 24'
Elevation of the eye 25½ feet.	Depression	-	-	— 4 51'
Apparent altitude	-	-	-	53° 19' 9"
Refraction of altitude, Table V.	-	-	-	- 43"
Thermometer 60°·17	} Table VI. Correction.	-	-	0
Apparent alt. 53° 19' 9"		-	-	0
Barometer - 30·3 inches	} Table VII. Correction.	-	-	+ 1
Apparent alt. - 53° 19' 9"		-	-	+ 1
Corrected Refraction	-	-	-	- 44"
Apparent altitude of Arcturus	-	-	-	53° 19' 9"
TRUE ALTITUDE required	-	-	-	53° 19' 23"

Example 73.

The observed altitude of *Aldebaran* is $45^{\circ} 28'$, and the height of the observer's eye $18\frac{1}{2}$ feet above the surface of the sea; required its true altitude, independently of the temperature and pressure of the atmosphere.

Ans. $45^{\circ} 22' 52''$.

Example 74.

Suppose the observed altitude of *Regulus* to be $34^{\circ} 51'$, the height of the eye 24 feet, that of the barometer 29.5 inches, and of the thermometer $35^{\circ}.6$; required its true altitude.

Ans. $34^{\circ} 44' 45''$.

Example 75

The observed altitude of the star *Pollux* being $67^{\circ} 16\frac{1}{2}'$, when corrected for the depression of the horizon, the height of the barometer 29.2 inches, and that of the thermometer $30^{\circ}.34$; required the true altitude of this star.

Ans. $67^{\circ} 16' 5''$.

Correction of the less of two Altitudes taken out of the Meridian.

This subject naturally consists of two parts; viz. the method of finding these corrections, and that of applying them; each of these shall be illustrated by examples.

*Method of finding the Correction. Art. 40.**Example 76.*

The altitude of the sun having been observed in North latitude $47^{\circ} 25'$, and found to be $23^{\circ} 6'$ when his declination was $13^{\circ} 2' N$. Some time afterwards the sun's altitude was observed to be $36^{\circ} 54'$; and it was ascertained that the way made in latitude during the interval between the observations was $12' 30''$. Required the multiplier of the difference of latitude.

Less alt. of \odot $23^{\circ} 6'$ } 1st term, Table XII. 1.46 Argum. 1.58
 Latitude north $47 25$ }

Declin. North $13^{\circ} 2'$ } 2nd term, Table XIII. 0.36 .
 Argument - 1.58 }

2nd term $+ 2 -$ 1st term $= 2.36 - 1.46 = .9$, the required multiplier.

Example 77.

Suppose that on board a vessel in latitude $56^{\circ} 38' N$. the altitude of the sun was found to be $46^{\circ} 54'$; and that some hours afterwards his altitude was taken again, and found equal to $24^{\circ} 46''$; and his declination at the moment of this last observation was $18^{\circ} 31' S$. The way made in latitude during the interval between the observations was $23'$ towards the South; required the multiplier of the difference of latitude.

Ans. 1.68 .

Example 78.

The observed altitude of the sun, in latitude $15^{\circ} 20' S$. being $32^{\circ} 45'$, when his declination was $21^{\circ} 45' S$.; and after the vessel had arrived at latitude $15^{\circ} 38' 54''$, the altitude was again

observed, and found to be $52^{\circ} 20'$; what is the multiplier of the difference of latitude?

Ans. $\cdot 711$.

Example 79.

The sun's altitude being observed at two different places on the same day, the latitudes of which were $49^{\circ} 57' N.$ and $50^{\circ} 22' N.$ the less altitude being equal to $14^{\circ} 44'$, and corresponded to the less latitude, and his declination at the time of observation equal to $8^{\circ} 56' S.$ Required the multiplier of the difference of latitude.

Ans. $\cdot 43$.

Method of applying this Correction. Art. 41.

Example 80.

The less altitude of the sun taken at 8^h A.M. on the 7th of May, 1814, was $24^{\circ} 43'$, and the latitude of the place of observation $38^{\circ} 41' N.$ Some time afterwards the sun's altitude was again taken and found to be $43^{\circ} 14'$, in latitude $38^{\circ} 27' N.$ Required the less corrected altitude, or what the less altitude would have been if observed in the place of the greater.

The declination of the sun at the time of the least observation was $16^{\circ} 39' 19'' N.$ the difference of latitude $14'$, and the multiplier of the difference of latitude, found, as in the preceding examples, is $1\cdot 036$; therefore the correction to be applied to the less altitude is $14' \times 1\cdot 036 = 14' 30''$. As the declination and latitude of the place of observation are both North, the sun passes the meridian South of the observer; and as the latitude of the place where the greater altitude was taken, ought to be greater than that where the less was observed, the meridian altitude at the former place is therefore greater than at the latter: hence,

Less altitude of the sun	-	-	-	-	$24^{\circ} 43'$
Difference of latitude	-	-	-	<i>Add.</i>	$\underline{\quad 14 \quad}$
				Sum	$24^{\circ} 57'$
Correction	-	-	-	<i>Subtract</i>	$\underline{\quad 14 \ 30 \quad}$
LESS ALTITUDE referred to the place of the greater					$24^{\circ} 42' 30''$

Example 81.

The sun's altitude being found equal $46^{\circ} 20'$, in South latitude $18^{\circ} 58'$; and after the ship had sailed towards the North-West until her latitude was reduced to $18^{\circ} 30'$, the altitude of the sun was found to be $32^{\circ} 3'$, and his declination at the time of the last observation equal to $5^{\circ} 12' N$. Required the less corrected altitude.

Ans. $32^{\circ} 11' 32''$.

Example 82.

The sun's altitude and declination were found to be $25^{\circ} 38'$ and $20^{\circ} 45' N$, respectively, in North latitude $10^{\circ} 21'$; and after a diminution of latitude equal to $34'$, his altitude was ascertained to be $46^{\circ} 24'$. What is the corrected altitude of the first observation?

Ans. $25^{\circ} 25' 25''$.

Example 83.

Suppose that, on the 2nd of February, 1815, at $9^h 12'$ A.M. civil time at Greenwich, the altitude of the sun was observed to be $8^{\circ} 50'$, in North latitude $6'$; and that after an elapse of some hours his altitude was again ascertained to be $19^{\circ} 58'$, in South latitude $16\frac{1}{4}$. Required the altitude at the former place of observation when referred to the latter.

Ans. $8^{\circ} 21' 38''$

PRACTICAL EXAMPLES TO

CHAPTER III,

Latitude from the meridian altitude of the Sun. Art. 45.

Example 84.

March 18th, 1814, being in longitude $56^{\circ} 24'$ W. and having found the altitude of the sun's lower limb when he passed the meridian towards the south to be $48^{\circ} 35'$, and ascertained that the elevation of the eye above the surface of the sea was $26\frac{1}{4}$ feet; required the latitude of the place of observation.

By converting the longitude into time, it is found that the time of the observation reduced to the first meridian is $3^h 45' 36''$ P.M. March 18th; and, art. 16, the sun's semi-diameter taken from the Nautical Almanac and reduced to the time proposed, is $15' 57''$; hence the

Observed altitude of the sun's lower limb	-	$48^{\circ} 35'$	
Elevation of the eye $26\frac{1}{4}$ feet.	Depression, Tab. I.	- 5 1	
	Remainder	-	$48^{\circ} 29' 59''$
Sun's semi-diam. 18th of March, 1814, at $3^h 45' 36''$		+ 15 57	
	Sum		$48^{\circ} 45' 56''$
Refraction—Parallax of the sun, Table V.	-	-	45
True altitude of the sun towards the South	-	$48^{\circ} 45' 11''$	
Sun's declination	- - - -	Add.	1 1 6
Height of the equator	- - -	Sum.	$49^{\circ} 46' 17''$
Complement. LATITUDE, NORTH	-		$50^{\circ} 13' 43''$

Example 85.

In $73^{\circ} 24'$ of East longitude, the altitude of the sun's upper limb was observed to be $63^{\circ} 55'$, when he passed the meridian northward of the observer, on the 4th of November, 1814; the

elevation of whose eye was 29 feet above the level of the sea, and the barometer and thermometer standing at 30.1 inches and $42^{\circ} 73'$ respectively. Required the true altitude of the place of observation.

Ans. $41^{\circ} 40' S.$

Example 86.

Suppose, on the 8th of April, 1815, the altitude of the sun's lower limb to be found equal $39^{\circ} 53'$ when he passed the meridian South of the observer, in longitude $45^{\circ} 22' W.$ and the eye was elevated $16\frac{1}{2}$ feet above the surface of the sea; the barometer standing at 28.9 inches, and thermometer at $55^{\circ} 38'$. Required the true latitude.

Ans. $56^{\circ} 47' 42'' N.$

Example 87.

The sun being supposed to pass the meridian between the observer's zenith, and the North pole, on the 12th of August, 1815, and the meridian altitude of his upper limb found to be $78^{\circ} 58'$. The latitude of the place of observation is required, admitting the height of the eye to be 27 feet above the level of the sea, and the place of observation to be in $54^{\circ} 35'$ of West longitude.

Ans. $3^{\circ} 45' 52'' N.$

Latitude from the meridian altitude of the Moon. Art. 45.

Example 88.

On the 10th of May, 1814, being in latitude $38^{\circ} 35' N.$ and longitude $54^{\circ} 42' W.$ by account, the moon was observed to pass the meridian towards the South at $9^h 33'$ P.M.; at the same time the altitude of her lower limb was ascertained to be $71^{\circ} 46'$, and the height of the eye 29.5 feet above the level of the sea. Required the true latitude of the place of observation, admitting the height of the barometer to have been 29.46 inches, and that of the thermometer $73^{\circ} 24'$.

The time of the moon's passage over the meridian of the place of observation reduced to the first meridian, and taken to the nearest minute, is $13^h 19'$, May 10th, astronomical time, or $1^h 19'$, A.M. May 11th, civil time. The corresponding declination

200 LATITUDE FROM MERIDIAN ALTITUDES OF THE MOON.

of the moon is $20^{\circ} 21' N.$ her horizontal semi-diameter $14' 52''$, and her horizontal parallax, taken from the Nautical Almanac, $54' 20''$.

Observed altitude of the moon's lower limb	-	$71^{\circ} 46'$	
Height of the eye 29.5 feet	-	Depression.	$- 5 20''$
		Remainder.	$71 40 40$
Hor. semi-diameter $14' 52''$	}	Moon's semi-diameter	$+ 15 6$
Augmentation, Tab. II. 13			
Apparent altitude of the moon's centre	-	-	$71^{\circ} 55' 46''$
Horizontal paral. $54' 20''$.	Paral.—Refract. Tab. VIII.	+	$16 32$
Barometer - 29.46 inches	}	Corrrection. Table VII.	$+ 0$
Apparent alt. $71^{\circ} 55' 46''$			
Thermometer $73^{\circ} 24$	}	Correction. Table VI.	$+ 1$
Appar. altitude $71^{\circ} 55' 46''$			
True altitude of the moon towards the S.	-		$72 12 19$
Declination of the moon, North	-	-	$20 21$
		Difference.	$51 51 19$
Complement. LATITUDE, NORTH	-	-	$38^{\circ} 8' 41''$

Example 89.

Being in $61^{\circ} 5' E.$ longitude, by account, on the 13th of July, 1814, when the moon passed the meridian towards the North, at $3^h 55' 40'' A.M.$ and the altitude of her lower limb was equal to $30^{\circ} 10'$. Required the correct latitude, independently of the temperature and pressure of the atmosphere, supposing the height of the observer's eye to have been 25 feet above the level of the sea.

Ans. $45^{\circ} 25' 20'' S.$

Example 90.

Suppose the observed meridian altitude of the moon's upper limb to be $74^{\circ} 32'$, on the 14th of April, 1815, at $7^h 27' P.M.$ The moon passing the meridian North of the observer, whose eye is 20 feet above the sea; and the longitude, by account, $47^{\circ} 30' West.$ Required the true latitude. Ans. $7^{\circ} 11' 3'' N.$

Example 91.

Suppose, that on the 13th of June, 1815, at 10^h 19' P. M. the moon was observed to pass the meridian northward of the observer, who was at that time in East longitude 160° 45', by account, and the altitude of her lower limb ascertained to be 69° 50'. The height of the observer's eye above the surface of the water was also found to be 28 feet, that of the barometer 30·6 inches, and of the thermometer 86°·16: it is required to find the correct latitude of the vessel at the moment of the observation.

Ans. 7° 29' 34" S.

Latitude from meridian altitudes of the Stars. Art. 45.

Example 92.

On the 14th of February, 1814, the star α Pegasi passed the meridian northward of the observer, and its altitude was observed at that moment to be 56° 36', and the height of the eye 21½ feet; what was the latitude of the place of observation?

The declination of *Pegasi* at the proposed time has been found in Example 45, to be 14° 12' 37" N. nearly.

Observed altitude of Pegasi towards the N.	-	56°	36'	
Elevation of the eye 21½ feet.	Depression.	-	— 4	32"
	Remainder	56	31	21
Refraction. Table V.	- - - - -	-	—	33
True altitude of α Pegasi towards the N.	-	56	30	50
Declination, North	- - - - -	-	14	12 37
Height of the equator	- - - - -	Sum.	70	43 27
Complement.	LATITUDE REQUIRED.	- -	19°	16' 33" S.

Example 93.

The star *Fomalhaut* was observed to pass the meridian on the 25th of June, 1814, South of the observer, when its altitude was 68° 34', and the height of the eye above the level of the sea 26 feet. Required the latitude of the place. Ans. 9° 4' 11" S.

Example 94.

If the star *Sirius* pass the meridian on the 1st of April, 1815, North of the observer, and its altitude be found at the instant of its passage to be $53^{\circ} 49'$, and the height of the eye 15 feet; what is the latitude of the place of observation?

Ans. $52^{\circ} 43' 30''$ S.

Example 95.

Suppose the bright star *Capella* pass the meridian on the 31st of August, 1815, at the moment its altitude is ascertained to be $64^{\circ} 58'$, the observer facing the North pole, and the height of his eye being 18 feet above the surface of the sea: required the latitude of the place of observation; supposing the barometer to stand at 30.3 inches, and the thermometer at $78^{\circ} 7'$.

Ans. $21^{\circ} 41' 16''$ N.

Latitude from several Altitudes of the Sun taken near the Meridian. Art. 48.

Example 96.

The chronometer having been compared with the sun at half-past 9, on the morning of the 21st of March, 1814, and found to be $20' 15''$ before true time; the latitude of the place was $23^{\circ} 44'$ North, and its longitude $73^{\circ} 10'$ West. At a place, which was $11' 30''$ of a degree West, and $7' 20''$ North of the former, according to the reckoning, the following observations of the sun's lower limb were taken, the height of the eye being 18 feet above the level of the sea; the barometer at 30.4 inches, and the thermometer at $76^{\circ} 5'$: required the latitude of this last place of observation.

The time, by the chronometer, at which the sun would have passed the meridian in the situation where its gain was ascertained would have been $12^h 20' 15''$; but as the place of which the latitude is required is $11' 30''$ of a degree, or $46''$ of time, West of the former, the passage of the sun over the meridian of this latter place will be at $12^h 21' 1''$. The time of this passage

reduced to the first meridian is, therefore, March 21st, at 4^h 53' 26", P.M. The declination of the sun at that moment is therefore 11' 9" N.

Time of passing the Meridian 12^h 21' 1".

Time of obs. by the watch.	Altitudes.	Intervals.	Squares of inter. or Multipliers.
12 ^h 16' 50"	65° 1' 15"	4' 11"	17·5
18 14	65 46 8	2 47	7·75
20 40	66 19 33	0 21	0·1
21 54	66 23 36	0 53	0·8
23 16	66 42 44	2 15	5·05
24 10	67 4 33	3 9	9·9
Divide by 6)397 17 49			Sum 46'·05
Mean altitude 66° 12' 58"			The sixth. Multiplier 7·68
Change in altitude during the last minute before the sun passes the meridian. Table IX.			} 4·6
			30·72
			4·608
Number to be added to the mean altitude.			Product. 35·328
Mean observed altitude			66° 12' 58"
Elevation of the eye 18 feet			Depression — 4 9
			Remainder 66 8 49
Semi-diameter of the ☉			+ 16 4
			66 24 53
Refraction—Parallax. Table V.			— 23
True mean altitude of the ☉			66 24 30
Correction to be added			— 35
Meridian altitude, towards the S.			66 25 5
Declination N.			11 9
Altitude of the equator.			Difference. 66 13 56
Complement. LATITUDE required N.			22° 46' 4"

Example 97.

On the 18th of September, 1814, suppose the following series

of observations to have been made on the sun's lower limb; and the time indicated by a watch that was 53' 44" before true time at the first meridian. The height of the observer's eye being 22 feet above the level of the sea, the latitude $36^{\circ} 41' N.$ and the longitude $25^{\circ} 32' E.$ by account: required the correct latitude of the place of observation.

Times of observation				Observed altitudes
11 ^h	7'	0"	- -	55° 19' 2"
	8	48	- -	19 39
	10	12	- -	19 55
	12	30	- -	20 2
	12	55	- -	20 5
	13	38	- -	20 11
Mean altitude				55° 19' 41"
Answer. $36^{\circ} 30' 40'' N.$				

Example 98.

On the 27th of April, 1815, suppose the following altitudes of the sun's upper limb to be observed, in latitude $12^{\circ} 14' S.$ and longitude $103^{\circ} 52' E.$ according to the ship's reckoning. The height of the eye being 24 feet; the barometer standing at 30.6 inches; Fahrenheit's thermometer at $80^{\circ}.1$; and the chronometer by which the respective times of the observations were noted, known to be 1^h 26' behind true time at the place of observation. Required the correct latitude.

Times of observation.				Observed altitud
10 ^h	31'	25"	- -	75° 9' 9
	33	15	- -	9 56
	34	58	- -	10 2
	35	27	- -	10 16
Mean altitude				75° 9' 51"
Answer. $12^{\circ} 50' 46'' S.$				

Example 99.

Suppose that, on the evening of the 8th of July, 1815, on board a vessel at anchor near one of the Society Islands, and in

South latitude $18^{\circ} 10'$, and West longitude $150^{\circ} 20'$, according to her reckoning; the subsequent observations were made on the sun's lower limb, for the purpose of ascertaining the latitude of the vessel with more correctness. The watch by which the time of each observation was indicated being found to agree with true time at the place of observation, the barometer at 29.9 inches, the thermometer $82^{\circ} 4$, and the height of the eye $16\frac{1}{2}$ feet. What is the true latitude?

Times of observation.				Observed altitudes	
11 ^h	54'	52"	- -	49°	30' 50"
	57	32	- -		31 44
	59	7	- -		31 58
12	1	16	- -		32 4
	3	54	- -		32 40
	5	18	- -		33 15
Mean altitude				49°	32' 5"
Answer $17^{\circ} 45' 21''$ South.					

Latitude from two altitudes of the Sun taken out of the meridian and the interval of time between the observations. Art. 62, &c.

Example 100.

Suppose that on the 12th of April, 1814, on board a vessel in N. latitude $33^{\circ} 30'$, and W. longitude $67^{\circ} 30'$, by account, the altitude of the sun's lower limb was found to be $28^{\circ} 36'$, when a good chronometer, regulated to true time at the first meridian, indicated $7^h 27' 12''$. When the same chronometer gave $11^h 51'$, the altitude of the sun's lower limb was again taken, and found to be $63^{\circ} 49'$. The elevation of the eye in each of these observations was $16\frac{1}{2}$ feet above the surface of the sea; and during the interval between them the vessel sailed towards the South-East until her change in latitude was $15' 20''$ of a degree, and that of her longitude $18' 24''$. Required the correct latitude of the place where the greater altitude was observed.

Time by the watch.

At the place of the less altitude	- - -	7 ^h 27' 12"
Less altitude taken to the West of the greater	- -	— 1 13.6
Corrected time of the less altitude	- -	7 ^h 25' 58".4
Time at the place of the greater altitude	-	11 51
Interval in time	- - - - -	4 ^h 25' 1".6
Interval in degrees	- - - - -	66° 15' 24"
Half interval	- - - - -	33 7 42

Reduced time, Declination and Latitude.

Estimated time of the less altitude	- -	7 ^h 27' 12"
Longitude West, in time	- - add	4 30
Time reduced to the first meridian	- -	11 57 12
Estimated time of the greater altitude	-	11 ^h 51' 0"
Longitude West, in time, 4 ^h 30'—1' 14", add	-	4 28 46
Time at the first meridian	- - -	16 ^h 19' 46"
Declination. { Less altitude	- - -	8° 11' 9" N.
Declination. { Greater altitude	- - -	8 15 9 N.
Mean declination	- - - - -	8 13 9 N.
Mean decl. taken from 90°. Polar distance.	-	81 46 51
Estimated latitude of the less altitude	-	33 30 0
Difference of latitude	- - - Subtract.	15 20
Estimated latitude of the greater altitude	-	33° 14' 40"

Less Altitude.

Observed altitude of the sun	- - -	28° 36' 0"
Elevation of the eye 16½ feet	- Depression	— 3 58
		28 32 2
Sun's semi-diameter	- - - - -	+ 15 50.6
		28 47 52.6
Refraction—Parallax	- - - - -	— 1 37.6
True altitude of the sun	- - - - -	28 46 15
Diff. of latitude of the places of observation	-	+ 15 20
Sum		29° 1' 35'

	Sum	29° 1' 35"
Product of the diff. of latitude by the multiplier found by Tables XII. and XIII, according to Arts. 40 and 41, or $15' 20'' \times .84$	}	- 12 52
		- - -
Less altitude corrected for the place of the greater		28° 48' 43"

Greater Altitude.

Observed altitude of the sun	- - -	63° 49' 0"
Elevation of the eye $16\frac{1}{3}$ feet.	- Depression	- 3 58
		63 45 2
Sun's semi-diameter	- - - -	+ 15 50 .6
		64 0 52 .6
Refraction—Parallax	- - - -	- 24
True altitude of the sun	- - - -	64° 0 28 .6

Azimuths.

The azimuth corresponding to the multiplier found above, viz. .84, taken from Table XIV, is 81° , which is the greater azimuth, or that answering to the less altitude. The multiplier agreeing with the greater altitude is found in the same manner as that for the less, and is .05, and the corresponding azimuth 18° : which is less than a fourth of the greater: hence,

Half interval	33° 7' 42"	sin. 9.7376030	cotang. 10.1353551
Polar distance	81 46 51	sin. 9.9955020	comp. cos. 0.8447679
Half dist. of sun's places	} 32° 44' 38'	sin. 9.7331050	tang. 11.0301230
Double Distance		65 29 16	First angle at the sun 84° 40' 22"

Second angle at the Sun.

Greatest alt. of the sun	64° 0' 29"		
Least corrected ditto	28 48 43	comp. cos.	0.0573937
Dist. of the sun's places	65 29 16	comp. sin.	0.0410193
Sum	158° 18' 28"		
Half sum	79 9 14	cos.	9.2745544
Half sum — greater alt.	15 8 45	sin.	9.4171007
	Sum - - -		18.7900681
	Half sum - -	sin.	9.3950340
	Half 2nd angle at sun		14° 22' 44"

Angle of variation.

Half the 1st angle at the sun	- - -		42° 20' 11"
Half the 2nd ditto	- - -	Subtract.	14 22 44
Half the angle of variation	- - -	Difference	27° 57' 27"

Calculation of Latitude.

Half the angle of variation	27° 57' 27"	cos.	9.9461052
Sun's polar distance	- 81 46 51	$\frac{1}{2}$ sin.	4.9977580
Less corrected alt. of sun	- 28 48 43	$\frac{1}{2}$ cos.	4.9713031
Polar distance—less altitude	52° 58' 8"		
Difference from 90°	- 37 1 52		
Half difference	- 18 30 56	comp. cos.	0.0230829

Auxiliary angle 63° 9' 37"	{ sin.	9.9382492
	{ cos.	9.6968589

Cos. Auxil. angle — comp. cos. half difference.	Cos.	9.6737760
Half sum of latitude + 90°	- - -	61° 56' 52"
Double — 90	{ LATITUDE of the place of the } { greater altitude - - - }	33° 43' 44"

Example 101.

On the 22nd of January, 1814, being in north latitude $13^{\circ} 22' \frac{1}{2}$, and 35° of west longitude, by account, the altitude of

the sun's upper limb was taken at $1^h 6'$ by a well regulated watch, and found to be $57^{\circ} 6' 13''$. 10 afterwards, the altitude of his lower limb was found to be $30^{\circ} 31'$. The height of the observer's eye in both these observations was 24 feet above the level of the sea, and the difference in latitude and longitude during the interval was 12 miles towards the South, and 15 towards the East. Required the true latitude of the place where the greater altitude was observed. Ans. $15^{\circ} 23' N$.

Example 102.

Suppose that on the 15th of February, 1815, in South latitude $28^{\circ} 42'$, according to the ship's reckoning, at a time when the sun's declination was $12^{\circ} 55'$ South, the altitude of his lower limb was found to be $32^{\circ} 16'$; and four hours and a quarter afterwards the altitude of the same limb was again observed, and found to be 73° . The course of the vessel, during the interval, was South-East by East, at the rate of seven knots an hour, and the height of the eye at each observation 19 feet. Required the corrected latitude of the place where the last observation was made? Ans. $29^{\circ} 12' 18'' S$.

Example 103.

Suppose, that on the 21st of October, 1815, on board a vessel in North latitude $20^{\circ} 34'$, and East longitude $115^{\circ} 42'$, by account, the altitude of the sun's lower limb to be $17^{\circ} 35'$, at $8^h 10'$ A.M. by a watch that had been ascertained to be $38' 15''$ before true time, on the preceding evening, in East longitude $115^{\circ} 17'$. At $12^h 30'$ by the same watch, suppose the observed altitude of the sun's lower limb was again taken and found to be $58^{\circ} 48'$; the height of the eye in both these observations being 18 feet above the level of the sea; and the ship sailing at the rate of 6 knots an hour, on a North-East course; the height of the mercury in the barometer at the time of the last observation being 29.3 inches, and the thermometer at $76^{\circ} 46'$. Required the corrected latitude of the place of the last observation. Ans. $20^{\circ} 42' 18'' N$.

PRACTICAL EXAMPLES TO
CHAPTER IV.

Calculation of the horary angle from Altitudes of the Sun.
Art. 75.

Example 104.

SUPPOSE that on the 24th of May, 1814, about $7^h \frac{1}{4}$, A.M. civil time, in North latitude $43^\circ 15'$, and East longitude $23^\circ 30'$, the following observations of the sun's lower limb was made, when the elevation of the eye was 18 feet above the level of the sea. it is required to determine the time at the place of observation.

Times by the watch.				Observed altitudes.	
7^h	14'	38"	- -	29°	$2' 15''$
	15	21	- -		3 16
	16	0	- -		4 10
	17	25	- -		5 55
	18	37	- -		7 37
	19	37	- -		9 5
Sum	-	101' 36"	-	Sum	- 32' 18"
Mean time	7^h	16' 29"	Mean altitude	29°	$5' 23''$

The mean astronomical time reduced to the first meridian is the 23rd at $17^h 42' 29''$; and the true altitude of the sun's centre obtained by correcting the mean altitude, $29^\circ 5' 23''$, for depression of the horizon, semi-diameter, refraction and parallax is $29^\circ 15' 28''$. The corresponding declination is $20^\circ 51'$ North; and as the latitude is North also, the distance of the sun from the elevated pole is $69^\circ 9'$. Hence

True altitude of the sun	20° 15' 28"		
Latitude	43 13' 0"	- comp. cos.	0.8376454
Polar distance	69 9' 0"	- comp. sin.	0.0203654
Sum	111 20' 28"		
$\frac{1}{2}$ Sum	55 49' 44"	cos.	9.5169915
$\frac{1}{2}$ Sum — altitude	35 34' 16"	- sin.	9.8212707
Sum			19.4962750
$\frac{1}{2}$ Sum. sin.			9.7481375
Half the horary angle	34° 3' 4"		
Multiplying by	-	-	8
Horary angle in time			4 ^h 32' 24" ⁵
The observation, being made in the morning, } subtract the time from 12 hours			7 ^h 27' 35" ⁵
Time by the watch			7 16 29
Watch too slow			0 ^h 11' 6" ⁵

Example 105.

Being in latitude 40° 2' North, longitude 85° 50' West, on the 5th of August, 1814, at 6^h 30' A.M. by the watch, the observed altitude of the sun's lower limb was, from the mean of six observations, found to be 15° 49' 44", and the height of the observer's eye 16 feet above the level of the sea. Required the difference between the time by the watch and true time.

Ans. 3' 7" very nearly, too fast.

Example 106.

Suppose that on the 15th of November, 1814, at 3^h 0' 5" P.M. the mean observed altitude of the sun's upper limb was found, from a series of observations, to be 16° 4' 22", corresponding to the above time; the latitude of the vessel being 51° 42' North, and the longitude 33° 1' West, by account, and the height of the eye 14 feet above the surface of the water; what was the error of the watch?

Ans. 23' 33" too slow.

Example 107.

Suppose that in South latitude 33° 56', and East longitude 18° 12', by account, several altitudes of the sun's upper limb were observed, and from ascertaining the time of each by a

212 HORARY ANGLE FROM THE ALTITUDE OF A STAR.

good chronometer, the mean time is found to be 40 minutes past 3, P.M. on the 27th of March, 1815, and the mean corresponding altitude $25^{\circ} 20' 19''$. Required the time at the place of observation, and the error of the chronometer; the height of the eye being $17\frac{1}{2}$ feet above the level of the sea; the barometer $30\frac{1}{4}$ inches, and Fahrenheit's thermometer $77^{\circ} 82$.

Ans. $\left\{ \begin{array}{l} \text{True time} \quad - \quad 3^h \ 23' \ 34''. \\ \text{Chronometer too fast} \ 16 \ 26. \end{array} \right.$

Horary angle from the Altitude of a Star. Art. 77.

Example 108.

At 30 minutes past 4, P.M. by the watch, on the 14th of December, 1814, being on board a vessel in latitude $37^{\circ} 46'$ North and longitude $21^{\circ} 15'$ East, by account, the mean altitude of *Arcturus*, when East of the meridian, was found, from a series of observations, to be $34^{\circ} 7' 12''$; at the same time that the height of the observer's eye was 15 feet above the level of the sea. Required the true time at the place, and the error of the watch.

The time of the observation reduced to the meridian is $3^h \ 51'$. The declination of *Arcturus* was at that time $20^{\circ} 9' 32''$ North; and, as the latitude and declination are both of the same name, the polar distance of the star was $69^{\circ} 50' 28''$; and its right ascension, in time, $14^h \ 7' 11''$. The sun's right ascension at the same moment is $17^h \ 25' 27''$. Hence

The apparent altitude of <i>Arcturus</i>	-	$34^{\circ} \ 7' \ 12''$
Elevation of the eye 15 feet.	Depression	$- \ 3 \ 48$
		$34 \ 3 \ 24$
Refraction	-	$- \ 1 \ 24$
True altitude of <i>Arcturus</i>	-	$34^{\circ} \ 2' \ 0''$

HORARY ANGLE FROM THE ALTITUDE OF A STAR. 213

True altitude of the star	34° 2' 0"																																						
Latitude of the place	37 46 0	comp. cos.	0.1020918																																				
Polar distance of the star	69 50 28	comp. sin.	0.0274529																																				
Sum	141 38 28																																						
Half Sum	70 49 14	cos.	9.5165661																																				
Half Sum — Altitude	36 47 14	sin.	9.7773172																																				
Sum			19.4234280																																				
Half Sum		sin.	9.7117140																																				
Half the horary angle			30° 59' 24"																																				
Multiplying by			8																																				
Star East. Subtract	<table> <tr> <td>{</td><td>Horary angle in time</td><td>{</td><td>6^h 7' 55"</td></tr> <tr> <td>{</td><td>Right ascension of the star</td><td>{</td><td>14 7 11</td></tr> <tr> <td></td><td>Right ascen. of the meridian</td><td></td><td>9 59 16</td></tr> <tr> <td></td><td>Add 12 hours</td><td></td><td>12</td></tr> <tr> <td></td><td></td><td></td><td>21 59 16</td></tr> <tr> <td></td><td>Sun's right ascension</td><td></td><td>17 25 27</td></tr> <tr> <td></td><td>True time at the place</td><td></td><td>4 33 49</td></tr> <tr> <td></td><td>Time by the watch</td><td></td><td>4 30</td></tr> <tr> <td></td><td>Watch too slow</td><td></td><td>0^h 3' 49"</td></tr> </table>			{	Horary angle in time	{	6 ^h 7' 55"	{	Right ascension of the star	{	14 7 11		Right ascen. of the meridian		9 59 16		Add 12 hours		12				21 59 16		Sun's right ascension		17 25 27		True time at the place		4 33 49		Time by the watch		4 30		Watch too slow		0 ^h 3' 49"
{	Horary angle in time	{	6 ^h 7' 55"																																				
{	Right ascension of the star	{	14 7 11																																				
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	True time at the place		4 33 49																																				
	Time by the watch		4 30																																				
	Watch too slow		0 ^h 3' 49"																																				

Example 109.

Being at sea on the 7th of July, 1814, and in S. latitude 29° 12', and E. longitude 55° 15'; according to the ship's reckoning, at 21 minutes past three o'clock, A.M. by a watch that was about 20' too slow, when the apparent altitude of *Antares*, West of the meridian, was ascertained to be 7° 50' 58", and the height of the observer's eye was 21 feet above the surface of the water. It is required to calculate the true time at the place of observation, and to ascertain the error of the watch.

Ans. { True time 3^h 40' 3" A.M.
 { Watch too slow 0 19 3

Example 110.

Suppose it should be ascertained, from a series of six observations, taken on the evening of the 1st of February, 1815, that

the mean altitude of *Pollux*, when East of the meridian, was equal to $36^{\circ} 45' 32''$, and the mean corresponding time equal to $6^h 12' 40''$, P.M. by a good seconds watch. The latitude being $53^{\circ} 24'$ North, and longitude $25^{\circ} 16'$ West, by account, and the height of the eye 18 feet; also the height of the barometer equal to 30.2 inches, and Fahrenheit's thermometer $28^{\circ} 4'$. Required the difference between the true time at the place of observation and that indicated by the watch.

Ans. Watch too fast, $0^h 0' 37''$.

Example 111.

Suppose, that on the 20th of September, 1815, in South latitude 40° , and East longitude 110° , when the chronometer on board the vessel, which was about $1^h 53'$ before true time at the place of observation, was $9^h 13'$ P.M. the mean altitude of *Fomalhaut*, East of the meridian, was, from a series of six observations, ascertained to be $45^{\circ} 11' 12''$; the height of the mercury in the barometer 30.2 inches, Fahrenheit's thermometer standing at 72° , and the height of the observer's eye being 19 feet above the level of the sea. Required the true time at the place of observation, and the error of the chronometer.

Ans. $\left\{ \begin{array}{l} \text{True time at the place of observ. } 7^h 21' 4'' \\ \text{Chronometer too fast} \quad \quad \quad 1 \quad 51 \quad 56. \end{array} \right.$

CALCULATION OF ALTITUDES.

True Altitudes of the Sun. Arts. 78, 80.

Example 112.

Required the true altitude of the sun on the 14th of July, 1814, at $3^h 42' 20''$ P.M. in latitude $5^{\circ} 55' 45''$ S. and longitude $152^{\circ} 3' E$.

As the time for which the altitude is required is after noon, the time expressed in degrees will give the horary angle; and the other elementary quantities of the calculation will be easily found by the preceding rules: hence,

The horary angle - $55^{\circ} 36' 30''$ The given latitude - $5^{\circ} 55' 45''$ Sun's declination - $21^{\circ} 44' 48''$ Polar distance - $111^{\circ} 44' 48''$ Half the horary angle - $27^{\circ} 48' 15''$ - cos. 9.9467210 Polar distance - $111^{\circ} 44' 48''$ - $\frac{1}{2}$ sin. 4.9839684 Latitude - $5^{\circ} 55' 45''$ - $\frac{1}{2}$ cos. 4.9988352 Polar dist. — latitude - $105^{\circ} 49' 3''$ Difference from 90° - $15^{\circ} 49' 3''$ Half difference from 90° - $7^{\circ} 54' 16''$ comp. cos. 0.0041458 Auxiliary angle - sin. 9.9836704 Ditto - cos. 9.7101529 (Cos. auxil. angle — comp. cos. $\frac{1}{2}$ diff. 90°) - cos. 9.7060071 $\frac{1}{2}$ (90° + altitude) - $59^{\circ} 27' 36''$ (Double — 90°). TRUE ALTITUDE of the sun - $28^{\circ} 55'$ *Example 113.*

Required the sun's altitude on the 27th of October, 1814, at $9^h 10' 15''$ A.M. in North latitude $48^{\circ} 24'$, and West longitude $58^{\circ} 33' 21''$.

Ans. $12^{\circ} 5' 38''$.*Example 114.*

What will be the true altitude of the sun at 10 minutes past four, P.M. on the 12th of May, 1815, in South latitude $23^{\circ} 30'$, and East longitude $12^{\circ} 48'$?

Ans. $19^{\circ} 3' 40''$.*Example 115.*

Required the sun's true altitude on the 4th of August, 1815, at $7^h 50' 50''$, A.M. by a watch that had been ascertained to be $27\frac{1}{2}$ too slow. The latitude of the place being $15^{\circ} 40'$ North, and the longitude $72^{\circ} 0' 45''$ West.

Ans. $37^{\circ} 0' 56''$.*True Altitudes of the Moon. Arts. 79, 80.**Example 116.*

Being in North latitude $26^{\circ} 47'$, and East longitude $36^{\circ} 45'$ on the 4th of May, 1814, at 58 minutes past ten in the evening,

by a watch that was 13' 5" before true time. Required the true altitude of the moon's centre at that moment.

Time by the watch	-	-	10 ^h 58'
Watch too fast	-	Subtract	13' 5"
<hr/>			
True time at the place of the req. alt.	-	-	10 44 55
Longitude, East, in time	-	-	2 27 0
Time reduced to the first meridian	-	-	8 ^h 17' 55"

Time at the first place of the required alt. 10^h 44' 55"

Sun's right ascension - - Add. 2 44 49

Right ascension of the meridian - 13 29 44

Ditto in degrees - - - 202 26 0

Right ascension of the moon - - 227 11 0

Horary angle of the moon, to the East - 24 45 0

Declination of the moon S. - - 12 51 0

Distance from the elevated pole - 102 51 0

Half the horary angle - 12° 23' - cos. 9.9897766

Polar distance - { 102 51 - ½ sin. 4.9944924

Latitude - { 26^h 47 - ½ cos. 4.9753569

Polar dist. — latitude - 76 4

Difference from 90° - 13 56

Half diff. from 90° - 6 58 comp. cos. 0.0032183

Auxiliary angle - { sin. 9.9628442
cos. 9.5983679

(Cos. auxil. angle — comp. cos. ½ diff.) - cos. 9.5951496

½ (90 + altitude) - - 66° 49'

(Double — 90°). TRUE ALTITUDE of the moon - 43 38'.

Example 117.

Required the true altitude of the moon's centre, on the 8th of September, 1814, at 4^h 30' 35", A.M. by the watch, in latitude 11° 6' South, and longitude 72° 13' 21" East; the watch by which the time was ascertained having been found, from altitudes of the sun taken on the preceding evening, to be 21' 14" too slow.

Ans. 49° 11' 46".

Example 118.

It is required to calculate the true altitude of the moon's centre on the 12th of April, 1815, at 5^h 50' 10", P.M. in latitude 37° 44' S. and longitude 47° 27' W. Ans. 18° 48' 52".

Example 119.

Suppose that on the 24th of August, 1815, the true altitude of the moon's centre was required; but that the horizon could not then be sufficiently distinguished to admit of its being ascertained by observation. Also, that at the moment for which the altitude is required, a good watch, which, at 20 minutes past nine on the preceding morning, had been found to be 11' 31 $\frac{1}{2}$ " too fast, and to have regularly gained 3 $\frac{3}{4}$ " per day since the last time it had been regulated, indicated 11^h 35' 42" P.M. The latitude of the place of the required altitude being 18° 41' 10" N. and the longitude 63° 16' 20" W. and the height of the eye 16 feet. The true altitude is required from calculation.

Ans. 48° 12'.

*True Altitudes of the Stars. Arts. 79, 80.**Example 120.*

On the 26th of February, 1814, at 40 minutes past three in the afternoon, being in North latitude 47° 23', and West longitude 32° 48' 14", the chronometer on board was found to be 12' 29" too fast with respect to true time. The vessel then pursued a NW. by W. course, at the rate of six knots an hour for the space of three hours and ten minutes. Required the true altitude of the star *Pollux* at the termination of this course.

Length of course 19 miles	{ Change in latitude	-	10' 36" W.
	{ Ditto in longitude	-	15 48 W.
Latitude of the vessel at the first observation	-	47° 23' 0" N.	
Change in latitude	- Add.	10 36	
Latitude of the place of the required altitude	-	47 33 36	
Longitude at the place of the first observation	-	32° 48' 14" W.	
Change	- Add.	15 48	
Longitude of the place of the required altitude	-	33° 4' 2'	

Time by the watch at the first observation	-	3 ^h 40' 3"
Duration of course	- - - Add.	3 10 0
Watch too fast	- - - Subtract.	12 29
True time at the place of the required altitude		6 37 31
Longitude West, in time	- - - Add.	2 12 16
True time reduced to the first meridian	-	8 ^h 49' 47"
Time at the place of the required altitude	-	6 ^h 37' 31"
Sun's right ascension	- - - Add.	20 59 44
		27 37 15
	* Subtract.	24
Right ascension of the meridian	- -	3 ^h 37' 15"
In degrees	- -	47° 29' 0"
Right ascension of the star <i>Pollux</i>	- -	107 15 40
Horary angle	- - - -	} 59 46 40
The star to the East of the meridian	- -	
Declination of <i>Pollux</i>	- - - N.	28 27 58
Distance from the elevated pole	- -	61 32 2
Half the horary angle	- 29° 53' 20" -	cos. 9.9380158
Polar distance	- 61 32 2 -	$\frac{1}{2}$ sin. 4.9720189
Latitude	- - - 47 33 36 -	$\frac{1}{2}$ cos. 4.9145932
Polar dist. — latitude	- 13 58 26	
Difference from 90°	- 76 1 34	
Half diff. from 90°	- 38 0 47	comp. cos. 0.1035468
Auxiliary angle.	- {	sin. 9.9281747
	- {	cos. 9.7248392
(Cos. auxil. angle — comp. cos. $\frac{1}{2}$ difference)	-	cos. 9.6212924
$\frac{1}{2}$ (90° + altitude)	- -	65° 17' 5"
(Double — 90°). TRUE ALTITUDE of <i>Pollux</i>	-	40 34 10

Example 121.

Required the altitude of the star *Fomalhaut*, on the 21st of September, 1814, at midnight, in South latitude 8° 49', and East longitude 87° 21' 15".

Ans. 56° 30'.

Example 122.

Calculate the altitude of *Aldebaran*, in South latitude $25^{\circ} 23'$ and West longitude $30^{\circ} 10' 12''$, on the 25th of January, 1815, at 20 minutes past ten at night.

Ans. $35^{\circ} 24'$.

Example 123.

Find the altitude of *Antares* on the 20th of August, 1815, at 32 minutes past eleven at night, in latitude $25^{\circ} 31'$ South, and longitude $36^{\circ} 31'$ East.

Ans. $21^{\circ} 13'$.

PRACTICAL EXAMPLES TO

CHAPTER V.

*Regulation of Marine Chronometers. Art. 88.**Example 124.*

Being in South latitude $30^{\circ} 25'$, and East longitude $78^{\circ} 27'$, on the 5th of February, 1814, the mean altitude of the sun was found, from a series of six observations, to be $34^{\circ} 20'$, and the mean corresponding time by the chronometer, $7^h 58' 5''$ A.M. On the 11th of the same month, a second series of observations was made, from which the sun's mean altitude was ascertained to be $29^{\circ} 31' 48''$, and its corresponding time by the same chronometer $7^h 20''$ A.M. The latitude of the place of the second observation was $32^{\circ} 48'$ South, and its longitude $83^{\circ} 37'$ East. Whether had the chronometer gained or lost, with respect to mean time, during the interval between the observations, and what was its rate?

True time of the first observation, found by	}	-	$8^h 3' 5''$
the rules in art. 75	- - - -		
Equation of time February 5	- - - -	+	$14 22$
Mean time of the first observation	- -		$8 17 27$
Time by the chronometer at ditto	- - -	-	$7 58 5$
Chronometer too slow Feb. 5th at $7^h 58' 5''$ A.M.			$0^h 19' 22''$
True time of the 2nd observation, Feb. 11th	-		$7^h 45'' 3''$
Difference of longitude in time	- - -	-	$20 40$
True time of the second observation reduced to	}		
the place of the first	- - - -		$7 24 23$
Equation of time February 11th	- - -	+	$14 36$
Mean time of the second observation	- -		$7^h 38' 59''$

Mean time of the 2nd observation	-	-	-	7 ^h 38' 59"
Time by the chronometer	-	-	-	7 20 0
Chronometer too slow, February 11th, at 7 ^h 20' A.M.	-	-	-	18 59"
Ditto February 5th at 7 ^h 58' 5", A.M.	-	-	-	19 22
Gain in 6 days	-	-	-	23"
Divide by 6. GAIN in 24 ^h , or RATE	-	-	-	3 ⁷ / ₈

Example 125.

On the 30th of August, 1814, the true altitude of the sun's centre, found from a series of observations, was $11^{\circ} 52'$, and the corresponding time by the chronometer $5^h 27' 16''$ P.M.; the latitude of the place of observation being $48^{\circ} 35'$ North, and the longitude $62^{\circ} 43'$ West. On the 7th of the following month, at $5^h 30' 31''$ P.M. the altitude was again found equal to $8^{\circ} 54' 25''$, the latitude and longitude being the same as before. Required the gain or loss of the chronometer with respect to mean time, and its rate during the interval between the observations.

Ans. { Gain during the interval $48'' \cdot 8$.
 { Daily rate increasing $6 \cdot 1$.

Example 126.

Suppose, on the 1st of May, 1815, at $5^h 2'$ P.M. by the chronometer, the mean altitude of the sun's lower limb, found from a series of 6 observations, to be $20^{\circ} 26' 35''$; the latitude of the place of observation being $53^{\circ} 21'$ North, and longitude $3^{\circ} 57' 20''$ East. Again, on the 12th of May, at the same place, the mean altitude being $14^{\circ} 11' 53''$, and the corresponding time, by the same chronometer, 6^h P.M. The height of the eye, above the surface of the sea in both cases being 18 feet; required the variation of the chronometer from mean time at each observation, and its daily rate during the interval between the observations.

Ans. { At the first obs. Chronom. too fast $3' 45'' \cdot 8$.
 { At the second obs. chronom. do. $3 31 \cdot 6$
 { Daily rate during the interval, loss $0 1 \cdot 17$.

Example 127.

On the 3rd of October, 1815, at $8^h 35'$ A.M. suppose the mean

altitude of the sun's centre to be $38^{\circ} 53'$, in South latitude $5^{\circ} 59'$, and East longitude $105^{\circ} 32'$. And again on the 15th of the same month, at $10^h 20'$ A.M. suppose his mean altitude from a series of observations to be $65^{\circ} 4' 40''$, the latitude of the place of observation being $9^{\circ} 10'$ South, and longitude $104^{\circ} 54'$ E. Required the rate of the chronometer by which the time was observed, the height of the eye at each observation being 20 feet above the level of the sea.

Ans. Daily gain $6^m 9^s$ nearly.

Longitude by Marine Chronometers. Art. 93.

Example 128.

On the 20th of February, 1814, that is 9 days after the chronometer had been ascertained to be $18' 59''$ too slow, and to gain $3^m 5$ in 24 hours, (See example 124,) six altitudes of the sun's upper limb were taken, and the mean altitude found to be $38^{\circ} 50'$, and the mean corresponding time of the observation, by the chronometer, $9^h 7' 10''$ A.M. the longitude of the place where the chronometer was regulated was $83^{\circ} 37'$ East, and the height of the observer's eye 21 feet above the level of the sea. The latitude of the place where the mean altitude of the sun was taken was $34^{\circ} 10'$ South, and the longitude, by account, $75^{\circ} 21'$ E. Required the true longitude of the vessel at the place of the last observation.

Latitude, by account	-	-	-	-	-	$34^{\circ} 10'$	0
Longitude by ditto	-	-	-	-	-	$75^{\circ} 21'$	0
Mean observed altitude of the sun's upper limb						39	11 46
Elevation of the eye 21 feet. Depression					-	-	4 30
Sun's apparent altitude						39	7 16
Sun's semi-diameter	-	-	-	-	-	-	16 12
						38	51 4
Refraction — Parallax	-	-	-	-	-	-	1 4
True altitude of the sun	-	-	-	-	-	38	50 0

Time by the chronometer corresponding to the mean altitude, February 20th	}	9 ^h 7' 10"
Chronometer too slow when regulated, February 11th at 7 ^h 20' A.M.	}	+ 18 59
		<hr/> 9 26 9
Gain in nine days, from the rate 3 ^h 3 ^m 3 ^s		- 34
		<hr/> 9 25 35
Equation of time		- 14 8
		<hr/> 9 11 27
True time at the place of regulation		
Longitude East 83° 37' in time		- 5 34 28
		<hr/> Time at the first meridian, A.M. 3 ^h 36' 59"
Declination of ☉, South		11° 9' 7"
Distance from the elevated pole		78 50 53
True altitude ☉.	38° 50' 0"	
Latitude	34 10 0	comp. cos. 0.0822806
Polar distance	78 50 53	comp. sin. 0.0082790
Sum	161 50 53	
$\frac{1}{2}$ Sum	75 55 26	cos. 9.3859840
Half sum — altitude	37 5 26	sin. 9.7803724
		sum 19.2569160
	$\frac{1}{2}$ Sum	sin. 9.6284580
Half the horary angle		25° 9' 18"
	Multiplying by	8
Horary angle in time		3 ^h 21' 14"
Comp. to 12 ^h . Time at the vessel		8 38 46
Time at the place, where the chronometer was regulated	}	9 11 27
		<hr/> Vessel to the West of the place of regulation 0 ^h 32' 41"
	In degrees	8° 10' 15"
Longitude of the place where the chronometer was regulated	}	83 37
		<hr/> LONGITUDE required - - - - - E. 75° 26' 45"

Example 129.

Suppose, that on the 10th of July, 1814, the true time was found to be $2^h 52' 12''$ P.M. at the moment that a well regulated chronometer, which was known to be $12^h 27''$ before true time at the first meridian, gave $5^h 50' 10''$. Required the longitude of the place.

Ans. $41^\circ 7' 45''$ West.

Example 130.

On the 15th of May, 1815, suppose the true time, found by an altitude of the sun, to be $7^h 56' 54''$ A.M. civil time, at the moment that a well regulated chronometer, which was $13\frac{1}{2}$ behind mean time at the first meridian, gave $6^h 8' 30''$. Required the longitude of the place of observation.

Ans. $23^\circ 1' 45''$ East.

Example 131.

Suppose, on the 21st of August, 1815, at $6^h 25'$ P.M. true time, obtained by an altitude of the sun, that a chronometer, which was then with mean time at the first meridian, indicated $10^h 39' 33''$, and was found to gain $3''$ daily. Also, on the 2nd of September following, in latitude $34^\circ 28'$ North, and longitude $75^\circ 45'$ West. Suppose the altitude of the sun's lower limb to be $21^\circ 50' 20''$ when the same chronometer gave $9^h 37' 20''$ P.M. the height of the eye above the surface of the sea being 16 feet. Required the true longitude of the vessel at this last station.

Ans. $75^\circ 29'$ West.

Correction of Longitude found by Chronometers. Art. 97.

Example 132.

On the 22nd of March, 1814, at 3^h P.M. from observations of the sun's altitude, the chronometer on board a vessel was found to be $37' 15''$ too fast, in longitude $57^\circ 24'$ West; and to have a daily rate of increase equal to $2''.1$. On the 2nd of May following, at ten minutes past five in the afternoon, the same chronometer was found to be too slow with respect to

mean time at the place of observation, by $1^h 18' 22''.5$; and the daily rate of increase was then $3''$. Required the correction to be applied to the longitude of this last place of observation, as found from the first daily variation of the chronometer, and also the corrections of the longitude found by the same means on the 30th of March, and the 12th and 21st of April.

Daily variation of the chronometer at the 1st obser.	-	$2''.1$
Ditto at the second observation	-	$3''.6$
	Sum	$5''.7$
Mean variation	$\frac{1}{2}$ Sum	$+ 2''.85$
Chronometer too fast at the first observation	-	$37' 15''.4$
Accumulated gain from March 22nd to May 2nd	}	$+ 1 26.3$
41 days $2^h 10'$ at $2''.1$ per day		
Chronometer too slow at the 2nd observation	+	$1^h 18' 22''.5$
Diff. of long. in time, between the two places	}	$1^h 57' 4''.2$
of obs. according to the first variation, $2''.1$		
Difference of longitude in degrees	-	$29^\circ 16' 3''$
Difference of longitude calculated in the same	}	$29 23 45$
manner with the mean variation, $2''.85$		

Since, by the nature of the question, the vessel was evidently sailing eastward, the correction of the diff. of long. on the 2nd of May, calculated with the first daily variation, $2''.1$, is } *diff.* $7' 42''$

The place arrived at is therefore East of that which is found by means of the first diurnal variation.

Correction of longitude $7' 42''$, or $462''$	-	log.	2.6646420
Multiple, from Table XI, answering	}	365, comp. log.	7.0629839
to 41 days $2^h 10'$, between March 22nd, and May 2nd			
Constant logarithm	Sum	-	1.7276259
From the 22nd to 30th March, 8 days,	}	36 log.	1.5563025
Multiple from Table XI.			
	Sum		1.2839284

Sum 2839284

Correction for the longitude found March 30th - 29' 23"

By adding the logarithm of the multiple answering
 to 21 days, from March 22nd to April 12th, } = 2' 3".4
 Table XI. the correction - - - - - }

Also for the 21st of April the correction is found } = 4' 8".4
 in the same manner and is - - - - - }

Example 133.

On the 12th of August, 1814, the watch was found to lose 5".3 in 24 hours, and to correspond to mean time at 8^h 23 A.M. in longitude 30° 14' 22" West. On the 1st of September following, the same watch was found to lose 3" per day, and to be 1^h 14' 36" behind mean time at the place of observation. Required the corrections to be applied to the longitude found by this watch on the 22nd and 27th of August, on account of the variation in its rate.

Ans. { On the 22nd 16".76.
 { On the 27th 36".57.

Example 134.

Having regulated the chronometer to mean time at Plymouth harbour, previously to sailing for the West Indies, on the 12th of November, 1814, at 9^h 42' A.M. when it corresponded with mean time at that port, and the variation for the last 24 hours was nothing: but on the 27th of the same month, at a quarter before four P.M. the same chronometer was found to be 53' 24".8 before mean time at the place of observation, and to be gaining at the rate of 4".6 per day. Required the correction to be applied to the longitude of this last place of observation found from the chronometer, as regulated at Plymouth harbour, and also that which must be applied to the longitude found by the same means on the 20th of the month, the place of departure being in longitude 4° 8' 10" West.

Ans. { On the 20th 1' 38".
 { On the 27th 5' 57".

Example 135.

Suppose that on the 6th of June, 1815, before sailing from

the Cape of Good Hope, longitude $18^{\circ} 24'$ East, the watch was found to gain $2'' 4'$ per day, with respect to mean time, but after 25 days sailing it was ascertained to lose $3'' 7'$ per day. Required the corrections which it is necessary to apply to the longitudes found by this watch on the 15th, 20th, and 28th of the same month.

Ans. On the 15th $0' 23''$.
On the 20th $0' 53''$.
On the 28th $2' 7''$.

PRACTICAL EXAMPLES.

CHAPTER VI.

Longitude from distances of the Moon from the Sun and Stars.

Method of finding the Altitude of the heavenly bodies, the distance of which has been observed. Art. 104.

Example 136.

ON the morning of the 15th of March, 1814, before taking the distance between the sun and moon, the mean altitude of the sun's lower limb, from a series of observations, was found to be $12^{\circ} 40' 3''$, and the corresponding time $7^h 10' 14''$. The altitude of the moon's upper limb, when West of the meridian, was likewise ascertained to be $37^{\circ} 59' 31''$, and the time, by the watch, answering to this observation $7^h 11' 2''$. The time answering to the mean observed distance between the nearest limbs of these bodies was $7^h 13' 9''$. After this, the altitude of the sun's lower limb was again found to be $13^{\circ} 35' 39''$, and the corresponding time $7^h 14' 53''$. Also the altitude of the moon's upper limb, and the corresponding time of the observation were again found to be $37^{\circ} 25' 56''$, and $7^h 16' 5''$. Required the true altitudes of the centres of these bodies at the instant of the mean observed distance, the height of the observer's eye being $21\frac{1}{2}$ feet above the level of the sea.

	Times.		Altitudes of the Sun.	
1st observation	-	$7^h 10' 14''$	-	$12^{\circ} 40' 3''$
2nd observation	-	$7^h 14' 53''$	-	$13^{\circ} 35' 39''$
1st interval	-	$0^h 4' 39''$	Difference	$0^{\circ} 55' 36''$

Time of the first observation	7 ^h 10' 12"
Time of observing the mean distance	7 ^h 13' 9"
2nd interval	0 ^h 2' 55"

Then

1st inter. 4' 39" : 2nd inter. 2' 55" :: 1st change in alt. 55' 36" :
2nd change in altitude. By logarithms

1st interval	4' 39" = 279"	comp. log.	7.5543958
2nd interval	2' 55" = 175	log.	2.2430380
1st change in altitude	55' 36" = 3336	log.	3.5232260
2nd change in altitude	34' 52" = 2092	log.	3.3206598

Observed altitude of the sun	12° 40' 3"
2nd change in altitude. <i>Sun ascends.</i>	Add 34' 52"
Altitude of the sun at the time of the observed dist.	13° 14' 55"
Sun's semi-diameter	+ 16' 6"
	Sum 13° 31' 1"
Height of the eye 21½ feet	Depression — 4' 33"
Refraction — Parallax	— 3' 23"
True altitude of the SUN	13° 28' 5"

	Times.		Altitudes of the Moon.
1st observation	7 ^h 11' 2"		37° 59' 31"
2nd observation	7 ^h 16' 5"		37° 25' 56"
1st interval	0 ^h 5' 3"	Difference	0° 33' 35"
Time of the first observed altitude			7 ^h 11' 2"
Time of observing the mean distance			7 ^h 13' 9"
2nd interval			0 ^h 2' 7"

Then

1st inter. 5' 3" : 2nd inter. 2' 7" :: 1st change in alt. 33' 35" :
2nd change in altitude. By logarithms,

1st interval	5' 3" = 303"	comp. log.	7.5185574
2nd interval	2' 7" = 127	log.	2.1038037
1st change in altitude	33' 35" = 2015	log.	3.3042751
2nd change in altitude	14' 5" = 845	log.	2.9266362

236 ALTITUDE OF THE HEAVENLY BODIES, &c.

1st observed altitude of the moon	- - -	37° 59' 31"
2nd change in altitude. Moon descends. Subtract		5
Altitude of the moon at the time of the obs. distance		37° 45' 26"
Semi-diameter of the moon	- - -	14 58
	Difference	37 30 28
Height of the eye 21½ feet	- - -	Depression - 4 33
Parallax — Refraction of moon	- - -	+ 41 52
True altitude of the moon	- - -	38° 7' 47"

Example 137.

Suppose the distances and altitudes of the sun and moon were observed to be as follow: it is required to find the true altitudes at the time corresponding to the mean distance; supposing the observations to have been made on the morning of the 19th of July, 1814, and the height of the eye 15 feet above the level of the sea.

Times.	Observed alt. ☉'s lower limb.		Times.	Observed alt. ☾'s lower limb.
8 ^h 2' 30"	- 39° 42'	-	8 ^h 2' 0"	- 20° 46'
7 0	- 40 20	-	6 10	- 21 20
Diff. 0 ^h 4' 30"	Diff. 0° 38'		Diff. 0 ^h 4' 10"	Diff. 0° 34'

Time.		Obs dist. Sun and Moon's nearest limbs.
8 ^h 3' 20"	-	40° 0' 0"
4 20	-	0 30
5 50	-	1 30
Mean 8 ^h 4' 30"	-	Mean 40° 0 40"

Ans. True altitude. { Sun's centre 40° 9' 50'.
Moon's centre 22 12 25.

Example 138.

Let it be supposed that the distance between the nearest limbs of the sun and moon was observed at 6^h 10' 21" P.M. when they were both West of the meridian, and that the altitude of the sun's lower limb was found to be 18° 12' 16" at 6^h 5' 14"; and that of the moon's lower limb 31° 11' 24", at 6^h 7' 12". Also

that the altitude of the same limb of the sun at 6^h 24', was 17° 56' 38"; and likewise that the altitude of the moon's lower limb was 30° 17' 15", at 6^h 12'. Required the true altitude of the centre of each of these bodies at the time of observing the distance between them, the height of the observer's eye being 18 feet above the surface of the sea; and the observations made on the 5th of April, 1815.

Ans. { Altitude sun's centre 18° 56' 51".
{ Ditto Moon's centre 31° 31' 57".

Example 139.

Suppose the distance between the farthest limb of the moon and the star Antares to have been observed, at 10^h 2' 41" P.M. on the 12th of October, 1815; when the former was East, and the latter West of the meridian; and that 2' 3" before this observation, the altitude of the moon's lower limb was ascertained to be 43° 12' 14", and that of the star at 10^h 1' 3", was 57° 4'. Also, that at 10^h 3' 56", the altitude of the moon's lower limb was again found to be 44° 3', and that of Antares 56° 25' 7". Required the respective altitudes of these bodies at the moment of observing the distance between them, the height of the eye being 19 feet.

Ans. { True altitude of moon's centre 41° 32' 16".
{ True altitude of Antares - 56° 37' 27".

Calculation of the true distance, the time at the first Meridian, and the Longitude. Art. 109, &c.

Example 140.

Suppose that on the 8th of August, 1814, about 6^h 30' A.M. in North latitude 51° 30', and East longitude, by account, 24°, the mean observed distance between the nearest limbs of the sun and moon was 99° 8' 20", as ascertained by a series of six observations. The observed altitude of the sun's upper limb being 8° 27' 39", and that of the moon's lower limb 54° 27' 4". The height of the observer's eye being 16 feet; that of the barometer 30.3 inches, and of Fahrenheit's thermometer 67° 7'. Required the true longitude of the vessel at the time of the observation.

Elements of the Calculation.

Latitude North	51° 30' 0"	Moon's observed altitude	54° 27' 4"
Time by account	6 ^h 30' 0"	Depression of the horizon	— 3 55
Longitude in time E.	1 36 0	Diff.	54 23 9
Time at the 1st merid.	4 54 0	Moon's semi-diameter	+ 16 0
Sun's semi-diameter	15 48	Moon's apparent altitude	54 39 9
Moon's semi-diameter	15 47	Parall. — Refraction	32' 40"
Aug. of moon's semi-dia.	13	Thermometer 67° F	+ 1
Moon's semi-dia. corrected	16 0	Barometer 30.3 in.	— 1
Moon's equatorial paral.	57 50	Moon's true altitude	55 11 49
Diminution of parallax	— 7	Sun's Declin. { 7th August at noon	16 33 50
Moon's parallax corrected	57 43	8th August date	16 17 1
Sun's observed altitude	60 27' 39"	Difference in 24 hours	16 49
Depression of horizon	— 3 55	Proportional diff. for 17 hours	12 10
Diff.	8 23 44	Declin. at the time of observ. N.	16 11 40
Sun's semi-diameter	— 15 48	Diff. from 90°. Polar distance	73 48 20
Sun's apparent altitude	8 7 56	Observed distance ☉ ☾	99° 8' 20"
Refract — Par. 5' 52"		Sun's semi-diameter	15 48
Therm. 67° F — 9	— 5 48	Moon's ditto	16 0
Baromet. 30.3 in. — 5		Apparent distance ☉ ☾	99° 40' 8"
Sun's true altitude	8° 2' 8"		

Calculation of the true distance.

Apparent distance ☉ ☾	99° 40' 8"		
Sun's appar. altitude	8 7 56	comp. cos. 0.0042156	
Moon's ditto	54 39 9	comp. cos. 0.2376331	
Sum	162 27 13		
$\frac{1}{2}$ Sum	81 13 36	cos. 9.1837853	
Appar. dist. — $\frac{1}{2}$ Sum	18 26 32	cos. 9.9771029	
Sun's true altitude	8 2 8	cos. 9.9957148	
Moon's ditto	55 11 49	cos. 9.7564515	
Sum	163 13 57	Sum	39.1550992
$\frac{1}{2}$ Sum	81 36 58	$\frac{1}{2}$ Sum	19.5775496
Auxiliary angle	—	cos. { 9.9202252	9.6473244 sin. aux. angle.
Sum — 10	—	cos. { 9.9523248	26° 21' 30" aux. angle.
Half distance	—	sin. { 9.8825300	
Double TRUE DISTANCE Required		49° 44'	
		99 23	

CALCULATION OF TIME AT THE FIRST MERIDIAN.

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Calculation of the time at the first Meridian.

Dist. in table	at 3 ^h A.M. 100° 22' 6"	1st diff 1° 53' 45" = 5625"	comp. log. 6-2498775
	at 6 ^h 93 53.21	1st interval 3 ^h = 10800	log. 4-0334238
True distance	99 26.0	2nd diff 59' 6" = 3546	log. 3-5497387
	2nd interval	1 ^h 53' 37" = 6808	Sum 9-8530400
Time of the first distance in the Table		3 ^h	
Second Interval		(Add.) 1 53' 37"	
True Time at the first Meridian		Sum 4 ^h 53' 28"	

Calculation of the time at the place of observation, and of the Longitude.

Sun's true altitude	8° 2' 36"		
Latitude of the place	51 30 0	comp. cos.	0-2058504
Polar distance	73 48 20	comp. sin.	0-0175837
Sum	133 20 28		
Half Sum	66 40 14	cos.	9-5982997
Half Sum — Sun's true alt.	58 38 6	sin.	9-9513912
		Sum	19-7531250
		Half Sum — sin.	9-8765625
		Half horary angle	48° 49'
		Multiplying by	
True Time at the place of observation			6 ^h 30' 39"
Time at the first meridian			4 53 28
Longitude in time			1 ^h 37' 41"
Longitude required, in degrees			24° 16' E.

Example 141.

Suppose that on the 25th of April, 1814, about 4^h P.M. in latitude 19° 59' South, and longitude 60° East, by account, the following observations of the sun and moon were taken. Required the true longitude of the vessel at the time of the mean observation.

234. CALCULATION OF TIME AT THE FIRST MERIDIAN.

Observed distance ☉'s nearest limb	Observed altitude ☉'s lower limb.	Observed altitude ☉'s upper limb.
69° 43' 0"	24° 18'	45° 51'
44 10	24 0	52
44 40	23 36	53
45 30	23 0	55
46 0	22 42	56
46 30	22 18	57

Ans. 60° 14' 43" East from Greenwich, or the first meridian.

Example 142.

On the 18th of October, 1844, being in North latitude $34^{\circ} 47'$, and West longitude $37^{\circ} 30'$, by account, it was ascertained by a series of observations at 4^h 12' P.M. that the distance between the nearest limbs of the sun and moon was $61^{\circ} 55' 8''$. The observed altitude of the sun's upper limb at the time of taking the distance, found by a simultaneous observation, was $16^{\circ} 2' 48''$; and that of the moon's lower limb, found by the same means, $32^{\circ} 51' 45''$. The height of the thermometer was $63^{\circ} 8'$; that of the barometer 29.2 inches, and the eye 18 feet above the level of the sea. Required the true longitude of the vessel at the time answering to the mean observed distance.

Ans. $37^{\circ} 51' W.$

Example 143.

On the 4th of May, 1845, at 6^h 52' 40" A.M. civil time, in latitude $48^{\circ} 10'$ North, and longitude $8^{\circ} 7'$ East, by account, suppose the distance between the nearest limbs of the sun and moon to be $59^{\circ} 36'$, the altitude of the sun's lower limb being at the time of observing the distance, $16^{\circ} 51' 35''$, and that of the upper limb of the moon $25^{\circ} 2' 52''$. Also the heights of the barometer and thermometer such as to counterbalance each other's influence, and the elevation of the eye 21 feet. Required the true longitude of the place of observation.

Ans. $7^{\circ} 47' 15'' E.$

Longitude from the distance between the Moon and a Star. Art. 120.

Example 144.

On the 2nd of March, 1814, at 10^h 1' 44" P.M. by a watch that had been found from an observation of the sun, taken a few hours before, to be 17' 34" too fast, being in South latitude 15° 21', and West longitude 103° 34' 42", on account, the distance between the nearest limb of the moon and the star *Regulus* was found to be 29° 20'. Required the true longitude of the place of observation.

Elements of the calculation of the Altitudes.

Latitude - 15° 21' 0" S. Right ascens. of ☾ 119° 58' 15"
 Right ascen. of * 149 36 58 Declination of ☾ 20 20 17 N.
 Declination of * 12 52 21 N. Polar dist. of ☾ 110 20 17
 Polar dist. of * 102 52 21

Altitude of the Star.

Time by the watch - 10^h 1' 44"
 Watch too fast - - - 17 34
 True time at the place - 9 44 10
 Sun's right ascension - 22 51 7
 Right ascen. of the merid. 128 35 17
 In degrees - - - 128° 49' 15"
 Right ascen. of the star 149 36 58
 Horary angle of the star }
 Star to the East - } 20. 47 43

Half horary angle - 10° 24' - cos. 9.9928059
 Polar distance of the star 102 52 - sin. 4.9944780
 Latitude - - - 15 21 - - ½ cos. 4.9921121
 Polar dist. — latitude - 87 31
 Difference from 90° - 2 29
 Half diff. from 90° - 1 15 comp. cos. 0.0001034
 Sum. Auxiliary angle sin. 9.9794994

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Sum	Auxiliary angle.	-	sin.	9° 9794994
	Auxiliary angle.	-	cos.	9° 4778896
(Cos. auxil. angle — comp. cos. 1 difference.)		-	cos.	9° 4772362
	$\frac{1}{2}$ (90° + altitude)	-		72° 28' 15"
(Double — 90°).	TRUE ALTITUDE of the star	-		54° 56' 30"

Moon's Altitude

Right ascension of meridian in degrees				128° 49' 15"
Right ascension of the moon.	-	-		119° 58' 15"
Hourly angle of the moon				
Moon to the West	-	-		8° 51' 0"
Half the hourly angle	-	4° 25'	-	cos. 9° 9987084
Polar distance of ☾	{	110 20	-	$\frac{1}{2}$ sin. 4° 9860289
Latitude	-	15 21	-	$\frac{1}{2}$ cos. 4° 9921121
Polar dist. — latitude		94 59		
Difference from 90°		4 59		
Half diff. from 90°	-	2 30	-	comp. cos. 0° 0004135
Sum	Auxiliary angle	-	sin.	9° 9772629
	Auxiliary angle	-	cos.	9° 4988245
(Cos. auxil. angle — comp. cos. 2 difference.)		-	cos.	9° 4984110
	$\frac{1}{2}$ (90° + altitude)	-		71° 22' 5"
(Double — 90°).	TRUE ALTITUDE of the ☾	-		52° 44' 10".

Elements for calculating the true distance.

True time at the place	9 ^h 44' 10"	True altitude of star	54° 56' 30"
Longitude W. in time	+ 38 19	Refraction	- 40
True time at first meridian	10 ^h 22' 29"	Apparent altitude of star	54° 57' 10"
Moon's semidiameter	{ 16' 12	Moon's true altitude	- 52° 44' 10"
Augmentation of semid.	{ + 14	Approximate par.—ref.	- 35 14
	Sum 16 26	Approximate altitude	52° 8' 59"
Moon's horiz. parallax	{ 59 21	Parallax — Refraction	- 35 42
Diminution of parallax	{ - 1	Moon's apparent alt.	52° 8' 28"
Parallax corrected	59 20		

Observed distance moon and star	29° 20' 0"
Moon's semi-diameter	16' 33"
Moon's apparent distance	29 36 26

Calculation of the true distance.

Appar. dist. moon and star	29° 36' 26"
Apparent altitude	54 57 10 comp. cos. 0.245985
Apparent altitude	52 44 28 comp. cos. 0.215007

Sum	156 42 4	
$\frac{1}{2}$ Sum	68 21 2	cos. 9.5669402
Half sum — distance	38 44 36	cos. 9.8920708
True altitude of *	54 56 30	cos. 9.7592221
True altitude of ☾	52 44 10	cos. 9.781047
Sum	107 40 40	Sum 39.4532665

		$\frac{1}{2}$ sum	19.7266332	} 9.9557386 sin. aux. ang. 64° 34' 40"
$\frac{1}{2}$ sum	53 50 20	cos.	{ 9.7708946	
Auxiliary angle		cos.	{ 9.6320793	
Sum — 10		sin.	9.4039739	
Half the true distance		-	14° 39'	
Double TRUE DISTANCE		-	29 18.	

Longitude.

Dist. in Tab.	{	2nd at 9h, 30' 38" 58"	1st diff. 1° 46' 44" = 6404"	comp. log. 6.1935487
		at 12, 28 22 14	1st interval 3h = 10800	log. 4.0834238
		True distance 29 18 0	2nd diff. 50' 58" = 3058	log. 3.4854375
			2nd int. 1° 52' 57" = 5157	Sum 3.7124100

True time at the first meridian - 10h 25' 57"

True time at the place of observ. - 9h 44' 10"

Longitude in time - Diff. 0 41 47

Required LONGITUDE, in degrees - 10h 26' 45" West.

Example 145.

On the 26th of September, at 10 minutes before 8, P.M. 1814, by a chronometer that had been ascertained on the 20th of the same month to be 24' 39" too slow, and to lose 3½" per day, the distance between the moon's nearest limb and the star

Antares was found to be $85^{\circ} 13' 52''$. The latitude of the place of observation was $24^{\circ} 10'$ North, and the longitude $33^{\circ} 45'$ West, by account. Required the true longitude of the place of observation. Ans. $33^{\circ} 30' 45''$ W.

Example 146.

Suppose that on the 29th of April, 1815, at $4^h 54^m$ P.M. civil time, in latitude $44^{\circ} 38'$ N. and longitude $46^{\circ} 30'$ W. by account, the distance between the farthest limb of the moon and the star *α Pegasi* was found equal to $68^{\circ} 37' 10''$. What was the true longitude of the place of observation?

Ans. $46^{\circ} 19'$ W.

Example 147.

Suppose, that at $10^h 8^m 48''$ P.M. on the 14th of August, 1815, in latitude $8^{\circ} 24'$ South, and longitude $62^{\circ} 12'$ East, by account, the observed distance between the nearest limb of the moon, and the star *Fomalhaut*, is found to be $53^{\circ} 52' 48''$; the apparent altitude of the moon's centre being $55^{\circ} 2' 36''$, and that of the star $41^{\circ} 32' 40''$. Required the true longitude of the place of observation. Ans. $62^{\circ} 36'$ E.

PRACTICAL EXAMPLES TO

CHAPTER VII.

*Azimuth and Declination of the Needle. Art. 132.**Example 148.*

REQUIRED the sun's true azimuth and the declination of the magnetic needle, about 6^h A.M. on the 7th of June, 1814, in latitude 26° 30' North; and longitude 29° 15' East, when the observed altitude of the sun's lower limb was 24° 11', the height of the observer's eye 16 feet above the level of the sea, and the azimuth observed with the compass 51° 36' from the North.

Polar distance	-	-	67° 19'		
Sun's true altitude	-	-	24 21	comp. cos.	0.9398912
Latitude	-	-	26 30	comp. cos.	0.0482081
		Sum	118 10		
		Half Sum	59 5	cos.	9.7107863
Polar distance — 1	Sum	10 14		cos.	9.9930359
		Sum			19.7919215
		Half Sum		cos.	9.8959607
		Half azimuth			38° 5'
Double.	Azimuth from N. toward E.				76 10
Azimuth taken with the compass N.					51 36
DECLINATION of the magnetic needle					24° 34' NE.

Example 149.

At the island of St. Helena, the sun's central altitude was

240. AMPLITUDE AND DECLINATION OF THE NEEDLE.

Found to be $30^{\circ} 23'$ in the forenoon, his declination at the same time was $22^{\circ} 58'$ South, and the sun's azimuth, as observed with the compass, $49^{\circ} 53'$. Required the true azimuth and the declination of the needle.

Ans. $\left\{ \begin{array}{l} \text{Azimuth } 72^{\circ} 21' \text{ from the South.} \\ \text{Declination } 22^{\circ} 28' \text{ NW.} \end{array} \right.$

Example 150.

Suppose that on the 10th of April, 1815, in North latitude $42^{\circ} 29'$, and West longitude 50° , the sun's morning azimuth was observed to be South $54^{\circ} 24'$ E.; and in the evening when the sun was at the same altitude, his azimuth was $89^{\circ} 46'$ W. The elapsed time between the observations being $6^{\text{h}} 20'$. Required the variation of the compass. Ans. $7^{\circ} 24'$ Easterly.

Example 151.

Suppose that in the afternoon of the 12th of October, 1815, the altitude of the sun's lower limb was found to be $7^{\circ} 52'$ about 10 minutes before five, in latitude $43^{\circ} 22'$ North, and longitude $30^{\circ} 16'$ W. The height of the eye being equal to 18 feet, and the azimuth, observed with the compass, $85^{\circ} 32'$ NW. Required the sun's true azimuth and the declination of the needle.

Ans. $\left\{ \begin{array}{l} \text{Sun's azimn. from the N. } - 108^{\circ} 58' \text{ W.} \\ \text{Declination of the needle N. } 23^{\circ} 32' \text{ W.} \end{array} \right.$

Amplitude and Declination of the Needle. Art. 136.

Example 152.

On the 22nd of June, 1814, about $7^{\text{h}} 45'$ in the evening, in latitude $45^{\circ} 32'$ North, and longitude $61^{\circ} 4'$ West, the amplitude of the sun was observed to be $4^{\circ} 44'$ from the West towards the North. Required the true amplitude and the declination of the magnetic needle.

The declination of the sun at the time of the observation was $23^{\circ} 27\frac{1}{2}'$; and therefore

Declination of the sun N.	-	$23^{\circ} 27\frac{1}{2}'$	sin.	9.5999725
Latitude	-	$45^{\circ} 32'$	comp. cos.	0.1545953
		Sum	- sin.	9.7545680

	Sun	-	sin.	9° 7545680
Amplitude of the sun W.	-	-		34° 38' N.
The observed azimuth W.	-	-		43 44 N.
DECLINATION of the magnetic needle.	-	-		9° 6 NW.

Example 153.

Required the moon's true amplitude at rising in North latitude $35^{\circ} 8'$, when her declination is 13° North.

Ans. E. $15^{\circ} 58' N.$

Example 154.

Required the sun's true amplitude at his setting in latitude $42^{\circ} 30'$ South, his declination being 20° South.

Ans. W. $27^{\circ} 38' N.$

Example 155.

In North latitude $30^{\circ} 48'$, the sun rises about 7^h in the morning on the shortest day, at which time suppose his amplitude was observed with the compass to be E. $49^{\circ} 52' S.$ Required the true amplitude and the declination of the needle in 1815, and in longitude $37^{\circ} 45'$ West.

Ans. { Amplitude at rising E. $27^{\circ} 37' S.$
{ Declination of the needle $22^{\circ} 15' NW.$

Astronomical Bearings of objects. Art. 142.

Example 156.

On the 18th of October, 1814, being in North latitude $34^{\circ} 17'$, and West longitude $37^{\circ} 30'$, by account, about $4^h 12'$ P.M. the altitude of the sun's lower limb was observed to be $16^{\circ} 3'$, that of the summit of a distant mountain, $2^{\circ} 58'$, and the distance between the summit and the nearest limb of the sun $65^{\circ} 33'$. These observations were simultaneous, the height of the eye 18 feet, and the mountain on the right hand of the sun's vertical. Required its bearing at the time of the observation.

Elements of the Calculation.

Latitude, North	34° 47'	Observed alt. of sun	16° 12'
Longitude, West	37 30	Depression of the horizon	— 4
In time	2 ^h 30'		
Estimated time at the place	4 12	Sun's semi-diameter	+ 16
Time at the first meridian	6 ^h 42'	Sun's apparent altitude	16 24
Sun's declination	9° 35' S.	Refraction—Parallax	— 3
Polar distance	99 35	Sun's true altitude	16° 21'
Observed dist. of the mount. from the sun's nearest limb	65 53	Obs. altitude of the mount.	2° 58'
Sun's semidiameter	+ 16	Depression of the horizon	— 4
Appar. dist. of sun's centre	66° 19'	Apparent alt. mountain	2° 54'

Calculation of the Sun's Azimuth.

Polar distance	99° 35'		
Sun's true altitude	16 21	comp. cos.	0.0179279
Latitude	34 47	comp. cos.	0.0854901
Sum	150 43		
Half sum	75 21	cos.	9.4029724
Polar dist. — Sum	24 14	cos.	9.9592884
Sum			19.4663288
Half Sum		cos.	9.7331644
Half azimuthal angle			57° 15'
Double. The sun's azimuth from the North			114 30 W.

Calculation of the Difference of the Azimuths.

Apparent distance of the sun	66° 19'		
Apparent altitude of the sun	16 24	comp. cos.	0.0180392
Apparent alt. of the mountain	2 54	comp. cos.	0.0055565
Sum	85 37		
Half Sum	42 48	cos.	9.8655362
Apparent dist. — Sum	23 22	cos.	9.9628358
Sum			19.8469677

Sun	-	-	19° 84' 69" 677
Half Sum	-	cos.	9° 92' 34" 838
Half diff. of azimuths			33° 2'
Mount, to the right of sun's vertical.		Diff. of azim.	66° 4'
Sun's azimuth from North	-	-	114° 30' W.
Subtract.		THE MOUNTAIN bears from the N.	48° 26' W.

Example 157.

Suppose, that on the 8th of August, 1814, at 6^h 30' A.M. in latitude 51° 30' N. and longitude 24° E. the altitude of the sun's upper limb was 17° 18' 30", and that of the top of an object, on the right of the sun's vertical, 3° 6'; at the same time that the distance between the nearest limb of the sun and the summit of the mountain was 81° 48', and the height of the observer's eye 15 feet. Required the bearing of the mountain.

Ans. Mountain bears from the S. 12° E.

Example 158.

On the 4th of May, 1815, at 6^h 35' A.M. civil time, and in latitude 48° 10' North, and longitude 8° 7' East, suppose the altitude of the sun's lower limb to be 16° 52', that of the summit of a mountain on the left of the sun's vertical 2° 57', and the observed distance between this summit and the sun's nearest limb 71° 14'. Required the bearing of the mountain, the height of the observer's eye being 18 feet.

Ans. Bearing of the mountain from the N. 13° 45' E.

Example 159.

On the 8th of September, 1815, at 5^h 10' P.M. near the coast of Mexico, latitude 19° 12' North, longitude 96° 4' West, suppose the altitude of the sun's lower limb to be 12° 32' 22", the distance of his nearest limb from the summit of one of the Mexican mountains 50° 25', and the observed altitude of the latter 5° 18' 30". Required the true bearing of the mountain, it being on the left of the sun's vertical circle, and the height of the observer's eye 21 feet above the surface of the sea.

Ans. Bearing of the mountain from the S. 40° 38' W.

Bearings from Altitudes taken near noon. Arts. 145 and 146.

Example 160.

Approaching the entrance of Boston harbour, on the coast of North America, the latitude of the vessel being $42^{\circ} 22' N.$ and the longitude $70^{\circ} 56' W.$, by account, on the 1st of March, 1814, when the time by the watch was $9^h 20'$ A.M. civil time, an object on the coast was observed to be on the right of the sun's vertical circle. It had also been found, a short time before, that the watch was too slow with respect to true time, by $1^h 23' 10''$. Now, suppose the observed elevation of the sun's lower limb, that of the top of the object, and the distance between the object and the nearest limb of the sun, found by simultaneous observations, to be $37^{\circ} 56'$, $3^{\circ} 12'$, and $96^{\circ} 22'$ respectively. Required the bearing of the object, the height of the observer's eye being 16 feet above the surface of the sea.

Elements of the Calculation.

Time by the watch	$9^h 20' 0''$	Sun's observed altitude	$37^{\circ} 56'$
Watch too slow	$1 23 10$	Depression of the horiz.	$— 3 55$
True time of the bearing	$10 43 10$		$37 52 5$
Subtract from	12	Sun's semi-diameter	$+ 16 10$
Hourly angle	$1^h 16 50$	Sun's apparent alt.	$38 8 15$
Hourly angle in degrees	$19^{\circ} 12' 30''$	Observed dist. sun and obj.	$96 22 0$
Latitude North	$42 22$	Sun's semi-diameter	$+ 16 10$
Complement latitude	$47 38$	App. dist. sun's centre	$96 38 10$
Longitude West	$70 56$		
Longitude in time	$4^h 43' 44''$	Obs. alt. of the object	$3^{\circ} 12' 0''$
Time of the bearing	$10 43 10$	Depression of the horizon	$— 3 55$
Time at 1st meridian, P.M.	$3^h 26' 54''$	Object's appar. altitude	$3^{\circ} 12' 0''$
Sun's declination, South	$7^{\circ} 33' 48''$		
Sun's dist. from the elev. pole	$97 38 48$		

Calculation of the Sun's Azimuth.

Sun's polar dist.	{	97° 38' 48"	
Complement lat.	{	47 38 0	
Sum	-	145 16 48	
Difference		50 0 48	
Half Sum	-	72 38 24	C. sin. 0.0202473 C. cos. 0.5252383
Half difference	-	25 0 24	sin. 9.6260566 cos. 9.9572521
Horary angle	-	19 15	
Half horary angle	9 37 30"	cot. 10.7706097	cot. 10.7706097
Sum	-	tang. 10.4169136	tang. 11.2531001
1st angle	-	69° 3'	2nd angle 86° 48'
		1st angle	69 3
Sun passes the meridian towards depressed pole.	Add		
Azimuth of the sun from the North			155° 51'

Calculation of the difference of the Azimuths.

Sun's apparent distance	-	96° 38'	
Sun's apparent altitude	-	38 8	comp. cos. 0.1042594
Object's apparent altitude	-	3 8	comp. cos. 0.0006497
Sum		137 54	
Half Sum		68 57	cos. 9.5553152
Appar. distance — half sum		27 41	cos. 9.9172027
	Sum		19.6074270
	1/2 Sum		cos. 9.8037185
Half difference of the azimuths	-		50° 29'
Double Difference of the azimuths	-		100 58
Object to the right of the sun's vertical circle,			
Add the sun's azimuth from the North			155 51 E.
	Sum		256 49
	Subtract		180
Bearing of the object from the South	-		76° 49' W.

Example 161.

Suppose, that near the Cape of Good Hope, in latitude $33^{\circ} 58'$ South, and longitude $18^{\circ} 25'$ East, on the 1st of May, 1814, at $11^h 15'$ A.M. civil time, by a watch that had been ascertained on the previous evening to be too slow by $1^h 3'$ it was found that the altitude of the sun's lower limb was $33^{\circ} 30'$, and the distance of his nearest limb from the summit of a mountain, on the left of his vertical circle, $58^{\circ} 22'$, the observed altitude of which was $4^{\circ} 21'$. The place where the watch had been found to be too slow was $21^{\circ} 30'$ of a degree to the West of that where the other observations were made. Required the bearing of the mountain, the height of the eye being 22 feet above the level of the sea.

Ans. $48^{\circ} 24'$ from the N. towards the W.

Example 162.

Suppose, that on the 23th of December, 1814, at 10 minutes after one o'clock in the afternoon, the Peak of Teneriffe was seen on the left of the sun's vertical circle, and the observed distance between its summit and the nearest limb of the sun was found to be $72^{\circ} 34'$; the altitude of the former being $47^{\circ} 47'$, and of the lower limb of the latter $34^{\circ} 46'$, at the same time. The latitude of the vessel was $29^{\circ} 5'$ N. and the longitude $16^{\circ} 32'$ West, by account, and the height of the eye 21 feet above the surface of the sea. Required the bearing of the Peak at the time of the observation.

Ans. From the South. $53^{\circ} 9'$ E.

Example 163.

Suppose, that on the 13th of February, 1815, soon after entering the Eastern extremity of the Straits of Magellan, and in latitude $52^{\circ} 40'$ S. and longitude $71^{\circ} 4'$ W. by account, it was required to ascertain the bearing of a remarkable object which then appeared on the left of the sun's vertical circle; and that, for this purpose, the altitudes of the sun's upper limb, and the top of the object, with the distance between the nearest

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limb of the sun and the object, taken at the same instant by three observers, were respectively $50^{\circ} 52'$, $3^{\circ} 36'$ and $84^{\circ} 24'$. The time of the observation was 28 P.M. by a chronometer, that 9 days before had been ascertained to be $58' 27''$ too fast, with respect to true time, and to gain at the rate of $2'' 9$ daily; and the height of the eye 21 feet above the level of the sea. Required the bearing of the object.

Ans. From the S. $76^{\circ} 2' W$.

ADDITION.

On clearing the Distance.

THE following concise and easy method of clearing the apparent distance between the moon and the sun or a star from the effects of parallax and refraction, has been extracted from a paper communicated by the learned Dr. Brinkley, Professor of Astronomy in the University of Dublin, to the Royal Irish Academy, and published in the 11th volume of their transactions. The facility which this method affords in solving this troublesome problem, strongly recommends it to mariners; and in order to render it independent of all other tables, than those given in this volume, a table of natural versed sines, corresponding to every minute of the quadrant, has been added. Whenever the versed sine of an arc greater than 90° is required, it is easily found by taking the versed sine of its supplemental arc from 2.

Thus, in the first of the following examples, where the arc is $103^\circ 29'$, the supplement of which is $76^\circ 31'$; we have $2 - \text{vers. } 76^\circ 31' = 2 - .766838 = 1.233162$.

PRACTICAL RULE.

1. Find, by help of a table, the parallax answering to the moon's altitude, and to the complement of the altitude. The latter will be the argument of table 1. Or Compute them by adding the proportional log. of the horizontal parallax to the arithmetical complement of the log. cos.

and log. sin. of alt., the sums will be the prop. logs. of the respective parallaxes.

2. Moon's par. — moon's refrac. = corr. of alt. Take diff. of (corr. of altitude + star's or sun's refraction + moon's alt.) and star's altitude (or sun's alt. + par.) This diff. is the diff. of true altitudes. Find also diff. of apparent altitudes.

3. When the sun is observed, add together the numbers in tables 1, 2, 4, and 5. When a star is observed, add the numbers in tables 1, 2, 3, and 5, log. of this sum (its index being always 3 + number of figures), + log. (vers. sin. observed distance — vers. sin. diff. observed altitudes) rejecting 10 from the index = log. of a number to be subtracted from the above diff. of versed sines.

4. The remainder + vers. sin. diff. of true altitudes = vers. sin. of true distance.

Observation. No distinction of cases occurs. No proportional parts but such as are taken out by inspection. The versed sines may be considered as whole numbers, the radius being (1,000,000). In taking out the versed sines of the observed distance, the seconds may be reserved and added to the conclusion. Also in finding the log. of (vers. sin. observed distance — vers. sin. diff. obs. alt.) the two last figures may be considered as cyphers.

For those conversant in contracted decimal multiplication, the third precept may stand as follows.

3. When the sun is observed, take the sum of the numbers in table 1, 2, 4, and 5. When a star, the sum of the numbers in table 1, 2, 3, and 5. Find also the excess of the versed sine of the observed distance, above the versed sine of the difference of observed altitudes. The figures in the above-mentioned sum must be increased to five, if necessary, by prefixing cyphers to the left hand of them. Place the first figure of the sum under the third figure of the excess from the right hand, the

second figure under the fourth figure of the excess, &c., thus inverting the figures of the sum. The product found according to the method of contracted decimal multiplication, is to be subtracted from the excess.

Example I.

Sun's altitude	19° 4'	Observed distance	-	103° 29' 27"
Moon's altitude	41° 6'	Horizontal parallax	-	58 35
Diff. ob. alt.	22 2			

Prop. log.	-	58' 35"	4875	4875
41° 6'	A. C. cos.		1229	sin. 1222
			6104	6697
				38' 30" arg. tab. 1.

Parallax in alt.	44' 9"	-	145
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Moon's refract.	1 5	Tab. 1	10497
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Correction of alt.*	43 4	Tab. 2	- 78
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41° 6'		Tab. 4	- 19
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Sun's refract.	2 44	Tab. 5	- 0
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	41 51 48	-	10739
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Sun's alt. + paral.	19 4 8		
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Diff. true alt.	-	22 47 40	
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* The correction of the moon's apparent altitude is found by inspection in Table VIII of the following tables. The refraction of all the heavenly bodies is given in Table V, in the column entitled refraction of the stars; and consequently the moon's parallax is the sum of the numbers in this column, and those corresponding to the same altitudes in Table VIII. The parallax of the sun is the difference of the numbers in the same column of Table V, and those in the preceding column, which may easily be taken by inspection. Thus, in the above example, the correction of the moon's apparent altitude, taken from Table VIII, is 43' 3", differing one second from that found by the above calculation; which, however, is not sufficient to cause any difference in the corresponding number (78) taken from the annexed Table 2. The refraction answering to 41° 6', in the third column of Table V, is 1' 6"; and consequently 43' 3" + 1' 6" = 44' 9", the moon's parallax as above. The correction of the sun's apparent altitude, in the second column of this table, is 2' 37", and his refraction, in the third column, is 2' 45"; and therefore his parallax is 2' 45" - 2' 37" = 8", as above.

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Vers. sin.	103° 29' .-	1233162	log.	10739	8.03100
Vers. sin.	22 2	73034	log.	1160100	6.06446
		1160128	log.	12458	4.02546

12458

1147670

Without logarithms.

Vers. sin.	22° 47'	78024	11601
	40"	74	93701

Vers. sin.	103° 2' 52"	1225768	11601
+	27		812

103° 3' 19" TRUE DIST. required 35

10

12458

Example II.

Star's obs. alt.	11° 17'	Observed distance	- 43° 35' 42"
Moon's obs. alt.	9 38	Horizontal parallax	- 54 42
Diff. obs. alt.	- 1 39		

Prop. log.	- 54.42"	5173	5173
A. C.	9° 38'	0062	sin. 7764
		5235	12937
			9' 9" arg. tab 1.

Parallax in alt.	53' 51"	44
Moon's refract.	5 26	Tab. 1. 2061
Correction of alt.	48 28	Tab. 2. 100
Star's refract.	4 40	Tab. 3. 11
Moon's alt.	9 38 .0	Tab. 5. 11
	10 31 18	2227
	11 17	
Diff. true alt.	0 45 52	

ON CLEARING THE DISTANCE.

Vers. sin. $43^{\circ} 35'$ 275628 Log. 2227 - 7.34772

Vers. sin. 1 19 415 Log. 275200 - 5.43966

275213 Log. 613 - 2.78737

613

274600

Without Logarithms.

Vers. sin. $0^{\circ} 45'$ 86

2752

52 3

72220

Vers. sin. $43^{\circ} 30' 19''$ 274689

550

+ 42

55

$43^{\circ} 31' 1''$ TRUE DISTANCE.

6

2

TABLE I.

ARGUMENT. *Parallax answering to the Complement of the Moon's apparent altitude.*

of arg.	Arg. &"	of arg.	of arg.	Arg. &"	of arg.
3	1		155	32	8752
10	2	25	160	33	9043
15	3	316	165	34	9233
19	4	606	169	35	9624
24	5	897	174	36	9915
29	6	1188	179	37	10206
34	7	1479	184	38	10497
39	8	1770	189	39	10788
44	9	2061	194	40	11079
48	10	2352	199	41	11370
53	11	2643	204	42	11661
58	12	2934	209	43	11952
63	13	3224	215	44	12244
68	14	3515	219	45	12533
73	15	3806	223	46	12824
78	16	4097	228	47	13115
83	17	4388	233	48	13406
88	18	4679	239	49	13697
93	19	4970	242	50	13988
97	20	5261	247	51	14279
102	21	5552	252	52	14570
107	22	5843	257	53	14861
112	23	6133	262	54	15151
116	24	6424	267	55	15442
121	25	6715	272	56	15733
126	26	7006	276	57	16024
131	27	7297	281	58	16315
135	28	7588	286	59	16606
140	29	7879	291	60	16896
145	30	8170		61	17187
150	31	8461		62	17478

TABLE II.

ARGUMENT. *Correction of the Moon's apparent altitude.*

Arg.		Arg.	
1	0	29	35
2	0	30	38
3	0	31	40
4	1	32	43
5	1	33	46
6	1	34	49
7	2	35	52
8	2	36	55
9	2	37	58
10	3	38	61
11	5	39	64
12	6	40	68
13	7	41	71
14	8	42	74
15	10	43	78
16	11	44	82
17	12	45	86
18	15	46	90
19	15	47	94
20	17	48	98
21	18	49	102
22	19	50	106
23	21	51	110
24	23	52	114
25	26	53	118
26	28	54	123
27	30	55	128
28	32	56	134

TABLE IV.

TABLE III.

Arg. O'	
3-00	65
3-15	57
3-30	51
3-45	46
4-00	42
4-30	33
5-00	27
6-0	18
7-0	15
9-0	13
10-0	11
15-0	6
20-0	4
25-0	2
30-0	0
90-0	0

ARGUMENT. *Sun's apparent altitude.*

Arg. O'	
3-0	67
3-15	59
3-30	53
3-45	48
4-00	45
4-30	38
5-0	33
6-0	27
7-0	22
8-0	20
9-0	19
10-0	18
15-0	17
20-0	19
25-0	20
30-0	21
35-0	24
40-0	27
50-0	33
60-0	37
70-0	41
80-0	42
90-0	43

TABLE V.

ARGUMENT. *Moon's apparent altitude.*

Arg. O'	
3-00	55
3-15	48
3-30	43
3-45	39
4-00	36
4-30	30
5-0	26
6-0	20
7-0	16
8-0	13
9-0	11
10-0	10
15-0	6
20-0	4
25-0	2
30-0	0
90-0	0

TABLE
OF
NATURAL VERSED SINES
TO EVERY MINUTE
OF THE
FIRST QUADRANT.

	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°
0	000000	000152	000609	001370	002456	003885	005578	007454	009473	012312
1	000000	000157	000613	001386	002456	003883	005569	007449	009472	012357
2	000000	000163	000620	001401	002477	003886	005599	007455	009483	012368
3	000000	000168	000640	001416	002497	003882	005570	007461	009484	012349
4	000001	000173	000650	001433	002518	003907	005605	007495	009494	012354
5	000001	000179	000663	001448	002538	003933	005631	007523	009522	012360
6	000002	000184	000672	001463	002559	003959	005662	007566	009570	012386
7	000002	000190	000682	001479	002580	003985	005693	007704	009617	012432
8	000003	000196	000692	001495	002601	004011	003724	007740	010022	012478
9	000003	000201	000704	001511	002622	004033	003755	007776	010050	012725
10	000004	000207	000715	001527	002643	004063	003786	007813	010141	012771
11	000005	000213	000726	001543	002664	004089	003818	007849	010182	012817
12	000006	000219	000737	001559	002685	004116	003849	007885	010224	012864
13	000007	000225	000748	001575	002707	004142	003880	007922	010265	012910
14	000008	000232	000760	001592	002728	004168	003912	007958	010307	012957
15	000010	000238	000771	001608	002750	004195	003944	007993	010349	013004
16	000011	000244	000782	001625	002771	004222	003975	008032	010390	013050
17	000012	000251	000794	001641	002793	004248	004007	008069	010432	013097
18	000014	000257	000806	001658	002815	004275	004039	008106	010474	013144
19	000015	000264	000817	001675	002837	004302	004071	008143	010516	013191
20	000017	000271	000829	001692	002859	004329	004103	008180	010558	013238
21	000019	000278	000841	001709	002881	004356	004135	008217	010601	013286
22	000020	000284	000853	001726	002903	004383	004167	008254	010643	013333
23	000022	000291	000865	001743	002925	004411	004200	008291	010685	013380
24	000024	000299	000877	001760	002947	004438	004232	008329	010728	013428
25	000026	000306	000889	001777	002970	004465	004264	008366	010770	013475
26	000029	000313	000902	001795	002992	004493	004297	008404	010813	013523
27	000031	000320	000914	001812	003015	004520	004330	008442	010855	013571
28	000033	000328	000927	001830	003037	004548	004362	008479	010898	013618
29	000036	000335	000939	001847	003060	004576	004395	008517	010941	013666
30	000038	000343	000952	001865	003083	004604	004428	008555	010984	013713
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43	000078	000449	001124	002103	003386	004973	004863	009056	011550	014346
44	000082	000458	001138	002122	003410	005002	004897	009095	011594	014395
45	000086	000466	001152	002141	003434	005031	004931	009134	011638	014444
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49	000102	000503	001208	002218	003531	005149	005069	009292	011816	014642
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	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	
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5	015446	018651	022156	025958	030057	034452	039140	044119	049394	054956	5
6	015497	018708	022217	026024	030128	034527	039221	044207	049484	055051	6
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19	016167	019442	023016	026888	031056	035519	040276	045326	050665	056295	19
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21	016271	019557	023141	027022	031200	035673	040440	045499	050849	056488	21
22	016324	019614	023203	027089	031272	035750	040522	045586	050940	056584	22
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24	016428	019729	023328	027224	031417	035905	040686	045760	051124	056777	24
25	016481	019786	023390	027292	031489	035982	040768	045847	051216	056874	25
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32	016851	020191	023830	027766	031998	036525	041346	046458	051861	057553	32
33	016904	020249	023893	027834	032071	036603	041428	046546	051954	057650	33
34	016958	020308	023956	027902	032144	036681	041511	046634	052046	057747	34
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38	017172	020542	024210	028176	032438	036994	041844	046985	052417	058138	38
39	017226	020601	024274	028245	032511	037072	041927	047074	052510	058236	39
40	017279	020659	024338	028313	032585	037151	042011	047162	052603	058334	40
41	017333	020718	024401	028382	032658	037230	042094	047250	052696	058431	41
42	017387	020777	024465	028451	032732	037308	042177	047338	052790	058529	42
43	017441	020836	024529	028520	032806	037387	042261	047427	052883	058628	43
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52	017931	021371	025109	029144	033474	038099	043017	048227	053726	059514	52
53	017986	021431	025173	029214	033549	038179	043102	048316	053820	059613	53
54	018041	021491	025239	029283	033624	038259	043186	048406	053915	059712	54
55	018096	021551	025304	029353	033699	038338	043271	048495	054009	059811	55
56	018151	021611	025369	029423	033774	038418	043356	048585	054103	059910	56
57	018207	021671	025434	029493	033849	038498	043440	048674	054198	060009	57
58	018262	021732	025499	029564	033924	038578	043525	048764	054292	060108	58
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	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°	
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3	060606	066732	073143	079836	086810	094061	101589	109394	117428	125804	3
4	060706	066837	073253	079950	086928	094185	101717	109523	117559	125945	4
5	060806	066942	073362	080064	087047	094308	101843	109651	117686	126086	5
6	060906	067046	073471	080178	087166	094431	101972	109782	117813	126228	6
7	061006	067151	073581	080293	087285	094555	102100	109920	117940	126369	7
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22	062515	068732	075232	082014	089076	096415	104030	111917	120075	128501	22
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25	062819	069050	075565	082366	089436	096789	104418	112318	120489	128929	25
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38	064132	070438	077013	083870	091006	098419	106106	114066	122296	130793	38
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44	142897	151878	160979	170391	180008	189855	199996	210498	221054	231987
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53	144719	153723	162824	172236	181853	191700	201841	212352	222908	233850
54	144922	153928	163029	172441	182058	191905	202046	212558	223114	234057
55	145124	154133	163234	172646	182263	192110	202251	212764	223320	234264
56	145327	154338	163439	172851	182468	192315	202456	212970	223526	234471
57	145529	154543	163644	173056	182673	192520	202661	213176	223732	234678
58	145732	154748	163849	173261	182878	192725	202866	213382	223938	234885
59	145934	154953	164054	173466	183083	192930	203071	213588	224144	235092
60	146137	155158	164259	173671	183288	193135	203276	213794	224350	235299

	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	
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2	234142	234476	234811	235146	235481	235816	236151	236486	236821	237156	1
3	233328	233662	234000	234335	234670	235005	235340	235675	236010	236345	2
4	232514	232848	233183	233518	233853	234188	234523	234858	235193	235528	3
5	231700	232034	232369	232704	233039	233374	233709	234044	234379	234714	4
6	230886	231220	231555	231890	232225	232560	232895	233230	233565	233900	5
7	230072	230406	230741	231076	231411	231746	232081	232416	232751	233086	6
8	229258	229592	230000	230335	230670	231005	231340	231675	232010	232345	7
9	228444	228778	229113	229448	229783	230118	230453	230788	231123	231458	8
10	227630	227964	228300	228635	228970	229305	229640	230000	230335	230670	9
11	226816	227150	227485	227820	228155	228490	228825	229160	229495	229830	10
12	226002	226336	226671	227006	227341	227676	228011	228346	228681	229016	11
13	225188	225522	225857	226192	226527	226862	227197	227532	227867	228202	12
14	224374	224708	225043	225378	225713	226048	226383	226718	227053	227388	13
15	223560	223894	224229	224564	224899	225234	225569	225904	226239	226574	14
16	222746	223080	223415	223750	224085	224420	224755	225090	225425	225760	15
17	221932	222266	222601	222936	223271	223606	223941	224276	224611	224946	16
18	221118	221452	221787	222122	222457	222792	223127	223462	223797	224132	17
19	220304	220638	220973	221308	221643	221978	222313	222648	222983	223318	18
20	219490	219824	220159	220494	220829	221164	221500	221835	222170	222505	19
21	218676	219010	219345	219680	220015	220350	220685	221020	221355	221690	20
22	217862	218196	218531	218866	219201	219536	219871	220206	220541	220876	21
23	217048	217382	217717	218052	218387	218722	219057	219392	219727	220062	22
24	216234	216568	216903	217238	217573	217908	218243	218578	218913	219248	23
25	215420	215754	216089	216424	216759	217094	217429	217764	218100	218435	24
26	214606	214940	215275	215610	215945	216280	216615	216950	217285	217620	25
27	213792	214126	214461	214796	215131	215466	215801	216136	216471	216806	26
28	212978	213312	213647	213982	214317	214652	214987	215322	215657	215992	27
29	212164	212498	212833	213168	213503	213838	214173	214508	214843	215178	28
30	211350	211684	212019	212354	212689	213024	213359	213694	214029	214364	29
31	210536	210870	211205	211540	211875	212210	212545	212880	213215	213550	30
32	209722	210056	210391	210726	211061	211396	211731	212066	212401	212736	31
33	208908	209242	209577	209912	210247	210582	210917	211252	211587	211922	32
34	208094	208428	208763	209098	209433	209768	210103	210438	210773	211108	33
35	207280	207614	207949	208284	208619	208954	209289	209624	209959	210294	34
36	206466	206800	207135	207470	207805	208140	208475	208810	209145	209480	35
37	205652	205986	206321	206656	206991	207326	207661	207996	208331	208666	36
38	204838	205172	205507	205842	206177	206512	206847	207182	207517	207852	37
39	204024	204358	204693	205028	205363	205698	206033	206368	206703	207038	38
40	203210	203544	203879	204214	204549	204884	205219	205554	205889	206224	39
41	202396	202730	203065	203400	203735	204070	204405	204740	205075	205410	40
42	201582	201916	202251	202586	202921	203256	203591	203926	204261	204596	41
43	200768	201102	201437	201772	202107	202442	202777	203112	203447	203782	42
44	199954	200288	200623	200958	201293	201628	201963	202298	202633	202968	43
45	199140	199474	199809	200144	200479	200814	201149	201484	201819	202154	44
46	198326	198660	198995	199330	199665	200000	200335	200670	201005	201340	45
47	197512	197846	198181	198516	198851	199186	199521	199856	200191	200526	46
48	196698	197032	197367	197702	198037	198372	198707	199042	199377	199712	47
49	195884	196218	196553	196888	197223	197558	197893	198228	198563	198898	48
50	195070	195404	195739	196074	196409	196744	197079	197414	197749	198084	49
51	194256	194590	194925	195260	195595	195930	196265	196600	196935	197270	50
52	193442	193776	194111	194446	194781	195116	195451	195786	196121	196456	51
53	192628	192962	193297	193632	193967	194302	194637	194972	195307	195642	52
54	191814	192148	192483	192818	193153	193488	193823	194158	194493	194828	53
55	190999	191334	191669	192004	192339	192674	193009	193344	193679	194014	54
56	190185	190519	190854	191189	191524	191859	192194	192529	192864	193199	55
57	189371	189705	190040	190375	190710	191045	191380	191715	192050	192385	56
58	188557	188891	189226	189561	189896	190231	190566	190901	191236	191571	57
59	187743	188077	188412	188747	189082	189417	189752	190087	190422	190757	58
60	186929	187263	187598	187933	188268	188603	188938	189273	189608	189943	59
61	186115	186449	186784	187119	187454	187789	188124	188459	188794	189129	60
62	185301	185635	185970	186305	186640	186975	187310	187645	187980	188315	61
63	184487	184821	185156	185491	185826	186161	186496	186831	187166	187501	62
64	183673	184007	184342	184677	185012	185347	185682	186017	186352	186687	63
65	182859	183193	183528	183863	184198	184533	184868	185203	185538	185873	64
66	182045	182379	182714	183049	183384	183719	184054	184389	184724	185059	65
67	181231	181565	181900	182235	182570	182905	183240	183575	183910	184245	66
68	180417	180751	181086	181421	181756	182091	182426	182761	183096	183431	67
69	179603	179937	180272	180607	180942	181277	181612	181947	182282	182617	68
70	178789	179123	179458	179793	180128	180463	180798	181133	181468	181803	69
71	177975	178309	178644	178979	179314	179649	180000	180335	180670	181005	70
72	177161	177495	177830	178165	178500	178835	179170	179505	179840	180175	71
73	176347	176681	177016	177351	177686	178021	178356	178691	179026	179361	72
74	175533	175867	176202	176537	176872	177207	177542	177877	178212	178547	73
75	174719	175053	175388	175723	176058	176393	176728	177063	177398	177733	74
76	173905	174239	174574	174909	175244	175579	175914	176249	176584	176919	75
77	173091	173425	173760	174095	174430	174765	175100	175435	175770	176105	76
78	172277	172611	172946	173281	173616	173951	174286	174621	174956	175291	77
79	171463	171797	172132	172467	172802	173137	173472	173807	174142	174477	78
80	170649	170983	171318	171653	171988	172323	172658	172993	173328	173663	79
81	169835	170169	170504	170839	171174	171509	171844	172179	172514	172849	80
82	169021	169355	169690	170025	170360	170695	171030	171365	171700	172035	81
83	168207	168541	168876	169211	169546	169881	170216	170551	170886	171221	82
84	167393	167727	168062	168397	168732	169067	169402	169737	170072	170407	83
85	166579	166913	167248	167583	167918	168253	168588	168923	169258	169593	84
86	165765	166099	166434	166769	167104	167439	167774	168109	168444	168779	85
87	164951	165285	165620	165955	166290	166625	166960	167295	167630	167965	86
88	164137	164471	164806	165141	165476	165811	166146	166481	166816	167151	87
89	163323	163657	163992	164327	164662	164997	165332	165667	166002	166337	88
90	162509	162843	163178	163513	163848	164183	164518	164853	165188	165523	89
91	161695	162029	162364	162699	163034	163369	163704	164039	164374	164709	90
92	160881	161215	161550	161885	162220	162555	162890	163225	163560	163895	91
93	160067	160401	160736	161071	161406	161741	162076	162411	162746	163081	92
94	159253	159587	159922	160257	160592	160927	161262	161597	161932	162267	93
95	158439	158773	159108	159443	159778	160113					

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2	357658	372132	385797	399650	413685	427900	442389	457049	471884	486766	2
3	357881	372358	386026	399882	413921	428139	442631	457293	472129	487009	3
4	358104	372584	386255	399115	414156	428377	442872	457537	472371	487250	4
5	358327	372811	386485	399347	414392	428616	443115	457781	472615	487490	5
6	358550	373037	386715	399580	414628	428854	443355	458026	472856	487731	6
7	358774	373263	386944	399812	414863	429093	443596	458270	473097	487972	7
8	358997	373490	387174	400045	415098	429331	443836	458514	473338	488213	8
9	359220	373716	387404	400278	415335	429570	444079	458758	473579	488454	9
10	59443	373943	387633	400511	415571	429809	444321	459002	473820	488695	10
11	59667	374169	387863	400744	415806	429048	444563	459244	474061	488937	11
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13	60114	374622	388322	401209	416278	429525	445046	459727	474543	489419	13
14	60337	374849	388552	401442	416514	429764	445288	459968	474784	489660	14
15	60561	375076	388783	401675	416750	430003	445530	460209	475025	489901	15
16	60785	375303	389013	401908	416985	430242	445772	460451	475266	490142	16
17	61008	375530	389243	402142	417222	430481	446014	460692	475507	490383	17
18	61232	375757	389473	402375	417455	430720	446256	460934	475748	490624	18
19	61456	375984	389703	402608	417689	430960	446498	461175	475989	490865	19
20	61680	376211	389933	402841	417923	431199	446740	461417	476230	491106	20
21	61904	376438	390163	403075	418156	431438	446982	461659	476471	491347	21
22	62128	376666	390394	403308	418390	431677	447224	461900	476712	491588	22
23	62352	376893	390624	403542	418623	431917	447466	462142	476953	491829	23
24	62576	377120	390855	403775	418857	432156	447708	462384	477194	492070	24
25	62800	377348	391085	404009	419090	432396	447950	462625	477435	492311	25
26	63024	377575	391316	404242	419323	432635	448192	462867	477676	492552	26
27	63248	377803	391546	404476	419557	432875	448434	463108	477917	492793	27
28	63472	378030	391777	404710	419790	433114	448676	463349	478158	493034	28
29	63696	378258	392008	404943	420023	433354	448918	463590	478399	493275	29
30	63920	378485	392239	405177	420257	433594	449160	463831	478640	493516	30
31	64144	378713	392469	405411	420490	433835	449402	464072	478881	493757	31
32	64368	378941	392699	405645	420723	434075	449644	464313	479122	494000	32
33	64592	379168	392930	405879	420957	434315	449886	464554	479363	494241	33
34	64816	379396	393162	406113	421190	434555	450128	464795	479604	494482	34
35	65040	379624	393393	406347	421423	434793	450370	465036	479845	494723	35
36	65264	379852	393624	406581	421657	435033	450612	465277	480086	494964	36
37	65488	380080	393855	406815	421890	435273	450854	465518	480327	495205	37
38	65712	380308	394086	407049	422123	435513	451096	465759	480568	495446	38
39	65936	380536	394318	407284	422357	435753	451338	466000	480809	495687	39
40	66160	380764	394549	407518	422590	435993	451580	466241	481050	495928	40
41	66384	380992	394780	407752	422823	436233	451822	466482	481291	496169	41
42	66608	381220	395012	407987	423057	436474	452064	466723	481532	496410	42
43	66832	381448	395243	408221	423290	436714	452306	466964	481773	496651	43
44	67056	381676	395474	408456	423523	436955	452548	467205	482014	496892	44
45	67280	381904	395706	408690	423757	437195	452790	467446	482255	497133	45
46	67504	382132	395938	408925	423992	437435	453032	467687	482496	497374	46
47	67728	382360	396169	409160	424226	437676	453273	467928	482737	497615	47
48	67952	382588	396401	409394	424460	437917	453515	468169	482978	497856	48
49	68176	382816	396633	409629	424693	438157	453757	468410	483219	498097	49
50	68400	383044	396864	409864	424927	438398	453999	468651	483460	498338	50
51	68624	383272	397096	410099	425161	438639	454241	468892	483701	498579	51
52	68848	383500	397328	410334	425395	438879	454483	469133	483942	498820	52
53	69072	383728	397560	410569	425629	439120	454725	469374	484183	499061	53
54	69296	383956	397792	410804	425863	439361	454967	469615	484424	499302	54
55	69520	384184	398024	411039	426097	439602	455209	469856	484665	499543	55
56	69744	384412	398256	411274	426331	439843	455451	470097	484906	499784	56
57	69968	384640	398488	411509	426565	440084	455693	470338	485147	500025	57
58	70192	384868	398720	411744	426799	440325	455935	470579	485388	500266	58
59	70416	385096	398953	411979	427033	440566	456177	470820	485629	500507	59

	60°	61°	62°	63°	64°	65°	66°	67°	68°	69°	
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1	500252	515445	530785	546269	561890	577645	593520	609537	625663	641904	1
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3	500756	515954	531299	546787	562413	578173	594061	610072	626192	642447	3
4	501008	516208	531556	547046	562675	578437	594327	610334	626454	642719	4
5	501260	516463	531815	547306	562937	578700	594592	610596	626716	642990	5
6	501512	516718	532070	547565	563198	578964	594858	610857	626978	643262	6
7	501764	516972	532327	547825	563460	579228	595124	611118	627240	643534	7
8	502017	517227	532584	548084	563722	579492	595390	611379	627502	643806	8
9	502269	517482	532842	548344	563983	579756	595656	611640	627764	644077	9
10	502521	517737	533099	548603	564245	580020	595922	611901	628026	644349	10
11	502774	517991	533356	548863	564507	580284	596189	612162	628289	644621	11
12	503026	518246	533613	549122	564769	580548	596455	612423	628552	644893	12
13	503278	518501	533871	549382	565031	580812	596721	612684	628815	645165	13
14	503531	518756	534128	549642	565293	581076	596988	612945	629078	645437	14
15	503783	519011	534385	549902	565555	581340	597253	613206	629341	645709	15
16	504036	519266	534642	550161	565817	581604	597520	613467	629604	645981	16
17	504289	519521	534900	550421	566079	581869	597786	613728	629867	646253	17
18	504541	519776	535158	550681	566341	582133	598052	614000	630130	646525	18
19	504794	520032	535416	550941	566603	582397	598319	614262	630393	646797	19
20	505047	520287	535673	551201	566865	582661	598585	614523	630656	647069	20
21	505300	520542	535931	551461	567127	582926	598851	614784	630919	647342	21
22	505552	520797	536188	551721	567390	583190	599118	615045	631182	647614	22
23	505805	521053	536446	551981	567652	583455	599384	615306	631445	647886	23
24	506058	521308	536704	552241	567914	583719	599651	615567	631708	648158	24
25	506311	521564	536962	552501	568177	583984	599917	615828	631970	648430	25
26	506564	521819	537220	552761	568439	584248	600184	616089	632232	648703	26
27	506817	522075	537477	553021	568701	584513	600451	616350	632494	648975	27
28	507070	522330	537735	553282	568964	584777	600717	616611	632756	649248	28
29	507323	522586	537993	553542	569226	585042	600984	616872	633018	649520	29
30	507576	522841	538251	553802	569489	585307	601251	617133	633280	649793	30
31	507830	523097	538509	554062	569751	585571	601518	617394	633542	650065	31
32	508083	523353	538767	554323	570014	585836	601784	617655	633804	650337	32
33	508336	523608	539026	554583	570277	586101	602051	617916	634066	650609	33
34	508589	523864	539284	554844	570539	586366	602318	618177	634328	650881	34
35	508843	524120	539542	555104	570802	586631	602585	618439	634590	651153	35
36	509096	524376	539800	555365	571065	586896	602852	618700	634852	651425	36
37	509350	524632	540058	555625	571328	587160	603119	618962	635114	651697	37
38	509603	524888	540317	555886	571590	587425	603386	619223	635376	651969	38
39	509857	525144	540573	556147	571853	587690	603653	619484	635638	652241	39
40	510110	525400	540833	556407	572116	587955	603920	620006	635900	652513	40
41	510364	525656	541092	556668	572379	588220	604187	620267	636162	652785	41
42	510617	525912	541350	556929	572642	588486	604454	620528	636424	653057	42
43	510871	526168	541609	557190	572905	588751	604722	620789	636686	653329	43
44	511125	526424	541867	557450	573168	589016	604989	621050	636948	653601	44
45	511379	526680	542126	557711	573431	589281	605256	621311	637210	653873	45
46	511633	526937	542385	557972	573694	589546	605523	621572	637472	654145	46
47	511886	527193	542643	558233	573957	589812	605791	621833	637734	654417	47
48	512140	527449	542902	558494	574221	590077	606058	622094	637996	654689	48
49	512394	527706	543161	558755	574484	590342	606325	622355	638258	654961	49
50	512648	527962	543420	559016	574747	590608	606593	622616	638520	655233	50
51	512902	528218	543678	559277	575010	590873	606860	622877	638782	655505	51
52	513156	528475	543937	559538	575274	591138	607128	623138	639044	655777	52
53	513410	528731	544196	559800	575537	591404	607395	623400	639306	656049	53
54	513665	528988	544455	560061	575801	591669	607663	623661	639568	656321	54
55	513919	529245	544714	560322	576064	591935	607930	623923	639830	656593	55
56	514173	529501	544973	560583	576327	592201	608198	624184	640092	656865	56
57	514427	529758	545232	560845	576591	592466	608466	624446	640354	657137	57
58	514682	530015	545491	561106	576854	592732	608732	624708	640616	657409	58
59	514936	530272	545750	561367	577118	592998	609001	624969	640878	657681	59

	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°	
0	657980	674432	690983	707628	724363	741181	758028	775049	792288	809691	0
1	658253	674707	691260	707906	724642	741462	758360	775332	792573	809977	1
2	658527	674982	691536	708183	724923	741743	758643	775616	792857	810262	2
3	658800	675257	691813	708463	725202	742024	758925	775899	793142	810548	3
4	659073	675532	692090	708741	725481	742305	759207	776183	793427	810833	4
5	659347	675807	692367	709019	725761	742586	759490	776466	793711	811119	5
6	659620	676083	692648	709298	726041	742867	759772	776750	793996	811406	6
7	659894	676358	692920	709576	726321	743148	760054	777033	794280	811690	7
8	660167	676633	693197	709854	726600	743429	760337	777317	794565	811976	8
9	660441	676908	693474	710133	726880	743711	760619	777601	794850	812261	9
10	660715	677184	693751	710411	727160	743992	760902	777884	795134	812547	10
11	660988	677459	694028	710690	727440	744273	761184	778168	795419	812833	11
12	661262	677735	694305	710968	727720	744554	761467	778451	795704	813119	12
13	661536	678010	694582	711247	728000	744835	761749	778735	795989	813404	13
14	661809	678285	694859	711525	728280	745117	762032	779019	796273	813690	14
15	662083	678560	695136	711804	728560	745398	762314	779303	796558	813976	15
16	662357	678836	695413	712082	728839	745679	762597	779586	796843	814261	16
17	662631	679111	695690	712361	729119	745961	762879	779870	797128	814547	17
18	662905	679387	695967	712639	729400	746242	763162	780154	797413	814833	18
19	663179	679663	696244	712918	729679	746523	763444	780438	797697	815119	19
20	663452	679938	696521	713197	729960	746805	763727	780721	797982	815405	20
21	663726	680214	696798	713475	730240	747086	764010	781005	798267	815691	21
22	663999	680489	697076	713754	730520	747368	764299	781289	798552	815977	22
23	664273	680765	697353	714033	730800	747649	764575	781573	798837	816263	23
24	664547	681041	697630	714312	731080	747931	764858	781857	799122	816549	24
25	664822	681316	697907	714590	731360	748212	765141	782141	799407	816835	25
26	665097	681592	698185	714869	731641	748494	765423	782424	799692	817121	26
27	665371	681868	698462	715148	731921	748775	765706	782708	799977	817406	27
28	665645	682144	698739	715427	732201	749057	765989	782992	800262	817692	28
29	665919	682419	699017	715706	732481	749338	766272	783276	800547	817978	29
30	666193	682695	699294	715985	732762	749619	766555	783560	800832	818264	30
31	666467	682971	699572	716264	733042	749901	766837	783844	801117	818550	31
32	666742	683247	699849	716542	733322	750183	767120	784128	801402	818836	32
33	667016	683523	700127	716821	733603	750465	767403	784412	801687	819122	33
34	667290	683799	700404	717100	733883	750747	767686	784696	801972	819408	34
35	667564	684075	700682	717379	734163	751028	767969	784981	802257	819693	35
36	667839	684351	700959	717658	734444	751310	768252	785265	802543	819979	36
37	668113	684627	701237	717938	734724	751592	768535	785549	802828	820265	37
38	668388	684903	701514	718217	735005	751874	768818	785833	803113	820551	38
39	668662	685179	701792	718496	735285	752155	769101	786117	803398	820837	39
40	668937	685455	702070	718775	735566	752437	769384	786401	803683	821123	40
41	669211	685731	702347	719054	735846	752719	769667	786685	803969	821409	41
42	669486	686008	702625	719333	736127	753001	769950	786970	804254	821695	42
43	669760	686284	702903	719612	736407	753283	770233	787255	804539	821981	43
44	670035	686560	703181	719892	736688	753565	770516	787536	804824	822267	44
45	670309	686836	703458	720171	736969	753847	770800	787822	805109	822553	45
46	670584	687112	703736	720450	737249	754129	771083	788107	805395	822839	46
47	670859	687389	704014	720730	737530	754411	771366	788391	805680	823125	47
48	671133	687665	704292	721009	737811	754693	771649	788673	805966	823411	48
49	671408	687941	704570	721288	738091	754975	771932	788959	806251	823697	49
50	671683	688218	704848	721568	738372	755257	772216	789244	806536	823983	50
51	671958	688494	705126	721847	738653	755539	772499	789528	806822	824269	51
52	672232	688771	705404	722126	738934	755821	772782	789813	807107	824555	52
53	672507	689047	705682	722406	739215	756103	773065	790097	807393	824841	53
54	672782	689323	705960	722685	739495	756385	773349	790381	807678	825127	54
55	673057	689600	706238	722965	739776	756667	773632	790666	807963	825413	55
56	673332	689877	706516	723244	740057	756949	773915	790950	808249	825699	56
57	673607	690153	706794	723524	740338	757231	774199	791235	808534	825985	57
58	673882	690430	707072	723803	740619	757514	774482	791519	808820	826271	58
59	674157	690706	707350	724083	740900	757796	774765	791804	809105	826557	59

	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	
0	826352	843565	860827	878131	895471	912844	930243	947664	965100	982548	0
1	826638	843853	861113	878419	895761	913134	930534	947954	965391	982838	1
2	826925	844140	861403	878708	896050	913424	930824	948245	965682	983129	2
3	827211	844427	861691	878997	896339	913714	931114	948527	965973	983420	3
4	827498	844715	861979	879286	896629	914003	931404	948807	966263	983711	4
5	827784	845002	862267	879574	896918	914293	931694	949100	966553	984002	5
6	828071	845290	862555	879863	897207	914583	931985	949407	966844	984293	6
7	828357	845577	862844	880152	897497	914873	932275	949698	967136	984583	7
8	828644	845864	863132	880441	897786	915163	932566	949988	967426	984874	8
9	828931	846152	863420	880730	898075	915453	932855	950279	967717	985165	9
10	829217	846439	863708	881018	898365	915742	933146	950569	968008	985456	10
11	829504	846727	863996	881307	898654	916032	933436	950860	968298	985747	11
12	829790	847014	864284	881596	898944	916322	933726	951150	968589	986038	12
13	830077	847302	864573	881885	899233	916612	934016	951441	968880	986329	13
14	830364	847589	864861	882174	899523	916902	934307	951731	969171	986619	14
15	830650	847877	865149	882463	899812	917192	934597	952022	969461	986910	15
16	830937	848164	865437	882751	900101	917482	934887	952312	969752	987201	16
17	831224	848452	865726	883040	900391	917772	935177	952603	970043	987492	17
18	831511	848739	866014	883329	900680	918061	935468	952893	970334	987783	18
19	831797	849027	866302	883618	900970	918351	935758	953184	970624	988074	19
20	832084	849314	866590	883907	901259	918641	936048	953475	970915	988365	20
21	832371	849602	866879	884196	901549	918931	936339	953765	971206	988656	21
22	832658	849889	867167	884485	901838	919221	936629	954056	971497	988946	22
23	832944	850177	867455	884774	902128	919511	936919	954346	971788	989237	23
24	833231	850465	867744	885063	902417	919801	937209	954637	972078	989528	24
25	833518	850752	868032	885352	902707	920091	937500	954928	972369	989819	25
26	833805	851040	868320	885641	902996	920381	937790	955218	972660	990110	26
27	834092	851328	868609	885930	903286	920671	938080	955509	972951	990401	27
28	834379	851615	868897	886219	903575	920961	938371	955799	973241	990692	28
29	834665	851903	869185	886508	903865	921251	938661	956090	973532	990983	29
30	834952	852191	869474	886797	904154	921541	938951	956381	973823	991273	30
31	835239	852478	869762	887086	904444	921831	939241	956671	974114	991564	31
32	835526	852766	870051	887375	904733	922121	939531	956962	974405	991855	32
33	835813	853054	870339	887664	905023	922411	939822	957252	974696	992146	33
34	836100	853342	870627	887953	905312	922701	940113	957543	974986	992437	34
35	836387	853630	870916	888242	905602	922991	940403	957834	975277	992728	35
36	836674	853917	871204	888531	905892	923281	940694	958124	975568	993019	36
37	836961	854205	871493	888820	906181	923571	940984	958415	975859	993310	37
38	837248	854492	871781	889109	906471	923861	941274	958706	976149	993600	38
39	837535	854780	872070	889398	906760	924151	941565	958996	976440	993891	39
40	837822	855068	872358	889687	907050	924441	941855	959287	976731	994182	40
41	838109	855356	872647	889976	907340	924731	942146	959578	977022	994473	41
42	838396	855644	872935	890266	907629	925021	942436	959868	977313	994764	42
43	838683	855932	873224	890555	907919	925311	942726	960159	977603	995055	43
44	838970	856219	873512	890844	908209	925601	943017	960449	977894	995346	44
45	839257	856507	873801	891133	908498	925891	943307	960740	978185	995637	45
46	839544	856795	874090	891422	908788	926182	943598	961031	978476	995928	46
47	839832	857083	874378	891711	909078	926472	943888	961321	978767	996219	47
48	840119	857371	874667	892001	909367	926762	944178	961612	979058	996509	48
49	840406	857659	874955	892290	909657	927052	944469	961903	979348	996800	49
50	840693	857947	875244	892579	909947	927342	944759	962193	979639	997091	50
51	840980	858236	875533	892868	910236	927632	945050	962484	979930	997382	51
52	841267	858523	875821	893157	910526	927922	945340	962775	980221	997673	52
53	841555	858811	876110	893447	910816	928212	945631	963066	980512	997964	53
54	841842	859099	876398	893736	911106	928503	945921	963356	980803	998255	54
55	842129	859387	876687	894025	911395	928793	946212	963647	981093	998546	55
56	842416	859675	876976	894314	911685	929083	946502	963938	981384	998836	56
57	842704	859963	877264	894604	911975	929373	946793	964228	981675	999127	57
58	842991	860251	877553	894893	912265	929663	947083	964519	981966	999418	58
59	843278	860539	877842	895182	912554	929953	947374	964810	982257	999709	59

INTRODUCTION

TO

THE TABLES,

EXPLANATORY OF THEIR CONSTRUCTION AND USE.

BY THE TRANSLATOR.

TABLE I.

Depression of the Horizon.

THE angular distance between the zenith and the horizon of an observer could only be equal to 90° if the surface of the earth were an extended plane, and the eye of the observer situated in that plane. Thus, fig. 6, if the surface of the earth coincided with the line AC, and the observer's eye were at A, a point in that line, and Z in the direction of the observer's zenith, the angle ZAC would be a right angle, or 90° . But as the earth's surface is curved, as shown in this figure, which represents a vertical section of the earth, by a plane passing through the eye of the observer and his zenith, any point P on the surface will be below the horizontal line AC, and consequently if AP be joined the angle ZAP will be greater than 90° . The eye of the observer is always more or less elevated above the point A; let it be at B, where the tangent drawn from the point B meets the vertical line AZ; then as the angle ZBP is equal to the sum of the angles ZAP, APB, and ZAP exceeds 90° , the angle ZBP also exceeds 90° ; and if the line BP be drawn parallel to AC, the angle ZBP will evidently be a right angle, and the angle PBP will be the depression of the horizon, or the quantity which the angle ZBP, the angular distance between the zenith and horizon of the observer, exceeds 90° .

Now if the vertical line ZA be produced to o , which represents the centre of the earth, and or be joined, the angles roB , PBD will evidently be equal to each other, and consequently the depression of the horizon may easily be found by the rules of plane trigonometry; for as $AB : rad. :: OP : \cos. POB = \cos. PBD$. But when the height AB is small, the common tables of logarithms are not sufficiently extensive to give this angle with the required accuracy, besides which its value still requires a correction on account of refraction.

To avoid both these inconveniences, let n denote the angle of depression PBD , r the mean terrestrial refraction in terms of the observed arc AP , H the height AB of the eye above the surface of the sea in English feet, and R the mean radius of the earth, or that corresponding to 45° of latitude; then, according to Delambre, (*Abbrégé d'Astronomie*, p. 626).

$$\tan. n = \left(\frac{1-r}{\sin. 45^\circ} \right) \times \left(\frac{H}{R} \right)^{\frac{1}{2}}.$$

As the arc denoted by n is always very small, it is not sensibly different from its tangent, and therefore may be substituted for it: hence

$$n = \left(\frac{1-r}{\sin. 45^\circ} \right) \times \left(\frac{H}{R} \right)^{\frac{1}{2}}.$$

Now the mean value of r is $\cdot 07876$, and that of $R = 20892710$ English feet, or equal to an arc of $206265''$; and by substituting these values for their respective letters in the preceding formula, it becomes

$$n = \frac{(1 - \cdot 07876) \times 206265''}{(2071068 / 20892710)^{\frac{1}{2}}} \sqrt{H} = 58'' \cdot 795 \sqrt{H},$$

which is very easily calculated, and does not require any correction.

Hence the following RULE.—Multiply the square root of the height of the eye in feet by $58'' \cdot 795$, and the product will be the depression in seconds.

Example.—Suppose $H = 16$ feet, then $58'' \cdot 795 \sqrt{16} = 240'' = 4' 0''$; the depression of the horizon when the observer's eye is 16 feet above the surface of the sea. And if $H = 25$ feet, $58'' \cdot 795 \sqrt{25} = 294'' = 4' 54''$, the depression in this case.

With respect to the distance that can be seen by an observer elevated above the earth's surface: let AO (fig. 6) the radius of the earth $= r$, AB the height of the eye $= h$, and BD the tangent from the point $B = d$. Then, by geometry, $AB^2 = AB \cdot BD = AB (AB + AD)$, or $d^2 = h (h + 2r)$ and $d = h \frac{1}{2} (h + 2r)^{\frac{1}{2}}$. But as the magnitude of h in all practical cases in navigation is so extremely small with respect to $2r$, the former seldom exceeding one millionth part of the latter, the quantity $(h + 2r)^{\frac{1}{2}}$ may be regarded as a constant quantity; and therefore the value of d will vary as $h^{\frac{1}{2}}$. But in all small arcs the tangent is not sensibly different from the arc itself, and in this case the arc never exceeds a few minutes, d may, without sensible error, be substituted for the arc AP , or the distance that can be seen from the point B ; hence this distance varies as the square root of the height of the eye.

Now as it is found from calculation that when the eye is 6 feet above the earth's surface the distance that can be seen is 3 miles, we have $\sqrt{6} : \sqrt{h} :: 3 : \frac{3}{\sqrt{6}} \sqrt{h} = \frac{1}{2} \sqrt{6} \times \sqrt{h} = 1.2247 \sqrt{h}$; which is an expression in miles for the distance that can be seen when the height of the eye above the level of the sea is equal to h .

Hence this RULE.—Multiply the square root of the height of the eye in feet by 1.2247, and the product will be the distance that can be seen, in English miles.

Example.—If $h = 25$ feet, then $1.2247 \sqrt{25} = 1.2247 \times 5 = 6.1235$, or very nearly $6\frac{1}{4}$ miles. And again, if $h = 18$ feet, $1.2247 \sqrt{18} = 5.1959 = 5.2$ nearly; and on this principle the numbers in the third column of Table I. have been calculated.

TABLE II.

Augmentation of the Moon's Semidiameter.

As the moon describes her diurnal circle about the earth as a centre, an observer situated on any part of its surface will see

her nearest to him in the zenith and most distant in the horizon; and the difference of these two distances is nearly equal to the radius of the earth, and about $\frac{1}{60}$ th of the moon's horizontal distance from the earth's centre. Observations of the moon's apparent diameter, occultations of the fixed stars, and various other circumstances, also prove that the moon's apparent diameter is subject to variation; and since the apparent magnitudes of bodies are inversely as their distances, we have $59:60$

$:: d: \frac{60a}{59} =$ to her semidiameter at the zenith, where d denotes

her horizontal semidiameter; and consequently $\frac{1}{59} d =$ the aug-

mentation at the zenith. But this augmentation varies according to the moon's altitude; hence if her altitude be denoted by

a , we have $90:a :: \frac{1}{59} d: \frac{1}{5310} ad = .000188 ad$ seconds, where a is in degrees and d in seconds of a degree.

Rule.—Multiply the altitude in degrees, the horizontal semidiameter in seconds of a degree, and the number .000188 together, and the product will be the augmentation required.

Example.—Required the augmentation of the moon's semidiameter when her altitude is 30° , and her horizontal diameter $15' 30''$. Hence $a = 30$, and $d = 15' 30'' = 930''$; and therefore $.000188 \times 30 \times 930'' = 5''$ nearly, the required augmentation, as given in the table.

TABLE III.

Diminution of the Equatorial Parallax at different Degrees of Latitude.

If the earth were truly spherical, the horizontal parallax of a heavenly body, the distance of which remained constant, would be the same in all latitudes. But this will not be the case if the earth's radii are unequal, for the horizontal parallax is the angle under which an observer situated at the centre of the heavenly body would see the terrestrial radius. Hence the sine of the horizontal parallax is equal to the quotient of the

Introduction to the Tables.

terrestrial radius divided by the distance between the centre of the heavenly body and that of the earth; and the equatorial radius being the greatest and the polar radius the least, the horizontal parallax consequently attains its *maximum* at the equator and its *minimum* at the poles. Now as $\frac{1}{309}$ expresses the ellipticity of the earth, the 309th part of the whole parallax will be the difference between these extremes; and the equatorial parallax also varies from about 53' to 61'; and consequently the diminution on account of latitude increases from 10''·3 to 11''·7, between the equator and the poles. But for all intermediate situations, this diminution varies as the square of the sine of the latitude: hence, if L denote the latitude, D the whole diminution, and d that required at the latitude L , its value will be obtained by this simple logarithmic formula

$$d = \sin.^2 L \times D.$$

Hence this easy RULE.—*Square the sine of the latitude and multiply it by the whole diminution, the product will be the diminution corresponding to that latitude.*

Examples.—If $L = 30^\circ$, and $D = 11''\cdot7$, the greatest diminution, then $\sin.^2 30^\circ = \frac{1}{4}$, and $\frac{1}{4} \times 11''\cdot7 = 3''$, very nearly, as given in the table.

Again, if $L = 55^\circ$, and $D = 10''\cdot3$, the least diminution, then

$$\text{Log. of } \sin.^2 55^\circ = -1\cdot8267290$$

$$\text{Log. of } 10''\cdot3 = \underline{1\cdot0128372}$$

$$\text{Log. of } 6''\cdot9114 = 0\cdot8395602$$

Or of 7'' nearly, as in the table.

TABLE IV.

Errors of the Surfaces of the large Mirror, when these Surfaces make an Angle of 1' with each other.

The rays of light are reflected by the quicksilver surface of the great mirror, which is the farthest from the objects from which these rays proceed; they consequently traverse the thickness of the glass, and experience a refraction on entering it and a second on emerging from it. Therefore, if the surfaces of this

mirror are not parallel to each other, these refractions will be unequal, and the angles formed by the incident and reflected rays will not be the same; and the observed angles will consequently participate in this defect. These errors also increase with the distance of the two observed bodies from each other, or as the incident and reflected rays are more inclined to the plane of the mirror; and astronomers determine their magnitude by observation. (See *Biot's Astronomy*, vol. i. p. 362.) The numbers in this table also furnish the means of calculating the errors of the observed distances for other inclinations of the surfaces, by proportion. For the error in the table, corresponding to the distance observed for verifying the instrument, is to the error in the same table for any other angle, as the error formed by the verification is to a fourth term, which is the error required.

Example.—Suppose the instrument had been verified by two objects 65' distant from each other, and the error ascertained to be 58", it is required to find the error corresponding to a distance of 95°, the observation being to the right.

Then as $38'' : 1' 43'' :: 58'' : 2' 37''$, the error required.

TABLE V.

Refraction less Parallax, for 29ⁱⁿ·92 of the Barometer, and 57°·2 of Fahrenheit's Thermometer.

Rays of light change their direction on passing obliquely from one medium to another of a different density; and this effect is called *Refraction*. If the luminous ray pass obliquely from one medium to another of the same nature, but of a different density, and at the point where it passes from the one to the other, a perpendicular be supposed to be drawn to their common surface, the ray on entering the denser medium will approach this perpendicular. Now the atmosphere being composed of an indefinite number of beds or strata of air, the densities of which increase as they approach the surface of the earth, the luminous rays that traverse them obliquely are in-

flected towards the centre of the earth; and hence all the heavenly bodies appear more elevated, on this account, than they really are. This astronomical refraction also varies according to the altitude of the heavenly body.

The refractions contained in this table have been calculated from the series given by the celebrated Laplace, in his great work, the *Mécanique Céleste*. The formulæ deduced from these series are,

$$\tan. u = \sin. 2nR \tan. z;$$

$$\text{and } \tan. nr = \tan. nR \cdot \tan. \frac{1}{2}u;$$

where $n = 3.78$, and $nR = 6867''$.

When z , which denotes the zenith distance, is given, the first equation will give the value of u ; and then the second equation will give that of r , the refraction, in seconds of a degree. But as this formula is adapted to the medium pressure of the atmosphere at the surface of the sea, or 29.92 inches, and the temperature of melting ice, or 32° of Fahrenheit's thermometer, it requires a further reduction to bring it to the temperature for which the table has been calculated, which is $57^{\circ}.2$ of Fahrenheit's thermometer. This may be done by multiplying n , the coefficient of r , by 1 added to as many times .00208 as there are degrees between the freezing point and the given temperature, as indicated by Fahrenheit's thermometer: thus, if these degrees be denoted by d , the formula becomes

$$\tan. (1.00208 dnr) = \tan. nR \cdot \tan. \frac{1}{2}u.$$

By substituting the given quantities in the two preceding formulæ, and expressing them in words, the following rule will be obtained.

RULE—1. Add the logarithm cotangent of the observed altitude to -2.8230506 , and the sum will be the log. tangent of $\frac{1}{2}u$.

2. Add the log. tangent of $\frac{1}{2}u$ to -2.5225024 , and the sum will be the log. tangent of an arc, which is to be taken from the table and reduced into seconds.

3. Then to the logarithm of this number of seconds, add -1.4003208 , and the sum will be the logarithm of the number of seconds in the required refraction.

Example.—Required the refraction corresponding to an observed altitude of 30° .

$$1st.—\text{Log. cotan. } 30^\circ = 10.2385606$$

$$\text{add } - 2.8230506$$

$$\text{tan. } u = 6' 34'' 22'' = 9.0010112$$

$$2d.—\text{Log. tan. } \frac{1}{2}u = 3' 17'' 11'' = 8.7590721$$

$$\text{add } - 2.5225024$$

$$\text{tan. } 6' 34'' \cdot 5 = 394'' \cdot 5 = 7.2815745$$

$$3d.—\text{Then log. } 391'' \cdot 5 = 2.5560470$$

$$\text{add } - 1.4003208$$

$$\text{Log. } 1' 39'' = 99'' \cdot 167 = 1.9963678$$

The required refraction at 30° of altitude is therefore equal to $1' 39''$, which is the number in the table.

But the second column of the table contains the refraction of the sun diminished by its parallax, or the results of $r - p$, where r denotes the parallax. Now the horizontal parallax of the sun is equal to the quotient obtained by dividing the mean radius of the earth by the mean distance between the centres of the earth and sun; and his parallax of altitude is proportional to the sine of his zenith distance or to the cosine of his altitude: therefore

$$\text{sine of the paral. in altitude} = \frac{\text{earth's radius} \times \cos. \text{altitude}}{\text{sun's mean distance.}} = \frac{r \cos. a}{d},$$

by using the initials of the words only. Expressing these quantities in terms of the earth's radius, the formula becomes

$$\sin. p = \frac{1}{23578} \cos. a = .000042413 \cos. a.$$

Hence this RULE.—To the number -5.6274935 , add the logarithm cosine of the altitude, and the sum will be the log. sine of the parallax in altitude.

Example.—Required the sun's parallax at 30° of altitude.

$$\text{Log. cos. } 30^\circ = 9.9375306$$

$$\text{Add } = -5.6274935$$

$$\text{Paral. reqd. } 7'' = \text{Sin. } 5.5050236$$

Consequently $1' 39'' - 7'' = 1' 32''$, the number answering to 30° in the second column of the table.

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TABLE VI.

Corrections of Refraction relative to Temperature.

The refractions of Table V. are calculated for the medium temperature; but when much accuracy is required, it becomes necessary to correct these refractions for the temperature at the time of the observations. These corrections are calculated by substituting the different values of d in the formula

$$\tan. (1.00208 \, d n r) = \tan. n r . \tan. \frac{1}{2} v,$$

and calculating the corresponding refractions; and the difference between these and the refractions answering to the medium temperature, or $57^{\circ}.2$, will give the corrections inserted in this table. These different values of d only cause a variation in the number to be added to the logarithms of the seconds found from the preceding formula. Thus, if the temperature was $78^{\circ}.8$ instead of $57^{\circ}.2$, the value of d would be $78^{\circ}.8 - 32^{\circ} = 46^{\circ}.8$, and the number to be added would be -1.3821650 . Hence if it were required to find the correction of the refraction at 30° of altitude corresponding to this temperature, by taking the number of seconds found in the preceding example, we have

$$\text{Log. of } 394^{\circ}.5 = 2.5984470$$

$$\text{add } -1.3821650$$

$$\text{Required refraction} = 95^{\circ}.107 = 1^{\circ}.982120$$

$$\text{Refract. at med. temper.} = 99^{\circ}.167$$

$$\text{Numb. in the Tab. Differ.} = 4^{\circ}.06 = 4'' \text{ very nearly.}$$

TABLE VII.

Corrections of Refraction relative to Atmospheric Pressure.

The refractions of Table V. are calculated for the medium pressure of the atmosphere, or 29.92 inches of the mercury in the barometer, and therefore require correction when the pressure is different from this, and great accuracy is requisite.

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Now, as the refracting power of the atmosphere is proportional to its density, and its density as its pressure, it follows that the refracting power is directly as the pressure: therefore if h denote the height of the mercury in the barometer, the refracting power of the atmosphere will vary as $\frac{h}{29.92}$: hence is derived this

RULE.—Multiply the mean refraction by this ratio, and the product will be the refraction answering to the given pressure. The difference between this and the mean refraction is the correction required.

Example.—Required the correction of the medium refraction on account of pressure, when the height of the barometer is 29.1 inches, and the altitude of the heavenly body 30° .

Here the medium refraction is $99''$, which being multiplied by $\frac{29.1}{29.92}$, gives $96''$ nearly for the refraction at the given pressure 29.1 inches; and $99'' - 96'' = 3''$, the required correction in the Table.

Remark.—When it is thought necessary to correct the medium refractions of Table V, both the corrections contained in this and the preceding table must be used; for the density of the atmosphere is in the direct ratio of its pressure and the inverse ratio of its temperature, and consequently in the compound ratio of the two. It is seldom necessary to make use of these corrections for small variations from the mean pressure and temperature corresponding to the refractions of Table V, when both these variations are either in excess or defect; for then, the one being additive and the other subtractive, the effective correction is only their difference, which is generally very small and frequently nothing. But when the one variation is in excess and the other in defect, the corrections are both additive or both subtractive, and the real correction is their sum. For example, if the thermometer were at the freezing point, and the barometer at 30.6 inches, the total correction at 10° of altitude would be $17'' + 8'' = 25''$ additive; and if the barometer were at 29.1 inches, and Fahrenheit's thermometer at $75^\circ.3$, the whole correction at the same altitude would be $9'' + 11'' = 20''$, subtractive.

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TABLE VIII.

Parallax of the Moon less Refraction.

The moon being much the nearest of the heavenly bodies, and subject to considerable variation in her distance, her parallax is not only the greatest, but also varies with respect to both time and place. The variations depending upon the latter are given in Table III; and with respect to the former, astronomers prove that the sine of the moon's horizontal parallax is equal to the ratio between the radius of the earth and the distance between the centres of the earth and moon at any given time; or by adopting the initial of these words only, $\sin. p = \frac{r}{d}$, which, according to Delambre, is $\frac{1635.5}{98650} = .0165788 = \sin. 57'$, for the mean distance of the moon from the earth; the extremes of the horizontal parallax being about $53'$ and $61'$. Then, if p denote the parallax at any altitude a , since this varies as the cosine of a , we easily obtain the following formula,

$$\sin. p = \sin. P \cos. a;$$

which converted into words gives this easy

RULE.—Add the log. sine of the horizontal parallax and the log. cosine of the moon's altitude together, omitting 10 in the index, and the sum will be the sine of the parallax corresponding to that altitude.

Example.—Required the moon's parallax at 30° of altitude, the horizontal parallax being $55'$.

$$\text{Log. sine } 55' = 8.2040703$$

$$\text{Log. cos. } 30^\circ = 9.9375306$$

$$\text{Parallax required } 47' 38'' = 8.116009$$

$$\text{Subtract refraction } 1' 39''$$

$$\text{Parallax—Refraction, } 45' 59'' \text{ of the Table.}$$

The right hand page of this table also contains the proportional parts for the odd minutes of altitude, and the seconds of the horizontal parallax; by means of which the whole of the required parallax may be obtained by inspection. Thus, the first

column entitled proportional parts contains those parts corresponding to 5, 10, 20, 30, 40, and 50 seconds of the horizontal parallax; the second column contains those answering to 1, 11, 21, 31, 41 and 51 seconds; and the third column, those for 2, 12, 22, 32, &c. The last two columns of the page contain the odd minutes of the altitude, from 1 to 9, with their corresponding proportional parts, and the proper sign at the top of the column. The use of these is evident by inspection.

TABLE IX.

Change in Altitude during the last Minute which precedes, and the first that follows, the Sun's Passage over the Meridian.

The altitude of the sun varies at every instant from his rising to his setting, increasing until he arrives at the meridian and then decreasing after he has passed it. But the variation in altitude is not uniform, owing to the different degrees of obliquity of the sun's path and the vertical circle. This change of altitude for any given time must therefore be found by calculating his zenith distances at the beginning and end of that time, and subtracting the one from the other. This altitude and its variations, however, are the same at equal intervals before and after the sun's passage over the meridian, and consequently the same calculations will answer for both the ascending and descending change. *M. Delambre*, in his "*Leçons élémentaires d'Astronomie*," page 207, has given the following formula for finding the zenith distance of the sun at any given time: viz.

$$\cos Z = \cos PA \cdot \cos PZ + \sin PA \cdot \sin PZ \cdot \cos P;$$

in which Z = the zenith distance; PA = the polar distance, equal to the declination, according as its denomination is the same with or different from that of the latitude; PZ = the distance between the pole and the zenith = the complement of the latitude; and P = the horary angle, in this case = $15'$ of a degree. This formula therefore furnishes the following

RULE.—1st. Add the log. cosines of $90 \mp$ declination and of the complement of the latitude together, subtract 20 from the index of their sum, and find the natural number answering to the remainder.

2d. Add together the log. sines of the same quantities and the log. cosine of $15'$, subtract 30 from the index, and find the natural number corresponding to the remainder.

3d. Add these to natural numbers together, take the logarithm of the sum, and increase the index by 10, which will give the log. cosine of ZA , the zenith distance, at one minute of time before or after the sun's passage over the meridian.

When the sun is on the meridian, the horary angle $r = 0$, the cosine of which is equal 1, and then the zenith distance is equal to the difference of the latitude and declination when they are of the same name, or to their sum when of a different denomination. Therefore the zenith distance obtained by the calculation taken from this sum or difference will give the change in altitude during the last minute which precedes, or the first which follows, the sun's passage over the meridian.

Example.—Required the increase or decrease in the sun's altitude during the last minute before, or the first after, his passage over the meridian, the latitude being 60° and the declination 18° , both of the same denomination.

Log. cos. $72^\circ = 9.4899824$	Log. sin. $72^\circ = 9.9782063$
Log. cos. $30 = 9.9375306$	Log. sin. $30 = 9.689700$
$- 1.4275130$	Log. cos. $15' = 9.9999959$
	$- 1.6771722$
Nat. Numbers { 2676165	
4755238	
Sum. 7431403	Log. $+ 10 = 9.8710708$
Correspond. zen. distance $= 42^\circ 0' 1''$	
Subtract $60^\circ - 18^\circ = 42$	
Change in alt. required, as in the table	$1''.4$

TABLE X.

Multipliers of the Numbers contained in Table IX.

This table depends upon the approximative principle, that the change in altitude during a short time before and after the sun passes the meridian, is proportional to the square of the time

included between the moment of observation and the instant of that passage; and the numbers it contains are therefore found by squaring this time expressed in minutes and decimals. This approximation is susceptible of being extended to about 8 minutes of time, or 2 degrees of space, before and after the sun's passage. If it were required to find the number in the table answering to $4^{\circ} 42'$, either before or after noon, it is equal to $(4^{\circ} 42')^2 = (4.7)^2 = 22.09$, or 22.1 nearly, as in the table.

TABLE XI.

Numbers for finding the Corrections of the Longitudes obtained by Marine Chronometers.

To obtain as near an approximation to the truth as possible, the gain or loss of the chronometer by which the difference of longitude is ascertained, is at first supposed to be nothing, and to increase uniformly, and therefore the gain on any given day from the commencement will be equal to the sum of the gains on all the preceding days, or to the sum of an arithmetical progression, having one for the first term, one for the common difference, and the given time in days for the number of terms: that is, the sum of the series of consecutive numbers $1 + 2 + 3 + 4 + 5 + \&c.$ Now as the number of units in the last term of this series is always equal to the number of terms, let this term be denoted by n , and the following simple formula will give the number in the table answering to the number of days expressed by n : viz. $\frac{1}{2} n (n + 1)$.

Example.—Required the number in the table answering to 57 days.

Here $n = 57$, and by substitution the formula becomes

$$\frac{1}{2} (57 + 1) = 57 + 29 = 1653, \text{ the number required.}$$

TABLE XII. and XIII.

For finding the Correction of the Loss of two Altitudes of the Sun taken out of the Meridian.

The principles upon which these tables are constructed are

investigated in Note VII, in the preceding pages; and the formula from which they are computed is

$$\text{versed sin. A} = \frac{\cos. (L \pm H)}{\cos. L \cdot \cos. H} \mp \frac{\sin. D}{\cos. L \cdot \cos. H};$$

where A denotes the azimuth, L the latitude, H the altitude, and D the declination of the sun. The upper sign is to be used when the latitude and declination are of the same denomination, and the lower when they are different. The left hand page of Table XII. contains the first term of this formula; and is to be entered with the latitude L and altitude H. The numbers in this table are therefore calculated by the following

RULE.—Add the complement log. cosines of the latitude and altitude to the log. cosine of their difference, subtract 10 from the index of the sum, and the remainder will be the logarithm of the required number in the left hand page.

And for the numbers in the right hand page, of the same table, entitled argument; Add the two complement log. cosines of the latitude and altitude together, and their sum will be the logarithm of the required number.

Example.—Let it be required to calculate the numbers in the table answering to 54° of latitude and 42° of altitude, when both the latitude and declination are of the same denomination.

$$\text{Comp. log. cos. } 54^\circ = 0.2307813 \quad \left. \begin{array}{l} \\ \end{array} \right\} 0.3597080 \text{ sum.}$$

$$\text{Comp. log. cos. } 42^\circ = 0.1289267 \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Nat. numb. } 2.29$$

$$\text{Log. cos. } 12^\circ = 9.9904044$$

$$\text{First term, nat. numb. } 2.24 = 0.3501124$$

Table XIII. contains the second member of the same formula, and is to be entered with the declination and the argument taken from the right hand page of Table XII; and therefore the numbers are readily calculated by the following

RULE.—Add the complements of the log. cosines of the latitude and altitude to the log. sine of the declination, diminishing the index by 10, and the remainder will be the logarithm of the required number.

Example.—Calculate the number in Table XIII. when the declination is 20° North, and the latitude and altitude the same as in the preceding example.

$$\text{Comp. log. cos. } 54^{\circ} = 0.2307813$$

$$\text{Comp. log. cos. } 42^{\circ} = 0.1289267$$

$$\text{Log. sin. decl. } 20^{\circ} = 0.5340517$$

$$\text{Nat. numb. required } 783 \quad 1.8937597$$

TABLE XIV.

Azimuth corresponding to the Day made in Latitude.

The numbers in this table are the versed sines obtained from the preceding formula and their corresponding azimuthal arcs. The multipliers in the table are the versed sines, and may either be immediately calculated from the formula, as above, or found in the Tables XII and XIII, as directed in art. 40; and then the corresponding arc found in a table of natural sines.

Example.—Required the azimuth, when the given quantities are the same as in the preceding example.

The first term corresponding to these numbers has been found = 2.24, and the second = .783; and therefore (art. 40) $(2 + .783) - 2.24 = .543$; the versed sine of the azimuth, the arc corresponding to which is $62^{\circ} 48'$, the number in the table very nearly; for 62° answers to the multiplier .53 and 60° to .55.

TABLE XV.

Altitude of the Sun when he passes the Prime Vertical.

Astronomers prove that when the sun passes the prime vertical, the sine of his altitude is equal to the sine of his declination divided by the sine of the latitude of the place of observation; the numbers in this table may therefore be easily calculated by the following simple formula, in which the respective words are denoted by their initials, viz.

$$\sin. A = \frac{\sin. D}{\sin. L}$$

That is.—Add the complement of the log. sine of the latitude to the log. sine of the declination, and the sum will be the log. sine of the altitude.

Example.—Required the sun's altitude when he passes the prime vertical in latitude 52° North, and his declination is 16° North.

$$\begin{aligned}\text{Comp. log. sine } 52^\circ &= 0.1034679 \\ \text{Log. sine } 16 &= 0.4403381 \\ \text{Altitude req. } 20^\circ 29' &= \text{sine } 0.5438060\end{aligned}$$

TABLE XVI.

Right Ascensions and Declinations of 36 of the principal fixed Stars for the 1st of January 1815, with their annual Variations.

Astronomers generally obtain the right ascensions and declinations of the fixed stars by observation, and calculate their other elements from these. The first column of this table contains the names of the stars with their characters and magnitudes annexed; and the annual variations contained in the third column are to be added to the right ascensions of the second column for every year. Thus the right ascension for any time subsequent to the date of the table will be obtained by multiplying the annual variation by the years and parts between that date and the given time, and adding the product to the right ascensions in the table; and for any time prior to the date of the table, by subtracting this product from the tabular right ascensions.

The annual variations in the last column are also to be multiplied in the same manner, and added to the corresponding declinations in the preceding column, or subtracted from them, according as the sign is + or -; and the result will be the required declination answering to the given time.

Example.—Required the right ascension and declination of the star *Rigel* for the 1st of July, 1818.

$$\begin{aligned}\text{Rt. ascen. 1 Jan. 1815} &= 5^h 55' 38'' 99 \\ \text{Variation } 2'' 876 \times 3.5 &= + 10 07 \\ \text{Right. ascen. req.} &= 5^h 55' 49'' 06\end{aligned}$$

$$\begin{aligned}\text{Decl. 1 Jan. 1815} &= 8^\circ 26' 43'' 65 \\ \text{Variation } 4'' 92 \times 3.5 &= - 17 22 \\ \text{Decl. required.} &= 8^\circ 26' 26'' 38 s.\end{aligned}$$

TABLE XVII.

Containing the Logarithms of Numbers, with their Arithmetical Complements, from 1 to 5500.

This table differs from those in common use, only in having the arithmetical complements of the logarithms placed in the same line with the logarithms themselves; and consequently does not require any particular explanation. It may be observed, however, that the index to any of the complements, though not inserted in the table, is always equal to the difference between 10 and the number of places of whole numbers in the natural number corresponding to the complement; except when the number is 10, 100, 1000, &c. when it is the difference between 11 and that number. The advantage of using the complements is, that in any proportion performed by logarithms, instead of adding the second and third terms together, and subtracting the first from their sum, the three terms are added together, and 10 is omitted in the index of the sum, which is done mentally, and which therefore reduces the whole to a single operation of addition; as in the subsequent example.

One of the principal uses of common logarithms in the calculations of Nautical Astronomy, is in finding the time answering to the true calculated distance between the moon and the sun or a star. This distance is given in the Nautical Almanac for every 3 hours; and the distance for any intermediate time, or the time for any intermediate distance, is found by proportion. Thus, if the distance were given and the corresponding time required, take the difference between the next greater and next less distances in the Nautical Almanac and also between the given distance and the nearest of these: then the time corresponding to this last difference may readily be found by proportion, or by adding the logarithms of 3 hours = 10800 seconds, and of the less difference to the complement logarithm of the greater difference, omitting 10 in the index; and the natural number answering to the sum will be the time, in seconds, corresponding to the less difference,

and which must be added to or subtracted from the time corresponding to the nearest distance given in the Nautical Almanac, in order to obtain the time required.

Example.—Suppose the distance between the centres of the sun and moon on the evening of the 8th of August 1814, was $91^{\circ} 54'$, what was the exact time of the observation?

$$\begin{array}{rcl} \text{Distance at 6 hours} & = & 92^{\circ} 35' 27'' \\ \text{Distance at 9 hours} & = & 91 \quad 0 \quad 14 \\ \hline \text{Difference in 3 hours} & = & 1^{\circ} 35' 13'' = 95' 2 \text{ nearly.} \end{array}$$

$$\begin{array}{rcl} \text{Distance at 6 hours} & 92^{\circ} 35' 27'' \\ \text{Given distance} & 91 \quad 54 \quad 0 \\ \hline \text{Difference} & 0^{\circ} 41' 27'' = 41' 45 \end{array}$$

$$\text{Comp. log. } 95' 2 = 8.0213631$$

$$\text{Log. of } 41' 45 = 1.6175245$$

$$\text{Log. of 3 ho.} = 10800'' = 4.0334238$$

$$\text{Corresp. time} = 4702'' 3 = 3.6723114 \quad \text{Sum, adding 10 in the index.}$$

Therefore, to the nearest time = 6^h

$$\text{Add } 4702'' 3 = 1 \quad 18' \quad 22'' 3$$

$$\text{Time required} = 7^h \quad 18' \quad 22'' 3$$

TABLE XVIII.

Containing the logarithm Sines and Cosines, with their Complements and Differences answering to every $10''$; also the logarithm Tangents and Cotangents, with their Differences corresponding to the same Arc of $10''$.

This table is different from those in common use, and will be found more convenient, as the complements of the logarithm sines and cosines can be taken from it by inspection in the same manner as the sines and cosines themselves. The differences for $10''$ instead of $1'$ or $60''$ will also be found convenient, by avoiding the proportion in finding the logarithm answering to any number of seconds. The figures on the right hand of the differences of the cosines of small arcs and the sines of larger ones,

and separated from the rest by points, are to be considered as decimals with respect to the other figures. Thus, if it were required to find the log. sine and cosine answering to any number of degrees, minutes, and seconds, the corresponding logarithm difference for 10" must be multiplied by the tens and units in the seconds separately, and the right hand figure of the last product omitted, carrying one, when it exceeds 5, and these two products added to the log. sine of the degrees and minutes or subtracted from the cosine.

Example.—Required the logarithm sine and cosine of $5^{\circ} 15' 37''$.

$$\text{Log. sine of } 5^{\circ} 15' = 9.9614288$$

$$\text{Log. } 30'' = 2288 \times 3 = \quad 6864$$

$$\text{Log. } 7 = 2288 \times 7 = \quad 16027$$

$$\text{Log. sin. } 5^{\circ} 15' 37'' = 9.9614755$$

$$\text{Log. cosine } 5^{\circ} 15' = 9.9981743$$

$$\text{Log. } 30'' = 19.5 \times 3 = \quad 58.5$$

$$\text{Log. } 7 = 19.5 \times 7 = \quad 13.65$$

$$\text{Log. cos. } 5^{\circ} 15' 37'' = 9.9981671$$

This operation of adding the two products answering to the tens and units of the seconds together and subtracting their sum from the log. sine of the degrees and minutes may be avoided, and the whole operation performed by addition, in the same manner as for the sine, by taking the log. cosine of the next greater minute, and also the number of seconds from 60, which may be done mentally, and then adding the log. of the remaining seconds to the log. cosine of the next greater degree and minute instead of subtracting it from the next less. Thus, resuming the latter part of the preceding example, viz. to find the cosine of $5^{\circ} 15' 37''$, since $60'' - 37'' = 23''$, we have

$$\text{Log. cosine of } 5^{\circ} 16' = 9.9981626$$

$$\text{Log. of } 20'' = 19.5 \times 2 = \quad 39$$

$$\text{Log. of } 3 = 19.5 \times 3 = \quad 5.85$$

$$\text{Log. cosine of } 5^{\circ} 15' 37'' = 9.9981671 \text{ as before.}$$

The same observations are equally applicable to the tangent and cotangent as to the sine and cosine.

The operations for finding the complements of the sines and cosines, answering to any number of seconds, are the reverse of those for the sines and cosines themselves; that is, the logarithm corresponding to the given number of seconds must be subtracted for the complement of the sine, and added for the complement of the cosine: or the subtraction may be avoided, as shown above, by taking the complement for the next greater minute and the given number of seconds from 60, and adding the corresponding logarithm. This will appear more clearly from the following.

Example.—Suppose the two sides of a spherical triangle to be $70^{\circ} 35'$ and $41^{\circ} 23'$, and the angle opposite the former $130^{\circ} 4' 28''$; required the angle opposite the latter side.

$$\begin{array}{rcl}
 \text{Comp. log. sine } 70^{\circ} 35' & = & 9.0254303 \\
 \text{Log. sine } 41^{\circ} 23' & = & 9.8202630 \\
 & & 49^{\circ} 55' = 9.8887232 \\
 \text{Log. sine } 120^{\circ} 4' 28'' \text{ or } 49^{\circ} 55' 32'' & = & \left\{ \begin{array}{rcl} 30'' & = & 531 \\ 2'' & = & 35 \end{array} \right. \\
 \text{Required angle} & = & 32^{\circ} 51' 37'' = \sin. 9.7344731
 \end{array}$$

Note. The logarithm secant and cosecant, though not inserted in this table, may easily be found when required; viz. the *secant*, by annexing the difference between the index of the cosine and 19, as an index, to the decimal part of the complement cosine; and the *cosecant*, by annexing the difference between the index of the sine and 19, in the same manner, to the decimal part of its complement. The *secant* and *cosecant*, however, are not required by the preceding rules.

TABLES

FOR FACILITATING THE CALCULATIONS

NAUTICAL ASTRONOMY.

TABLE I.

Depression of the Horizon.

Height of the eye in feet.	Depression.	D. difference.	Distance seen in miles and decimals.	Height of the eye in feet.	Depression.	D. difference.	Distance seen in miles and decimals.
1	0' 59"		1.2	31	5' 28"		6.8
2	1 23	24"	1.7	32	5 33	5"	6.9
3	1 42	19	2.1	33	5 38	5	7.0
4	1 58	16	2.4	34	5 43	5	7.1
5	2 12	14	2.7	35	5 48	5	7.2
6	2 24	12	3.0	36	5 53	5	7.3
7	2 35	11	3.2	37	5 58	5	7.4
8	2 46	11	3.4	38	6 3	5	7.5
9	2 56	10	3.7	39	6 8	5	7.6
10	3 6	9	3.9	40	6 13	5	7.7
11	3 15	9	4.1	41	6 17	14	8.0
12	3 24	8	4.3	46	6 29	13	8.3
13	3 32	8	4.4	49	6 32	13	8.6
14	3 40	8	4.6	52	7 4	12	8.8
15	3 48	8	4.7	55	7 36	12	9.1
16	3 55	7	4.9	58	7 28	11	9.3
17	4 2	7	5.0	61	7 39	11	9.6
18	4 9	7	5.2	64	7 50	11	9.8
19	4 16	7	5.4	67	8 1	11	10.0
20	4 23	7	5.5	70	8 12	10	10.2
21	4 30	6	5.6	73	8 22	10	10.3
22	4 36	6	5.7	76	8 32	10	10.4
23	4 42	6	5.8	79	8 42	10	10.9
24	4 48	6	6.0	82	8 52	10	11.1
25	4 54	6	6.1	85	9 2	10	11.3
26	5 0	6	6.2	88	9 12	9	11.5
27	5 6	6	6.4	91	9 21	9	11.7
28	5 12	5	6.5	93	9 30	9	11.9
29	5 17	5	6.6	97	9 39	9	12.1
30	5 23	5	6.7	100	9 48	9	12.3

TABLE II.

Augmentation of the Moon's Semidiameter.

Apparent altitude.	Horizontal semidiameter.		
	14' 30"	15' 30"	16' 30"
0°	0"	0"	0"
4	1	1	1
8	2	2	2
12	3	3	4
16	4	4	5
20	5	5	6
25	7	7	9
30	6	8	9
35	8	9	10
40	9	10	11
45	10	11	12
55	11	13	14
65	12	14	16
75	13	15	17
90	14	16	18

TABLE III.

Augmentation of the Equatorial Parallax, at different Latitudes.

Latitude.	Equatorial parallax.	
	53'	61"
0°	0'	0'
20	1	1
25	2	2
30	3	3
35	4	4
40	4	5
45	5	6
50	6	7
55	7	8
60	8	9
65	9	10
75	10	11

TABLE IV.

Errors from the Surfaces of the large Mirror when they make with each other an Angle of 1'.

Observed angles.	Observation to the right.	Observation to the left.	Cross observations.
0°	0"	0"	0'
16	2	1	2
20	6	2	4
30	10	1	6
40	16	0	8
45	19	1	9
50	23	2	11
55	28	4	12
60	33	6	14
65	38	8	15
70	47	10	18
75	55	13	21
80	1' 4	16	24
85	1 15	19	28
90	1 28	23	32
95	1 43	28	37
100	2 1	33	43
105	2 23	38	53
110	2 50	47	1' 2
115	3 23	55	1 12
120	4 5	1' 4	1 31
125	5 0	1 15	1 53
130	5 58	1 28	2 15

TABLE OF NAUTICAL ASTRONOMY.

TABLE V:

Refraction for 29.92 Inches of the Barometer, and 57° 2 of Fahrenheit Thermometer.

Apparent altitude.	Refraction less paral- lax of the ☉	Refraction of the stars.	Differ- ences.	Apparent altitude.	Refraction less paral- lax of the ☉	Refraction of the stars.	Differ- ences.
0° 0'	35' 7"	35' 16"	110"	6° 0'	8' 13"	8' 20"	11"
10	31 17	31 25	83	10	8 2	8 11	11
20	29 33	29 42	96	20	7 51	7 59	11
30	27 37	28 6	88	30	7 40	7 48	11
40	26 29	26 38	82	40	7 32	7 38	10
50	25 7	25 15	76	50	7 19	7 28	10
1 0	23 50	23 59	70	7 0	7 9	7 18	9
10	22 40	22 49	63	10	7 6	7 15	9
20	21 4	21 44	61	20	6 51	7 9	9
30	20 34	20 43	56	30	6 43	6 51	8
40	19 38	19 47	53	40	6 35	6 43	8
50	18 45	18 54	48	50	6 27	6 36	7
2 0	17 57	18 6	45	8 0	6 20	6 28	7
10	17 12	17 20	48	10	6 13	6 21	7
20	16 29	16 38	39	20	6 5	6 14	7
30	15 50	15 59	37	30	5 58	6 7	6
40	15 13	15 22	35	40	5 52	6 1	6
50	14 39	14 47	32	50	5 46	5 54	6
3 0	14 4	14 15	30	9 0	5 39	5 48	6
10	13 36	13 45	29	10	5 33	5 42	6
20	13 7	13 16	27	20	5 28	5 36	6
30	12 41	12 49	25	30	5 22	5 31	5
40	12 15	12 24	22	40	5 17	5 25	5
50	11 51	12 0	22	50	5 12	5 20	5
4 0	11 29	11 38	22	10 0	5 6	5 15	5
10	11 7	11 16	20	11 0	4 39	4 47	28
20	10 47	10 56	19	12 0	4 15	4 24	23
30	10 28	10 37	18	13 0	3 55	4 4	19
40	10 10	10 19	17	14 0	3 38	3 46	17
50	9 53	10 2	16	15 0	3 23	3 31	16
5 0	9 37	9 45	15	16 0	3 9	3 18	14
10	9 21	9 30	15	17 0	2 57	3 6	13
20	9 6	9 15	14	18 0	2 47	2 55	11
30	8 52	9 1	14	19 0	2 37	2 45	10
40	8 38	8 47	13	20 0	2 28	2 36	9
50	8 26	8 34	12	21 0	2 20	2 28	8
6 0	8 13	8 22					

TABLE V.

Correction for 29.92 Inches of the Barometer, and 57° of Fahrenheit's Thermometer.

Apparent altitude.	Refraction less paral- lax of the ☉	Refraction of the stars.	Differ- ences.	Apparent altitude.	Refraction less paral- lax of the ☉	Refraction of the stars.	Differ- ences.
21°	2' 20"	2' 28"	8"	56	0' 34"	0' 39"	5"
22	2 13	2 21	7	57	0 33	0 37	4
23	2 6	2 14	6	58	0 31	0 36	5
24	2 0	2 8	5	59	0 30	0 35	5
25	1 53	2 3	6	60	0 29	0 33	4
26	1 49	1 57	5	61	0 28	0 32	4
27	1 44	1 52	4	62	0 26	0 31	5
28	1 40	1 48	4	63	0 25	0 29	4
29	1 36	1 43	4	64	0 24	0 28	4
30	1 32	1 39	4	65	0 23	0 27	4
31	1 28	1 35	4	66	0 22	0 26	4
32	1 24	1 32	3	67	0 21	0 25	4
33	1 21	1 29	3	68	0 20	0 23	3
34	1 18	1 25	3	69	0 19	0 22	3
35	1 15	1 22	3	70	0 18	0 21	3
36	1 12	1 19	3	71	0 17	0 20	3
37	1 9	1 16	3	72	0 16	0 19	3
38	1 6	1 13	2	73	0 15	0 18	3
39	1 4	1 11	2	74	0 14	0 17	3
40	1 2	1 8	2	75	0 13	0 15	2
41	0 59	1 6	2	76	0 12	0 14	2
42	0 57	1 4	2	77	0 11	0 13	2
43	0 55	1 2	2	78	0 10	0 12	2
44	0 53	0 59	2	79	0 10	0 11	1
45	0 51	0 57	2	80	0 9	0 10	1
46	0 49	0 55	2	81	0 8	0 9	1
47	0 48	0 54	1	82	0 7	0 8	1
48	0 46	0 52	2	83	0 6	0 7	1
49	0 44	0 50	2	84	0 5	0 6	1
50	0 43	0 48	1	85	0 4	0 5	1
51	0 41	0 46	2	86	0 3	0 4	1
52	0 39	0 45	1	87	0 3	0 3	0
53	0 38	0 43	1	88	0 2	0 2	0
54	0 37	0 42	1	89	0 1	0 1	0
55	0 35	0 40	2	90	0 0	0 0	0
56	0 34	0 39	1				

TABLES OF NAUTICAL ASTRONOMY.

TABLE VI.

Connections of Refraction relative to Temperature.

The refractions of TABLE V answer to 57.2 of Fahrenheit's thermometer. Cold increases refraction.

Add the following numbers to the refractions of TABLE V:

Subtract them from the numbers of **TABLE VIII**, or from the parallax of the moon less refraction.

Fahrenheit's thermometer.

[illegible]

TABLE VI

Corrections of Refraction relative to Temperature.

The refractions of TABLE V answer to 57° 2 of Fahrenheit's thermometer. Heat diminishes refraction.

Subtract the following numbers from the refractions of TABLE V:

Add them to the numbers of TABLE VIII, or to the parallax of the moon less refraction.

Fahrenheit's thermometer.

[illegible]

TABLES OF NAUTICAL ASTRONOMY.

TABLE VII.

Corrections of Refraction, relative to Atmospheric Pressure.

The refractions of TABLE V are those which take place, when the atmosphere sustains a column of mercury of 29.92 inches.

Refraction increases with the pressure of the atmosphere.

Add the following numbers to the refractions of TABLE V.

Subtract them from the numbers of TABLE VIII, or the parallax of the moon less refraction.

Height of the barometer in inches.

Appa- rent al- titude.	in. 31.22	in. 31.12	in. 30.92	in. 30.72	in. 30.52	in. 30.32	in. 30.12	in. 29.92
5°	27"	23"	19"	16"	12"	8"	4"	0"
5½	25	22	18	14	11	7	4	0
6	24	20	17	13	10	7	3	0
7	21	18	15	12	9	6	3	0
8	18	15	13	10	7	5	3	0
9	16	14	12	9	7	5	2	0
10	15	12	11	8	6	4	2	0
12	12	11	9	7	5	4	2	0
14	10	9	8	6	5	3	2	0
16	9	8	7	5	4	2	1	0
18	8	7	6	5	4	2	1	0
20	7	6	5	4	3	2	1	0
25	6	5	4	3	3	2	1	0
30	5	4	3	3	2	1	1	0
40	3	3	2	2	1	1	1	0
50	2	2	2	1	1	1	0	0
60	2	1	1	1	1	1	0	0
70	1	1	1	1	0	0	0	0
80	0	0	0	0	0	0	0	0
90	0	0	0	0	0	0	0	0

TABLES OF NAUTICAL ASTRONOMY.

TABLE VII.

Corrections of Refraction, relative to Atmospheric Pressure.

The refractions of TABLE V are those which take place when the atmosphere sustains a column of mercury of 29.92 inches.

Refraction diminishes as the pressure of the atmosphere decreases.

Subtract the following numbers from the refractions of TABLE V,

and add them to the numbers of TABLE VIII, or the parallax of the moon less refraction.

Height of the barometer in inches.

Apparent altitude.	in. 29.92	in. 29.72	in. 29.52	in. 29.32	in. 29.12	in. 28.92	in. 28.72	in. 28.52
5°	0"	4"	8"	12"	16"	20"	24"	27"
5½	0	4	7	11	15	18	22	25
6	0	3	6	10	14	17	20	24
7	0	3	6	9	12	15	18	21
8	0	3	5	8	11	14	16	18
9	0	2	5	7	10	12	14	16
10	0	2	4	6	9	11	13	15
12	0	2	4	5	7	9	11	13
14	0	2	3	5	6	8	9	11
16	0	1	3	4	5	7	8	9
18	0	1	2	4	5	6	7	8
20	0	1	2	3	4	5	6	7
25	0	1	2	2	3	4	5	6
30	0	1	1	2	3	4	4	5
40	0	0	1	1	2	3	3	3
50	0	0	1	1	1	2	2	2
60	0	0	0	1	1	1	1	2
70	0	0	0	0	1	1	1	1
80	0	0	0	0	0	0	0	1
90	0	0	0	0	0	0	0	0

TABLE VIII.

Parallax of the Moon less Refraction.

Apparent altitude.	Horizontal parallax.						
	53'	54'	55'	56'	57'	58'	59'
0° 0'	19' 44"	20' 44"	21' 44"	22' 44"	23' 44"	24' 44"	25' 44"
10	21 36	22 36	23 36	24 36	25 36	26 36	27 36
20	23 18	24 18	25 18	26 18	27 18	28 18	29 18
30	24 54	25 54	26 54	27 54	28 54	29 54	30 54
40	26 22	27 22	28 22	29 22	30 22	31 22	32 22
50	27 45	28 45	29 45	30 45	31 45	32 45	33 45
1 0	29 1	30 1	31 1	32 1	33 1	34 1	35 1
10	30 9	31 9	32 9	33 9	34 9	35 9	36 9
20	31 16	32 16	33 16	34 16	35 16	36 16	37 16
30	32 16	33 16	34 16	35 16	36 16	37 16	38 16
40	33 12	34 12	35 12	36 12	37 12	38 12	39 12
50	34 5	35 5	36 5	37 5	38 5	39 5	40 5
2 6	34 52	35 52	36 52	37 52	38 52	39 52	40 52
10	35 38	36 38	37 38	38 38	39 38	40 38	41 38
20	36 19	37 19	38 19	39 19	40 19	41 19	42 19
30	36 58	37 58	38 58	39 58	40 58	41 58	42 58
40	37 35	38 35	39 35	40 35	41 35	42 35	43 35
50	38 9	39 9	40 9	41 9	42 9	43 9	44 9
3 0	38 41	39 41	40 41	41 41	42 41	43 41	44 41
10	39 10	40 10	41 10	42 10	43 10	44 10	45 10
20	39 38	40 38	41 38	42 38	43 38	44 38	45 38
30	40 5	41 5	42 5	43 5	44 5	45 5	46 5
40	40 30	41 30	42 30	43 30	44 30	45 30	46 30
50	40 53	41 53	42 53	43 53	44 53	45 53	46 53
4 0	41 14	42 14	43 14	44 14	45 14	46 14	47 14
10	41 36	42 36	43 36	44 36	45 36	46 36	47 36
20	41 55	42 55	43 55	44 55	45 55	46 55	47 55
30	42 13	43 13	44 13	45 13	46 13	47 13	48 13
40	42 30	43 30	44 30	45 30	46 30	47 30	48 30
50	42 47	43 47	44 47	45 47	46 47	47 47	48 47
5 0	42 3	43 3	44 3	45 3	46 3	47 3	48 3
10	43 17	44 17	45 17	46 17	47 17	48 17	49 17
20	43 32	44 32	45 32	46 32	47 32	48 32	49 32
30	43 44	44 44	45 44	46 44	47 44	48 44	49 44
40	43 57	44 57	45 57	46 57	47 57	48 57	49 57
50	44 9	45 9	46 9	47 9	48 9	49 9	50 9

TABLE VII.

Parallax of the Moon less Refraction.

		Proportional parts for the parallax.										For the
60'	61'	0"	1'	2'	3'	4'	5'	6'	7'	8'	9'	Parallax.
26'	44"	0"	1	2	3	4	5	6	7	8	9	
28	36	10	11	12	13	14	15	16	17	18	19	
30	18	20	21	22	23	24	25	26	27	28	29	
31	54	30	31	32	33	34	35	36	37	38	39	
33	24	40	41	42	43	44	45	46	47	48	49	
34	45	50	51	52	53	54	55	56	57	58	59	
36	1	0	1	2	3	4	5	6	7	8	9	
37	9	10	11	12	13	14	15	16	17	18	19	
38	16	20	21	22	23	24	25	26	27	28	29	
39	24	30	31	32	33	34	35	36	37	38	39	
40	32	40	41	42	43	44	45	46	47	48	49	
41	40	50	51	52	53	54	55	56	57	58	59	
41	52	0	1	2	3	4	5	6	7	8	9	
42	39	10	11	12	13	14	15	16	17	18	19	
43	19	20	21	22	23	24	25	26	27	28	29	
43	57	30	31	32	33	34	35	36	37	38	39	
44	34	40	41	42	43	44	45	46	47	48	49	
45	9	50	51	52	53	54	55	56	57	58	59	
45	40	0	1	2	3	4	5	6	7	8	9	
46	9	10	11	12	13	14	15	16	17	18	19	
46	38	20	21	22	23	24	25	26	27	28	29	
47	4	30	31	32	33	34	35	36	37	38	39	
47	29	40	41	42	43	44	45	46	47	48	49	
47	52	50	51	52	53	54	55	56	57	58	59	
48	13	0	1	2	3	4	5	6	7	8	9	
48	35	10	11	12	13	14	15	16	17	18	19	
48	54	20	21	22	23	24	25	26	27	28	29	
49	12	30	31	32	33	34	35	36	37	38	39	
49	29	40	41	42	43	44	45	46	47	48	49	
49	45	50	51	52	53	54	55	56	57	58	59	
50	1	0	1	2	3	4	5	6	7	8	9	
50	16	10	11	12	13	14	15	16	17	18	19	
50	30	20	21	22	23	24	25	26	27	28	29	
50	43	30	31	32	33	34	35	36	37	38	39	
50	56	40	41	42	43	44	45	46	47	48	49	
50	7	50	51	52	53	54	55	56	57	58	59	

TABLE VIII.

Parallax of the Moon less Refraction.

alt.		Horizontal parallax.									
		53'		54'		55'		56'		57'	
6°	0'	44'	21"	45'	21"	46'	20"	47'	2	48'	20"
	10'	44'	31	45'	31	46'	30	47'	30	48'	30
	20'	44'	41	45'	41	46'	41	47'	41	48'	40
	30'	44'	51	45'	51	46'	51	47'	51	48'	50
	40'	45'	0	46'	0	47'	0	47'	0	48'	59
	50'	45'	9	46'	9	47'	9	48'	8	49'	8
7		45'	18	46'	18	47'	17	48'	17	49'	26
	10'	45'	26	46'	26	47'	26	48'	25	49'	25
	20'	45'	34	46'	34	47'	34	48'	33	49'	33
	30'	45'	41	46'	41	47'	41	48'	40	49'	40
	40'	45'	48	46'	48	47'	48	48'	47	49'	47
	50'	45'	54	46'	54	47'	54	48'	53	49'	53
8	0	46'	1	47'	1	48'	0	49'	0	49'	59
	10'	46'	6	47'	6	48'	5	49'	5	50'	4
	20'	46'	12	47'	12	48'	11	49'	10	50'	10
	30'	46'	17	47'	17	48'	17	49'	16	50'	15
	40'	46'	23	47'	23	48'	22	49'	21	50'	21
	50'	46'	28	47'	28	48'	27	49'	26	50'	26
9	0	46'	33	47'	32	48'	32	49'	31	50'	30
	10'	46'	37	47'	37	48'	36	49'	36	50'	35
	20'	46'	42	47'	41	48'	41	49'	40	50'	39
	30'	46'	46	47'	45	48'	44	49'	43	50'	42
	40'	46'	49	47'	49	48'	48	49'	47	50'	46
	50'	46'	52	47'	52	48'	51	49'	50	50'	50
10	0	46'	57	47'	56	48'	55	49'	54	50'	53
	10'	47'	0	47'	59	48'	58	49'	57	50'	56
	20'	47'	3	48'	2	49'	1	50'	0	50'	59
	30'	47'	5	48'	5	49'	4	50'	3	51'	2
	40'	47'	8	48'	8	49'	7	50'	6	51'	5
	50'	47'	10	48'	11	49'	10	50'	9	51'	8
11	0	47'	14	48'	13	49'	12	50'	11	51'	10
	10'	47'	17	48'	16	49'	15	50'	14	51'	12
	20'	47'	19	48'	18	49'	17	50'	16	51'	15
	30'	47'	21	48'	20	49'	19	50'	18	51'	17
	40'	47'	23	48'	22	49'	21	50'	20	51'	19
	50'	47'	25	48'	24	49'	23	50'	22	51'	20

TABLE VIII.

Parallax of the Moon less Refraction.

Proportional parts for the parallax.														For the	
60'	61'	1'	1"	2"	3"	4"	5"	6"	7"	8"	9"	10"	+		
51'	19'	52'	18"	0	1	2	3	4	5	6	7	8	9		
51	29	52	28	10	11	12	13	14	15	16	17	18	19		
51	39	52	39	20	21	22	23	24	25	26	27	28	29		
51	49	52	49	30	31	32	33	34	35	36	37	38	39		
51	58	52	57	40	41	42	43	44	45	46	47	48	49		
52	7	53	6	50	51	52	53	54	55	56	57	58	59		
52	15	53	15	0	1	2	3	4	5	6	7	8	9		
52	23	53	23	10	11	12	13	14	15	16	17	18	19		
52	31	53	31	20	21	22	23	24	25	26	27	28	29		
52	38	53	38	30	31	32	33	34	35	36	37	38	39		
52	46	53	45	40	41	42	43	44	45	46	47	48	49		
52	51	53	50	50	51	52	53	54	55	56	57	58	59		
52	57	53	56	0	1	2	3	4	5	6	7	8	9		
53	7	54	6	10	11	12	13	14	15	16	17	18	19		
53	15	54	15	20	21	22	23	24	25	26	27	28	29		
53	23	54	23	30	31	32	33	34	35	36	37	38	39		
53	31	54	31	40	41	42	43	44	45	46	47	48	49		
53	38	54	38	50	51	52	53	54	55	56	57	58	59		
53	46	54	45	0	1	2	3	4	5	6	7	8	9		
53	51	54	50	10	11	12	13	14	15	16	17	18	19		
53	57	54	56	20	21	22	23	24	25	26	27	28	29		
53	59	54	58	30	31	32	33	34	35	36	37	38	39		
54	2	55	1	40	41	42	43	44	45	46	47	48	49		
54	4	55	3	50	51	52	53	54	55	56	57	58	59		
54	11	55	10	0	1	2	3	4	5	6	7	8	9		
54	13	55	12	10	11	12	13	14	15	16	17	18	19		
54	15	55	14	20	21	22	23	24	25	26	27	28	29		
54	16	55	15	30	31	32	33	34	35	36	37	38	39		
54	16	55	15	40	41	42	43	44	45	46	47	48	49		
54	16	55	15	50	51	52	53	54	55	56	57	58	59		

TABLES OF NAUTICAL ASTRONOMY.

TABLE VIII.

Parallax of the Moon less Refraction.

Horizontal parallax.										
Altitude.	53'		54'		55'		56'		57'	
12° 0'	47'	27"	48'	25"	49'	25"	50'	25"	51'	21"
10	47	29	48	27	49	26	50	26	51	22
20	47	30	48	29	49	28	50	28	51	23
30	47	32	48	30	49	29	50	29	51	24
40	47	33	48	32	49	30	50	30	51	25
50	47	34	48	33	49	31	50	31	51	26
13 0	47	35	48	34	49	32	50	31	51	27
10	47	36	48	35	49	33	50	32	51	28
20	47	36	48	36	49	34	50	33	51	29
30	47	38	48	36	49	35	50	34	51	30
40	47	39	48	37	49	35	50	34	51	31
50	47	39	48	38	49	36	50	34	51	32
14 0	47	40	48	38	49	36	50	35	51	33
10	47	40	48	38	49	37	50	35	51	33
20	47	41	48	39	49	37	50	35	51	33
30	47	41	48	39	49	37	50	35	51	33
40	47	41	48	39	49	37	50	35	51	33
50	47	41	48	39	49	37	50	35	51	33
15 0	47	41	48	39	49	37	50	35	51	33
10	47	41	48	39	49	37	50	35	51	33
20	47	41	48	39	49	37	50	35	51	33
30	47	41	48	39	49	37	50	35	51	33
40	47	41	48	39	49	37	50	35	51	33
50	47	40	48	38	49	36	50	34	51	31
16 0	47	40	48	38	49	35	50	33	51	31
10	47	40	48	37	49	35	50	32	51	30
20	47	39	48	36	49	34	50	32	51	29
30	47	38	48	36	49	33	50	31	51	29
40	47	38	48	35	49	33	50	30	51	28
50	47	37	48	34	49	32	50	29	51	27
17 0	47	36	48	34	49	31	50	28	51	26
10	47	35	48	33	49	30	50	27	51	25
20	47	34	48	32	49	29	50	26	51	24
30	47	34	48	31	49	28	50	25	51	22
40	47	33	48	30	49	27	50	24	51	21
50	47	31	48	29	49	26	50	23	51	20

TABLES OF NAUTICAL ASTRONOMY.

TABLE VMI.

Parallax of the Moon less Refraction.

		Proportional parts for the parallax.										the
60'	61'	0"	1"	2"	3"	4"	5"	6"	7"	8"	9"	+
54 18	55 17	0	1	2	3	4	5	6	7	8	9	0"
54 19	55 18	10	11	12	13	14	15	16	17	18	19	0
54 21	55 19	20	21	22	23	24	25	26	27	28	29	0
54 22	55 20	30	31	32	33	34	35	36	37	38	39	0
54 23	55 21	40	41	42	43	44	45	46	47	48	49	0
54 24	55 22	50	51	52	53	54	55	56	57	58	59	0
54 25	55 23	0	1	2	3	4	5	6	7	8	9	1
54 26	55 24	10	11	12	13	14	15	16	17	18	19	1
54 27	55 25	20	21	22	23	24	25	26	27	28	29	1
54 28	55 26	30	31	32	33	34	35	36	37	38	39	1
54 29	55 27	40	41	42	43	44	45	46	47	48	49	1
54 30	55 28	50	51	52	53	54	55	56	57	58	59	1
54 31	55 29	0	1	2	3	4	5	6	7	8	9	0
54 32	55 30	10	11	12	13	14	15	16	17	18	19	0
54 33	55 31	20	21	22	23	24	25	26	27	28	29	0
54 34	55 32	30	31	32	33	34	35	36	37	38	39	0
54 35	55 33	40	41	42	43	44	45	46	47	48	49	0
54 36	55 34	50	51	52	53	54	55	56	57	58	59	0
54 37	55 35	0	1	2	3	4	5	6	7	8	9	0
54 38	55 36	10	11	12	13	14	15	16	17	18	19	0
54 39	55 37	20	21	22	23	24	25	26	27	28	29	0
54 40	55 38	30	31	32	33	34	35	36	37	38	39	0
54 41	55 39	40	41	42	43	44	45	46	47	48	49	0
54 42	55 40	50	51	52	53	54	55	56	57	58	59	0
54 43	55 41	0	1	2	3	4	5	6	7	8	9	0
54 44	55 42	10	11	12	13	14	15	16	17	18	19	0
54 45	55 43	20	21	22	23	24	25	26	27	28	29	0
54 46	55 44	30	31	32	33	34	35	36	37	38	39	0
54 47	55 45	40	41	42	43	44	45	46	47	48	49	0
54 48	55 46	50	51	52	53	54	55	56	57	58	59	0
54 49	55 47	0	1	2	3	4	5	6	7	8	9	0
54 50	55 48	10	11	12	13	14	15	16	17	18	19	0
54 51	55 49	20	21	22	23	24	25	26	27	28	29	0
54 52	55 50	30	31	32	33	34	35	36	37	38	39	0
54 53	55 51	40	41	42	43	44	45	46	47	48	49	0
54 54	55 52	50	51	52	53	54	55	56	57	58	59	0
54 55	55 53	0	1	2	3	4	5	6	7	8	9	0
54 56	55 54	10	11	12	13	14	15	16	17	18	19	0
54 57	55 55	20	21	22	23	24	25	26	27	28	29	0
54 58	55 56	30	31	32	33	34	35	36	37	38	39	0
54 59	55 57	40	41	42	43	44	45	46	47	48	49	0
54 60	55 58	50	51	52	53	54	55	56	57	58	59	0
54 61	55 59	0	1	2	3	4	5	6	7	8	9	0
54 62	55 60	10	11	12	13	14	15	16	17	18	19	0
54 63	55 61	20	21	22	23	24	25	26	27	28	29	0
54 64	55 62	30	31	32	33	34	35	36	37	38	39	0
54 65	55 63	40	41	42	43	44	45	46	47	48	49	0
54 66	55 64	50	51	52	53	54	55	56	57	58	59	0
54 67	55 65	0	1	2	3	4	5	6	7	8	9	0
54 68	55 66	10	11	12	13	14	15	16	17	18	19	0
54 69	55 67	20	21	22	23	24	25	26	27	28	29	0
54 70	55 68	30	31	32	33	34	35	36	37	38	39	0
54 71	55 69	40	41	42	43	44	45	46	47	48	49	0
54 72	55 70	50	51	52	53	54	55	56	57	58	59	0
54 73	55 71	0	1	2	3	4	5	6	7	8	9	0
54 74	55 72	10	11	12	13	14	15	16	17	18	19	0
54 75	55 73	20	21	22	23	24	25	26	27	28	29	0
54 76	55 74	30	31	32	33	34	35	36	37	38	39	0
54 77	55 75	40	41	42	43	44	45	46	47	48	49	0
54 78	55 76	50	51	52	53	54	55	56	57	58	59	0
54 79	55 77	0	1	2	3	4	5	6	7	8	9	0
54 80	55 78	10	11	12	13	14	15	16	17	18	19	0
54 81	55 79	20	21	22	23	24	25	26	27	28	29	0
54 82	55 80	30	31	32	33	34	35	36	37	38	39	0
54 83	55 81	40	41	42	43	44	45	46	47	48	49	0
54 84	55 82	50	51	52	53	54	55	56	57	58	59	0
54 85	55 83	0	1	2	3	4	5	6	7	8	9	0
54 86	55 84	10	11	12	13	14	15	16	17	18	19	0
54 87	55 85	20	21	22	23	24	25	26	27	28	29	0
54 88	55 86	30	31	32	33	34	35	36	37	38	39	0
54 89	55 87	40	41	42	43	44	45	46	47	48	49	0
54 90	55 88	50	51	52	53	54	55	56	57	58	59	0
54 91	55 89	0	1	2	3	4	5	6	7	8	9	0
54 92	55 90	10	11	12	13	14	15	16	17	18	19	0
54 93	55 91	20	21	22	23	24	25	26	27	28	29	0
54 94	55 92	30	31	32	33	34	35	36	37	38	39	0
54 95	55 93	40	41	42	43	44	45	46	47	48	49	0
54 96	55 94	50	51	52	53	54	55	56	57	58	59	0
54 97	55 95	0	1	2	3	4	5	6	7	8	9	0
54 98	55 96	10	11	12	13	14	15	16	17	18	19	0
54 99	55 97	20	21	22	23	24	25	26	27	28	29	0
54 100	55 98	30	31	32	33	34	35	36	37	38	39	0

TABLE VIII.

Parallax of the Moon less Refraction.

Apparent distance.	Horizontal parallax.						
	53'	54'	55'	56'	57'	58'	59'
18° 0"	47' 30"	48' 27"	49' 24"	50' 22"	51' 19"	52' 16"	53' 13"
10	47' 29	48' 26	49' 23	50' 20	51' 17	52' 14	53' 11
20	47' 28	48' 25	49' 22	50' 19	51' 16	52' 13	53' 10
30	47' 27	48' 24	49' 20	50' 17	51' 14	52' 11	53' 8
40	47' 25	48' 22	49' 19	50' 16	51' 13	52' 9	53' 6
50	47' 24	48' 21	49' 17	50' 14	51' 11	52' 8	53' 5
19 0	47' 22	48' 19	49' 16	50' 13	51' 9	52' 6	53' 3
10	47' 21	48' 18	49' 14	50' 11	51' 8	52' 4	53' 1
20	47' 19	48' 16	49' 13	50' 9	51' 6	52' 2	53' 59
30	47' 18	48' 15	49' 11	50' 8	51' 5	52' 1	53' 57
40	47' 16	48' 13	49' 9	50' 6	51' 2	51' 59	52' 55
50	47' 14	48' 11	49' 7	50' 4	51' 0	51' 57	52' 53
20 0	47' 13	48' 9	49' 5	50' 2	50' 58	51' 55	52' 51
10	47' 11	48' 7	49' 3	50' 0	50' 56	51' 53	52' 49
20	47' 9	48' 5	49' 2	50' 58	50' 54	51' 50	52' 47
30	47' 7	48' 3	49' 0	50' 56	50' 52	51' 48	52' 44
40	47' 5	48' 1	48' 57	49' 54	50' 50	51' 46	52' 42
50	47' 3	47' 59	48' 55	49' 51	50' 48	51' 44	52' 40
21 0	47' 1	47' 57	48' 53	49' 49	50' 45	51' 41	52' 37
10	46' 59	47' 55	48' 51	49' 47	50' 43	51' 39	52' 35
20	46' 57	47' 53	48' 49	49' 45	50' 41	51' 36	52' 32
30	46' 55	47' 51	48' 46	49' 42	50' 38	51' 34	52' 30
40	46' 53	47' 48	48' 44	49' 40	50' 36	51' 31	52' 27
50	46' 50	47' 46	48' 42	49' 37	50' 33	51' 29	52' 25
22 0	46' 48	47' 44	48' 39	49' 35	50' 31	51' 26	52' 22
10	46' 46	47' 41	48' 37	49' 32	50' 28	51' 24	52' 19
20	46' 43	47' 39	48' 34	49' 30	50' 25	51' 21	52' 16
30	46' 41	47' 36	48' 32	49' 27	50' 23	51' 18	52' 14
40	46' 39	47' 34	48' 29	49' 25	50' 20	51' 15	52' 11
50	46' 36	47' 31	48' 27	49' 22	50' 17	51' 12	52' 8
23 0	46' 34	47' 29	48' 24	49' 19	50' 14	51' 10	52' 5
10	46' 31	47' 26	48' 21	49' 17	50' 12	51' 7	52' 2
20	46' 28	47' 23	48' 19	49' 14	50' 9	51' 4	51' 59
30	46' 26	47' 21	48' 16	49' 11	50' 6	51' 1	51' 56
40	46' 23	47' 18	48' 13	49' 8	50' 3	50' 58	51' 53
50	46' 20	47' 15	48' 10	49' 5	50' 0	50' 55	51' 50

TABLES OF NAUTICAL ASTRONOMY.

TABLE VIII.

Parallax of the Moon less Refraction.

		Proportional parts for the parallax.										The Moon's Parallax.	
60'	61'	0"	1"	2"	3"	4"	5"	6"	7"	8"	9"		
54' 10"	55' 7"	1	2	3	4	5	6	7	8	9	1'	0"	
54' 8"	55' 5"	9	10	11	12	13	14	15	16	17	18	2	0
54' 7"	55' 4"	19	20	21	22	23	24	25	26	27	27	3	0
54' 5"	55' 2"	28	29	30	31	32	33	34	35	36	37	4	1
54' 3"	55' 0"	38	39	40	41	42	43	44	45	46	46	5	1
54' 1"	54' 58"	47	48	49	50	51	52	53	54	55	56	6	1
												7	1
												8	1
54' 0"	54' 56"	0	1	2	3	4	5	6	7	8	8	9	1
53' 58"	54' 54"	9	10	11	12	13	14	15	16	17	18		
53' 56"	54' 52"	19	20	21	22	23	24	24	25	26	27		
53' 54"	54' 50"	28	29	30	31	32	33	34	35	36	37		
53' 52"	54' 48"	38	39	40	41	42	43	44	45	46	46		
53' 49"	54' 46"	47	48	49	50	51	52	53	54	55	56		
53' 47"	54' 44"	0	1	2	3	4	5	6	7	8	8	1	0
53' 45"	54' 42"	9	10	11	12	13	14	15	16	17	18	2	0
53' 43"	54' 40"	19	20	21	22	23	24	24	25	26	27	3	1
53' 41"	54' 37"	28	29	30	31	32	33	34	35	36	37	4	1
53' 38"	54' 34"	38	39	40	41	42	43	44	45	46	46	5	1
53' 36"	54' 32"	47	48	49	50	51	52	53	54	55	55	6	1
												7	2
												8	2
53' 33"	54' 29"	0	1	2	3	4	5	6	7	7	8	9	2
53' 31"	54' 27"	9	10	11	12	13	14	15	16	17	18		
53' 28"	54' 24"	19	20	20	21	22	23	24	25	26	27		
53' 26"	54' 21"	28	29	30	31	32	33	34	35	36	36		
53' 23"	54' 19"	37	38	39	40	41	42	43	44	45	46		
53' 20"	54' 16"	47	47	48	49	50	51	52	53	54	55		
53' 18"	54' 13"	0	1	2	3	4	5	6	6	7	8		
53' 15"	54' 10"	9	10	11	12	13	14	15	16	17	18	1	0
53' 12"	54' 7"	18	19	20	21	22	23	24	25	26	27	2	1
53' 9"	54' 4"	28	29	30	31	32	33	34	35	36	36	3	1
53' 6"	54' 1"	37	38	39	40	41	42	43	44	45	45	4	1
53' 3"	53' 58"	46	47	48	49	50	51	52	53	54	55	5	1
												6	2
												7	2
53' 0"	53' 55"	0	1	2	3	4	5	6	6	7	8		
52' 57"	53' 52"	9	10	11	12	13	14	15	16	17	17		
52' 54"	53' 49"	18	19	20	21	22	23	24	25	26	27	9	2
52' 51"	53' 46"	28	28	29	30	31	32	33	34	35	36		
52' 48"	53' 43"	37	38	39	40	41	42	43	44	45	45		
52' 45"	53' 40"	46	47	48	49	50	51	52	53	54	55		

TABLE VIII.

Parallax of the Moon less Refraction.

Apparent altitude.	Horizontal parallax:						
	53'	54'	55'	56'	57'	58'	59'
24° 0'	46' 38"	47' 12"	48' 7"	49' 2"	50' 52"	51' 46"	
10	46 15	47 9	48 4	48 59	50 48	51 43	
20	46 12	47 7	48 1	48 56	50 45	51 40	
30	46 9	47 4	47 58	48 53	50 42	51 37	
40	46 6	47 1	47 55	48 50	50 39	51 33	
50	46 3	46 57	47 51	48 47	50 36	51 30	
25 0	46 0	46 55	47 49	48 43	49 38	50 32	51 27
10	45 57	46 52	47 46	48 40	49 35	50 29	51 23
20	45 54	46 48	47 43	48 37	49 31	50 25	51 20
30	45 51	46 45	47 40	48 34	49 28	50 22	51 16
40	45 48	46 42	47 36	48 30	49 24	50 18	51 13
50	45 45	46 39	47 33	48 27	49 21	50 15	51 9
26 0	45 42	46 36	47 30	48 24	49 17	50 11	51 5
10	45 38	46 32	47 26	48 20	49 14	50 8	51 42
20	45 35	46 29	47 23	48 17	49 10	50 4	50 58
30	45 32	46 26	47 19	48 13	49 7	50 0	50 54
40	45 29	46 22	47 16	48 9	49 3	49 57	50 50
50	45 25	46 19	47 12	48 6	48 59	49 53	50 47
27 0	45 22	46 15	47 9	48 2	48 56	49 49	50 43
10	45 18	46 11	47 5	47 59	48 52	49 45	50 39
20	45 15	46 8	47 2	47 55	48 48	49 42	50 35
30	45 12	46 5	46 58	47 51	48 44	49 38	50 31
40	45 8	46 1	46 54	47 47	48 41	49 34	50 27
50	45 4	45 58	46 51	47 44	48 37	49 30	50 23
28 0	45 1	45 54	46 47	47 40	48 33	49 26	50 19
10	44 57	45 50	46 43	47 36	48 29	49 22	50 15
20	44 54	45 46	46 39	47 32	48 25	49 18	50 11
30	44 50	45 43	46 35	47 28	48 21	49 14	50 6
40	44 46	45 39	46 32	47 24	48 17	49 9	50 2
50	44 43	45 35	46 28	47 20	48 13	49 5	49 58
29 0	44 39	45 31	46 24	47 16	48 9	49 1	49 54
10	44 35	45 27	46 20	47 12	48 5	48 57	49 49
20	44 31	45 23	46 16	47 8	48 0	48 53	49 45
30	44 27	45 20	46 12	47 4	47 56	48 48	49 41
40	44 23	45 16	46 8	47 0	47 52	48 44	49 36
50	44 19	45 11	46 4	47 56	47 48	48 40	49 32

TABLE VIII.

Parallax of the Moon less Refraction.

		Proportional parts for the parallax										the altitude.	
60'	61'	0"	1"	2"	3"	4"	5"	6"	7"	8"	9"		
52' 41"	56' 36"	0	1	2	3	4	5	5	6	7	8	1'	0'
52' 38	53' 33	9	10	11	12	13	14	15	15	16	17	2	1
52' 35	53' 28	18	19	20	21	22	23	24	25	25	26	3	1
52' 31	53' 26	27	28	29	30	31	32	33	34	35	35	4	1
52' 28	53' 22	36	37	38	39	40	41	42	43	44	45	5	2
52' 24	53' 19	46	46	47	48	49	50	51	52	53	54	6	2
												7	2
52' 21	53' 15	0	1	2	3	4	5	5	6	7	8	8	3
52' 17	53' 12	9	10	11	12	13	14	15	16	17	17	9	3
52' 14	53' 7	18	19	20	21	22	23	23	24	25	26		
52' 10	53' 4	27	28	29	30	31	32	32	33	34	35		
52' 7	53' 1	36	37	38	39	40	41	41	42	43	44		
52' 3	52' 57	45	46	47	48	49	50	51	51	52	53		
51' 59	52' 53	0	1	2	3	4	4	5	6	7	8	1	0
51' 55	52' 49	9	10	11	12	13	13	14	15	16	17	2	1
51' 52	52' 45	18	19	20	21	22	23	24	25	26	26	3	1
51' 48	52' 42	27	28	29	30	31	32	33	34	35	35	4	1
51' 44	52' 38	36	37	38	39	40	41	42	43	44	44	5	2
51' 40	52' 34	45	46	47	48	49	50	51	52	53	53	6	2
												7	2
51' 36	52' 30	0	1	2	3	4	4	5	6	7	8	8	3
51' 32	52' 26	9	10	11	12	12	13	14	15	16	17	9	3
51' 28	52' 21	18	19	20	20	21	22	23	24	25	26		
51' 24	52' 17	27	27	28	29	30	31	32	33	34	35		
51' 20	52' 13	35	36	37	38	39	40	41	42	43	43		
51' 16	52' 9	44	45	46	47	48	49	50	51	51	52		
51' 14	52' 5	0	1	2	3	4	4	5	6	7	8	1	0
51' 8	52' 0	9	10	11	11	12	13	14	15	16	17	2	1
51' 3	51' 56	18	18	19	20	21	22	23	24	25	25	3	1
50' 59	51' 52	26	27	28	29	30	31	32	32	33	34	4	2
50' 55	51' 47	35	36	37	38	39	40	40	41	42	43	5	2
50' 50	51' 43	44	45	46	47	47	48	49	50	51	52	6	2
												7	3
50' 46	51' 39	0	1	2	3	3	4	5	6	7	8	8	3
50' 42	51' 34	9	10	10	11	12	13	14	15	16	17	9	4
50' 37	51' 30	17	18	19	20	21	22	23	23	24	25		
50' 33	51' 24	26	27	28	29	30	31	32	32	33	34		
50' 28	51' 20	35	36	37	37	38	39	40	41	42	43		
50' 24	51' 16	44	44	45	46	47	48	49	50	50	51		

TABLE VIII.

Parallax of the Moon less Refraction.

Apparent altitude	Horizontal parallax.						
	53'	54'	55'	56'	57'	58'	59'
30° 0'	44' 16"	45' 7"	45' 59"	46' 51"	47' 45"	48' 33"	49' 27"
10	44 12	45 3	45 55	46 47	47 29	48 31	49 23
20	44 7	44 59	45 51	46 43	47 35	48 26	49 18
30	44 3	44 55	45 47	46 39	47 30	48 22	49 14
40	43 59	44 51	45 43	46 34	47 26	48 17	49 9
50	43 55	44 47	45 38	46 30	47 21	48 13	49 4
31 0	43 51	44 43	45 34	46 25	47 17	48 8	49 0
10	43 47	44 38	45 30	46 21	47 12	48 4	48 55
20	43 43	44 34	45 25	46 17	47 8	47 59	48 50
30	43 39	44 30	45 21	46 12	47 3	47 54	48 46
40	43 34	44 25	45 16	46 8	46 59	47 50	48 41
50	43 30	44 21	45 12	46 3	46 54	47 45	48 36
32 0	43 16	44 17	45 8	45 58	46 49	47 40	48 31
10	43 21	44 12	45 3	45 54	46 15	47 35	48 26
20	43 17	44 8	44 58	45 49	46 40	47 31	48 21
30	43 13	44 3	44 54	45 45	46 35	47 26	48 16
40	43 8	44 59	44 49	45 40	46 30	47 21	48 11
50	43 4	44 53	44 44	45 35	46 26	47 16	48 6
33 0	43 59	44 50	44 40	45 30	46 21	47 11	48 1
10	43 55	44 45	44 35	45 26	46 16	47 6	47 56
20	43 50	44 39	44 31	45 21	46 11	47 1	47 51
30	43 46	44 36	44 26	45 16	46 6	46 56	47 46
40	43 41	44 31	44 21	45 11	46 1	46 51	47 41
50	43 37	44 26	44 16	45 6	45 56	46 46	47 36
34 0	43 32	44 22	44 12	45 1	45 51	46 41	47 30
10	43 27	44 17	44 7	44 56	45 46	46 36	47 25
20	43 23	44 12	44 2	44 51	45 41	46 31	47 20
30	43 18	44 7	43 57	44 46	45 36	46 25	47 15
40	43 13	44 3	43 52	44 41	45 31	46 20	47 9
50	43 8	44 58	43 47	44 36	45 25	46 15	47 4
35 0	43 4	44 53	43 42	44 31	45 20	46 9	46 59
10	43 59	44 48	43 37	44 26	45 15	46 4	46 53
20	43 54	44 43	43 32	44 21	45 10	45 59	46 48
30	43 49	44 38	43 27	44 16	45 5	45 53	46 42
40	43 44	44 33	43 22	44 10	44 59	45 48	46 37
50	43 39	44 28	43 17	44 5	44 54	45 43	46 31

TABLE VIII.

Parallax of the Moon less Refraction.

		Proportional parts for the parallax										For the altitude	
60'	51'	0"	1"	2"	3"	4"	5"	6"	7"	8"	9"		
50' 19"	51' 11"	0	1	2	3	3	4	5	6	7	8	1'	0"
50 15	51 6	9	10	11	12	13	14	15	16	17	18	2	1
50 10	51 2	17	18	19	20	21	22	23	24	25	26	3	1
50 5	50 57	26	27	28	29	30	31	32	33	34	35	4	2
50 1	50 51	34	35	36	37	38	39	40	41	42	43	5	2
49 56	50 47	43	44	45	46	47	48	49	50	51	52	6	3
												7	3
												8	4
												9	4
49 51	50 43	0	1	2	3	3	4	5	6	7	8		
49 46	50 38	9	10	11	12	13	14	15	16	17	18		
49 42	50 33	17	18	19	20	21	22	23	24	25	26		
49 37	50 28	26	27	28	29	30	31	32	33	34	35		
49 32	50 23	34	35	36	37	38	39	40	41	42	43		
49 27	50 18	43	44	45	46	47	48	49	50	51	52		
49 22	50 13	0	1	2	3	3	4	5	6	7	8	1	0
49 17	50 8	8	9	10	11	12	13	14	15	16	17	2	1
49 12	50 3	17	18	19	20	21	22	23	24	25	26	3	1
49 7	49 58	25	26	27	28	29	30	31	32	33	34	4	2
49 2	49 52	34	35	36	37	38	39	40	41	42	43	5	2
48 57	49 47	42	43	44	45	46	47	48	49	50	51	6	3
												7	3
												8	4
												9	4
48 52	49 42	0	1	2	3	3	4	5	6	7	8		
48 46	49 37	8	9	10	11	12	13	14	15	16	17		
48 41	49 31	17	18	19	20	21	22	23	24	25	26		
48 36	49 26	25	26	27	28	29	30	31	32	33	34		
48 31	49 21	34	35	36	37	38	39	40	41	42	43		
48 26	49 15	42	43	44	45	46	47	48	49	50	51		
48 20	49 10	0	1	2	3	4	5	6	7	8	9	1	0
48 15	49 4	8	9	10	11	12	13	14	15	16	17	2	1
48 9	48 59	16	17	18	19	20	21	22	23	24	25	3	2
48 4	48 54	25	26	27	28	29	30	31	32	33	34	4	2
47 59	48 48	33	34	35	36	37	38	39	40	41	42	5	3
47 53	48 42	41	42	43	44	45	46	47	48	49	50	6	3
												7	4
												8	4
												9	5
47 48	48 37	0	1	2	3	4	5	6	7	8	9		
47 42	48 31	8	9	10	11	12	13	14	15	16	17		
47 37	48 26	16	17	18	19	20	21	22	23	24	25		
47 31	48 20	24	25	26	27	28	29	30	31	32	33		
47 25	48 14	33	34	35	36	37	38	39	40	41	42		
47 20	48 8	41	42	43	44	45	46	47	48	49	50		

TABLES OF NAUTICAL ASTRONOMY.

TABLE VIII.

Parallax of the Moon less Refraction.

Apparent altitude.	Horizontal parallax.						
	53'	54'	55'	56'	57'	58'	59'
36° 0'	41' 34"	42' 23"	43' 11"	44' 0'	44' 49"	45' 37"	46' 26"
10	41' 29"	42' 18"	43' 6"	43' 55"	44' 43"	45' 32"	46' 20"
20	41' 24"	42' 13"	43' 1"	43' 49"	44' 38"	45' 26"	46' 14"
30	41' 19"	42' 8"	42' 56"	43' 44"	44' 32"	45' 21"	46' 9"
40	41' 14"	42' 2"	42' 51"	43' 39"	44' 27"	45' 15"	46' 3"
50	41' 9"	41' 57"	42' 46"	43' 33"	44' 21"	45' 9"	45' 57"
37° 0'	41' 4"	41' 52"	42' 40"	43' 28"	44' 16"	45' 4"	45' 52"
10	40' 59"	41' 47"	42' 35"	43' 22"	44' 10"	44' 58"	45' 46"
20	40' 54"	41' 42"	42' 29"	43' 17"	44' 5"	44' 52"	45' 40"
30	40' 49"	41' 36"	42' 24"	43' 12"	43' 59"	44' 47"	45' 34"
40	40' 44"	41' 31"	42' 18"	43' 6"	43' 53"	44' 41"	45' 28"
50	40' 38"	41' 25"	42' 13"	43' 0"	43' 48"	44' 35"	45' 23"
38° 0'	40' 33"	41' 20"	42' 8"	42' 55"	43' 42"	44' 29"	45' 17"
10	40' 28"	41' 15"	42' 2"	42' 49"	43' 36"	44' 24"	45' 11"
20	40' 22"	41' 10"	41' 57"	42' 44"	43' 31"	44' 18"	45' 5"
30	40' 17"	41' 4"	41' 51"	42' 58"	43' 25"	44' 12"	44' 59"
40	40' 12"	40' 59"	41' 45"	42' 32"	43' 19"	44' 6"	44' 53"
50	40' 6"	40' 53"	41' 40"	42' 27"	43' 13"	44' 0"	44' 47"
39° 0'	40' 1"	40' 48"	41' 34"	42' 21"	43' 8"	43' 54"	44' 41"
10	39' 56"	40' 42"	41' 29"	42' 15"	43' 2"	43' 48"	44' 35"
20	39' 50"	40' 37"	41' 23"	42' 9"	42' 56"	43' 42"	44' 29"
30	39' 45"	40' 31"	41' 17"	42' 4"	42' 50"	43' 36"	44' 23"
40	39' 39"	40' 25"	41' 12"	41' 58"	42' 43"	43' 30"	44' 16"
50	39' 34"	40' 20"	41' 6"	41' 52"	42' 38"	43' 24"	44' 10"
40° 0'	39' 28"	40' 14"	41' 0"	41' 46"	42' 32"	43' 18"	44' 4"
10	39' 23"	40' 8"	40' 54"	41' 40"	42' 26"	43' 12"	43' 58"
20	39' 17"	40' 3"	40' 48"	41' 34"	42' 20"	43' 6"	43' 51"
30	39' 11"	39' 57"	40' 43"	41' 28"	42' 14"	43' 0"	43' 45"
40	39' 5"	39' 51"	40' 37"	41' 22"	42' 8"	42' 53"	43' 39"
50	39' 0"	39' 46"	40' 31"	41' 16"	42' 2"	42' 47"	43' 33"
41° 0'	38' 54"	39' 40"	40' 25"	41' 10"	41' 56"	42' 41"	43' 26"
10	38' 49"	39' 34"	40' 19"	41' 4"	41' 50"	42' 35"	43' 20"
20	38' 43"	39' 28"	40' 13"	40' 58"	41' 43"	42' 28"	43' 13"
30	38' 37"	39' 22"	40' 7"	40' 52"	41' 37"	42' 22"	43' 7"
40	38' 32"	39' 16"	40' 1"	40' 46"	41' 31"	42' 16"	43' 0"
50	38' 26"	39' 11"	39' 55"	40' 40"	41' 25"	42' 9"	42' 54"

TABLE VIII.

Parallax of the Moon less Refraction.

		Proportional parts for the parallax.										For the altitude.	
60'	61'	0"	1"	2"	3"	4"	5"	6"	7"	8"	9"		—
47' 14"	48' 3"	0	1	2	3	4	5	6	7	8	9	1'	1"
47' 8	47' 57	8	9	10	11	12	13	14	15	16	17	2	1
47' 3	47' 51	16	17	18	19	20	21	22	23	24	25	3	2
46' 57	47' 45	24	25	26	27	28	29	30	31	32	33	4	2
46' 51	47' 39	32	33	34	35	36	37	38	39	40	41	5	3
46' 45	47' 33	40	41	42	43	44	45	46	47	48	49	6	3
												7	4
												8	4
												9	5
46' 40	47' 28	0	1	2	3	4	5	6	7	8	9		
46' 34	47' 22	8	9	10	11	12	13	14	15	16	17		
46' 28	47' 16	16	17	18	19	20	21	22	23	24	25		
46' 22	47' 10	24	25	26	27	28	29	30	31	32	33		
46' 16	47' 4	32	33	34	35	36	37	38	39	40	41		
46' 10	46' 57	40	41	42	43	44	45	46	47	48	49		
46' 4	46' 51	0	1	2	3	4	5	6	7	8	9	1	1
45' 58	46' 45	8	9	10	11	12	13	14	15	16	17	2	1
45' 52	46' 39	16	17	18	19	20	21	22	23	24	25	3	2
45' 46	46' 33	24	25	26	27	28	29	30	31	32	33	4	2
45' 40	46' 27	32	33	34	35	36	37	38	39	40	41	5	3
45' 34	46' 20	39	40	41	42	43	44	45	46	47	48	6	3
												7	4
												8	4
												9	5
45' 27	46' 14	0	1	2	3	4	5	6	7	8	9		
45' 21	46' 8	8	9	10	11	12	13	14	15	16	17		
45' 15	46' 1	15	16	17	18	19	20	21	22	23	24		
45' 9	45' 55	23	24	25	26	27	28	29	30	31	32		
45' 3	45' 49	31	32	33	34	35	36	37	38	39	40		
44' 56	45' 42	39	40	41	42	43	44	45	46	47	48		
44' 50	45' 36	0	1	2	3	4	5	6	7	8	9	1	1
44' 44	45' 29	8	9	10	11	12	13	14	15	16	17	2	1
44' 37	45' 23	15	16	17	18	19	20	21	22	23	24	3	2
44' 31	45' 17	23	24	25	26	27	28	29	30	31	32	4	2
44' 24	45' 10	30	31	32	33	34	35	36	37	38	39	5	3
44' 18	45' 3	38	39	40	41	42	43	44	45	46	47	6	4
												7	4
												8	5
												9	5
44' 11	44' 57	0	1	2	3	4	5	6	7	8	9		
44' 5	44' 50	7	8	9	10	11	12	13	14	15	16		
43' 58	44' 43	15	16	17	18	19	20	21	22	23	24		
43' 52	44' 37	23	24	25	26	27	28	29	30	31	32		
43' 45	44' 30	30	31	32	33	34	35	36	37	38	39		
43' 39	44' 23	37	38	39	40	41	42	43	44	45	46		

TABLES OF NAUTICAL ASTRONOMY.

TABLE VIII.

Parallax of the Moon less Refraction.

Apparent altitude.	Horizontal parallax.						
	53'	54'	55'	56'	57'	58'	59'
42° 0'	38' 20"	39' 45"	39' 49"	40' 34"	41' 18"	42' 3"	42' 48"
10'	38 14	38 59	39 43	40 28	41 12	41 56	42 41
20	38 8	38 53	39 37	40 21	41 6	41 50	42 34
30	38 2	38 47	39 31	40 15	40 59	41 44	42 28
40	37 57	38 41	39 25	40 9	40 53	41 37	42 21
50	37 51	38 35	39 19	40 3	40 47	41 31	42 15
43 0	37 45	38 29	39 12	39 56	40 40	41 24	42 8
10	37 39	38 22	39 6	39 50	40 34	41 17	42 1
20	37 33	38 16	39 0	39 44	40 27	41 11	41 55
30	37 27	38 10	38 54	39 37	40 21	41 4	41 48
40	37 21	38 4	38 47	39 31	40 14	40 58	41 41
50	37 15	37 58	38 41	39 24	40 8	40 51	41 34
44 0	37 9	37 52	38 35	39 18	40 1	40 44	41 28
10	37 2	37 44	38 28	39 11	39 54	40 37	41 20
20	36 56	37 39	38 22	39 5	39 48	40 31	41 14
30	36 50	37 33	38 16	38 59	39 41	40 24	41 7
40	36 44	37 27	38 9	38 52	39 35	40 17	41 0
50	36 38	37 20	38 3	38 46	39 28	40 11	40 53
45 0	36 32	37 14	37 58	38 39	39 21	40 4	40 46
10	36 25	37 8	37 50	38 32	39 15	39 57	40 39
20	36 19	37 2	37 44	38 27	39 8	39 50	40 32
30	36 13	36 55	37 37	38 19	39 1	39 43	40 25
40	36 7	36 49	37 31	38 12	38 54	39 36	40 18
50	36 0	36 42	37 24	38 6	38 48	39 29	40 11
46 0	35 54	36 36	37 18	37 59	38 41	39 22	40 4
10	35 48	36 29	37 11	37 52	38 34	39 15	39 57
20	35 41	36 23	37 4	37 46	38 27	39 8	39 50
30	35 35	36 16	36 58	37 39	38 20	39 1	39 43
40	35 29	36 9	36 51	37 32	38 13	38 54	39 36
50	35 22	36 3	36 44	37 26	38 6	38 47	39 28
47 0	35 16	35 57	36 38	37 18	37 59	38 40	39 21
10	35 9	35 50	36 31	37 12	37 52	38 33	39 14
20	35 3	35 43	36 24	37 5	37 45	38 26	39 7
30	34 56	35 37	36 17	36 58	37 38	38 19	38 59
40	34 50	35 30	36 10	36 51	37 31	38 12	38 52
50	34 43	35 23	36 4	36 44	37 24	38 5	38 45

TABLE VIII.

Parallax of the Moon less Refraction.

Proportional parts for the parallax.												For the altitude.	
60'	61'	0"	1"	2"	3"	4"	5"	6"	7"	8"	9"		—
43' 32"	44 17	0	1	2	3	4	5	6	7	8	9	1'	1"
43 25	44 10	7	8	9	10	11	12	13	14	15	16	2	1
43 19	44 3	15	16	17	18	19	20	21	22	23	24	3	2
43 12	43 56	22	23	24	25	26	27	28	29	30	31	4	3
43 5	43 49	29	30	31	32	33	34	35	36	37	38	5	3
42 59	43 43	37	38	39	40	41	42	43	44	45	46	6	4
												7	4
												8	5
												9	5
													6
42 52	43 36	0	1	2	3	4	5	6	7	8	9		
42 45	43 29	7	8	9	10	11	12	13	14	15	16		
42 38	43 22	15	16	17	18	19	20	21	22	23	24		
42 31	43 15	22	23	24	25	26	27	28	29	30	31		
42 24	43 8	29	30	31	32	33	34	35	36	37	38		
42 18	43 1	36	37	38	39	40	41	42	43	44	45		
42 11	42 54	0	1	2	3	4	5	6	7	8	9	1	1
42 4	42 47	7	8	9	10	11	12	13	14	15	16	2	1
41 57	42 40	14	15	16	17	18	19	20	21	22	23	3	2
41 50	42 33	21	22	23	24	25	26	27	28	29	30	4	3
41 43	42 25	29	30	31	32	33	34	35	36	37	38	5	3
41 36	42 18	36	37	38	39	40	41	42	43	44	45	6	4
												7	4
												8	5
												9	5
													6
41 29	42 11	0	1	2	3	4	5	6	7	8	9		
41 22	42 4	7	8	9	10	11	12	13	14	15	16		
41 14	41 57	14	15	16	17	18	19	20	21	22	23		
41 7	41 49	21	22	23	24	25	26	27	28	29	30		
41 0	41 42	28	29	30	31	32	33	34	35	36	37		
40 53	41 35	35	36	37	38	39	40	41	42	43	44		
40 46	41 27	0	1	2	3	4	5	6	7	8	9	1	1
40 39	41 20	7	8	9	10	11	12	13	14	15	16	2	1
40 31	41 13	14	15	16	17	18	19	20	21	22	23	3	2
40 24	41 5	21	22	23	24	25	26	27	28	29	30	4	3
40 17	40 58	28	29	30	31	32	33	34	35	36	37	5	4
40 9	40 50	34	35	36	37	38	39	40	41	42	43	6	4
												7	5
												8	5
												9	6
													6
40 2	40 43	0	1	2	3	4	5	6	7	8	9		
39 55	40 36	7	8	9	10	11	12	13	14	15	16		
39 47	40 28	14	15	16	17	18	19	20	21	22	23		
39 40	40 20	20	21	22	23	24	25	26	27	28	29		
39 33	40 13	27	28	29	30	31	32	33	34	35	36		
39 25	40 5	34	35	36	37	38	39	40	41	42	43		

TABLE VII.

Parallax of the Moon less Refraction.

Apparent altitude.	Horizontal parallax.						
	54'	55'	56'	57'	58'	59'	
46° 0'	34' 27"	35' 17"	35' 57"	36' 37"	37' 17"	37' 57"	38' 37"
10	34' 30	35' 20	35' 50	36' 30	37' 10	37' 50	38' 30
20	34' 33	35' 23	35' 53	36' 23	37' 3	37' 43	38' 23
30	34' 37	35' 26	35' 56	36' 16	36' 56	37' 36	38' 15
40	34' 40	35' 29	35' 59	36' 9	36' 49	37' 28	38' 8
50	34' 43	35' 32	35' 62	36' 2	36' 41	37' 21	38' 0
49 0	33' 57	34' 36	35' 16	35' 55	36' 34	37' 14	37' 53
10	33' 50	34' 29	35' 9	35' 48	36' 27	37' 6	37' 45
20	33' 43	34' 22	35' 2	35' 41	36' 20	36' 59	37' 38
30	33' 37	34' 16	34' 55	35' 34	36' 12	36' 51	37' 30
40	33' 30	34' 9	34' 48	35' 26	36' 5	36' 44	37' 23
50	33' 23	34' 2	34' 40	35' 19	36' 3	36' 37	37' 15
50 0	33' 16	33' 55	34' 33	35' 12	35' 51	36' 29	37' 8
10	33' 9	33' 48	34' 26	35' 5	35' 43	36' 22	37' 0
20	33' 2	33' 41	34' 19	35' 58	35' 36	36' 14	36' 52
30	32' 55	33' 34	34' 12	34' 50	35' 28	36' 7	36' 45
40	32' 48	33' 27	34' 5	34' 43	35' 21	35' 59	36' 37
50	32' 41	33' 20	33' 58	34' 36	35' 14	35' 51	36' 29
51 0	32' 34	33' 13	33' 51	34' 28	35' 6	35' 44	36' 22
10	32' 27	33' 6	33' 44	34' 21	34' 59	35' 36	36' 14
20	32' 20	32' 59	33' 36	34' 14	34' 51	35' 29	36' 6
30	32' 14	32' 52	33' 29	34' 6	34' 44	35' 21	35' 58
40	32' 7	32' 45	33' 22	33' 59	34' 36	35' 13	35' 51
50	32' 0	32' 37	33' 14	33' 52	34' 29	35' 6	35' 43
52 0	31' 53	32' 30	33' 7	33' 44	34' 21	34' 58	35' 35
10	31' 46	32' 23	33' 0	33' 37	34' 13	34' 50	35' 27
20	31' 39	32' 16	32' 53	33' 29	34' 6	34' 43	35' 19
30	31' 32	32' 9	32' 45	33' 22	33' 58	34' 35	35' 11
40	31' 25	32' 1	32' 38	33' 14	33' 51	34' 27	35' 3
50	31' 18	31' 54	32' 30	33' 7	33' 43	34' 19	34' 55
53 0	31' 11	31' 47	32' 23	32' 59	33' 35	34' 11	34' 48
10	31' 4	31' 40	32' 16	32' 52	33' 28	34' 4	34' 40
20	30' 57	31' 32	32' 8	32' 44	33' 20	33' 56	34' 32
30	30' 49	31' 25	32' 1	32' 36	33' 12	33' 48	34' 24
40	30' 42	31' 18	31' 53	32' 29	33' 4	33' 40	34' 15
50	30' 35	31' 10	31' 45	32' 21	32' 57	33' 32	34' 7

TABLES OF NAUTICAL ASTRONOMY.

TABLE VIII.

Parallax of the Moon less Refraction.

		Proportional parts for the parallax.										For the altitude.	
60'	61'	0"	1"	2"	3"	4"	5"	6"	7"	8"	9"		—
39' 16"	39' 58"	13	14	15	16	17	18	19	20	21	22	1	1
39 10	39 50	13	14	15	16	17	18	19	20	21	22	2	1
39 3	39 42	13	14	15	16	17	18	19	20	21	22	3	2
38 55	39 35	20	21	22	23	24	25	26	27	28	29	4	3
38 47	39 27	27	28	29	30	31	32	33	34	35	36	5	4
38 40	39 19	33	34	35	36	37	38	39	40	41	42	6	4
												7	5
												8	6
38 32	39 12	0	1	2	3	4	5	6	7	8	9		
38 25	39 4	6	7	8	9	10	11	12	13	14	15	9	6
38 17	38 56	13	14	15	16	17	18	19	20	21	22		
38 9	38 48	19	20	21	22	23	24	25	26	27	28		
38 2	38 41	25	26	27	28	29	30	31	32	33	34		
37 54	38 33	32	33	34	35	36	37	38	39	40	41		
												1	1
37 46	38 25	0	1	2	3	4	5	6	7	8	9	1	2
37 38	38 17	6	7	8	9	10	11	12	13	14	15	2	2
37 31	38 9	13	14	15	16	17	18	19	20	21	22	3	2
37 23	38 1	19	20	21	22	23	24	25	26	27	28	4	3
37 15	37 52	25	26	27	28	29	30	31	32	33	34	5	3
37 7	37 45	32	33	34	35	36	37	38	39	40	41	6	5
												7	5
												8	
36 59	37 37	0	1	2	3	4	5	6	7	8	9		
36 52	37 29	6	7	8	9	10	11	12	13	14	15		
36 44	37 21	12	13	14	15	16	17	18	19	20	21		
36 36	37 13	19	20	21	22	23	24	25	26	27	28		
36 28	37 5	25	26	27	28	29	30	31	32	33	34		
36 20	36 57	31	32	33	34	35	36	37	38	39	40		
												1	1
36 12	36 49	0	1	2	3	4	5	6	7	8	9	1	2
36 4	36 41	6	7	8	9	10	11	12	13	14	15	2	2
35 56	36 33	12	13	14	15	16	17	18	19	20	21	3	2
35 48	36 24	18	19	20	21	22	23	24	25	26	27	4	3
35 40	36 16	24	25	26	27	28	29	30	31	32	33	5	4
35 32	36 8	30	31	32	33	34	35	36	37	38	39	6	5
												7	5
												8	6
35 24	36 0	0	1	2	3	4	5	6	7	8	9		
35 15	35 51	6	7	8	9	10	11	12	13	14	15		
35 17	35 43	12	13	14	15	16	17	18	19	20	21		
34 59	35 35	18	19	20	21	22	23	24	25	26	27		
34 51	35 27	24	25	26	27	28	29	30	31	32	33		
34 43	35 18	30	31	32	33	34	35	36	37	38	39		

TABLE VIII.

Parallax of the Moon less Refraction.

Apparent altitude,	Horizontal parallax.						
	55'	56'	57'	58'	59'	60'	61'
51° 0'	30 28	31 38	31 36	32 13	32 42	32 24	33 59
10	30 21	30 56	31 31	32 6	32 41	32 16	33 51
20	30 13	30 48	31 23	31 58	32 33	32 8	33 43
30	30 5	30 41	31 16	31 51	32 25	32 0	33 35
40	29 59	30 33	31 8	31 43	32 18	32 52	33 27
50	29 51	30 25	31 1	31 35	32 10	32 44	33 19
52° 0	29 44	30 19	30 53	31 27	32 2	32 36	33 11
10	29 37	30 11	30 45	31 20	31 54	32 28	33 2
20	29 29	30 4	30 38	31 12	31 46	32 20	32 54
30	29 22	29 56	30 30	31 4	31 38	32 12	32 46
40	29 15	29 47	30 21	30 55	31 29	32 3	32 37
50	29 7	29 40	30 14	30 47	31 21	31 55	32 28
53° 0	29 0	29 35	30 7	30 40	31 14	31 48	32 21
10	28 52	29 26	29 59	30 3	31 6	31 39	32 14
20	28 45	29 18	29 51	30 25	30 38	31 1	32 4
30	28 37	29 11	29 44	30 17	30 50	31 23	31 56
40	28 30	29 3	29 36	30 9	30 42	31 15	31 48
50	28 22	28 55	29 28	30 1	30 34	31 7	31 39
54° 0	28 14	28 48	29 20	29 53	30 26	30 58	31 31
10	28 7	28 40	29 13	29 45	30 18	30 50	31 22
20	28 0	28 32	29 5	29 37	30 9	30 42	31 14
30	27 52	28 24	28 57	29 29	30 1	30 33	31 6
40	27 45	28 17	28 50	29 21	29 53	30 25	30 57
50	27 37	28 9	28 41	29 13	29 45	30 17	30 49
55° 0	27 30	28 1	28 33	29 5	29 37	30 9	30 40
10	27 22	27 53	28 25	29 57	29 29	30 0	30 32
20	27 14	27 46	28 17	28 49	29 20	29 52	30 23
30	27 7	27 38	28 9	28 41	29 12	29 44	30 15
40	26 59	27 30	28 1	28 33	29 4	29 35	30 6
50	26 51	27 22	27 53	28 24	28 56	29 27	29 58
56° 0	26 44	27 14	27 45	28 16	28 47	29 18	29 49
10	26 36	27 6	27 37	28 8	28 59	29 10	29 40
20	26 28	26 59	27 30	28 0	28 31	29 1	29 32
30	26 20	26 51	27 21	27 52	28 22	28 53	29 23
40	26 13	26 43	27 13	27 44	28 14	28 45	29 14
50	26 5	26 35	27 5	27 36	28 6	28 36	29 6

TABLES OF NAUTICAL ASTRONOMY

TABLE VIII.

Parallax of the Moon less Refraction.

		Proportional parts for an hour										For the altitude.	
60'	61'	0	1	2''	3	4	5	6	7	8	9		—
34' 35'	35' 10	10	11	1	2	2	3	3	4	5	5	1'	1"
34 26	35 2	6	6	7	8	8	9	9	10	10	11	2	2
34 18	34 53	12	13	13	13	14	15	15	16	16	17	3	3
34 10	34 45	17	18	19	19	20	20	21	21	22	23	4	4
34 2	34 36	23	24	25	25	26	26	27	27	28	29	5	5
33 53	34 28	29	30	30	31	31	32	32	33	34	35	6	6
												7	7
												8	8
												9	9
33 45	34 19	0	1	1	2	2	3	3	4	5	5		
33 37	34 11	6	6	7	7	8	8	9	10	10	11		
33 28	34 2	11	12	12	13	14	14	15	15	16	16		
33 20	33 54	17	18	18	19	19	20	20	21	22	22		
33 11	33 44	23	24	24	25	25	26	26	27	27	28		
33 2	33 36	29	29	29	30	31	31	32	32	33	33		
32 55	33 28	0	1	1	2	2	3	3	4	4	5	1	1
32 46	33 20	6	6	7	7	8	8	9	9	10	10	2	2
32 38	33 11	11	12	12	13	13	14	14	15	15	16	3	3
32 29	33 2	17	17	18	18	19	19	20	20	21	22	4	4
32 21	32 54	22	23	23	24	24	25	25	26	26	27	5	5
32 12	32 45	28	28	29	29	30	30	31	31	32	33	6	6
												7	7
												8	8
												9	9
32 4	32 36	0	1	1	2	2	3	3	4	4	5		
31 55	32 28	5	6	6	7	7	8	8	9	9	10		
31 47	32 19	11	11	12	12	13	13	14	14	15	16		
31 38	32 10	16	17	17	18	18	19	19	20	20	21		
31 29	32 1	21	22	22	23	23	24	24	25	25	26		
31 21	31 53	27	27	28	28	29	29	30	30	31	32		
31 12	31 44	0	1	1	2	2	3	3	4	4	5	1	
31 3	31 35	5	6	6	7	7	8	8	9	9	10	2	1
30 55	31 26	10	11	11	12	12	13	13	14	14	15	3	2
30 46	31 17	16	16	17	17	18	18	19	19	20	20	4	3
30 37	31 8	21	21	22	22	23	23	24	24	25	26	5	3
30 29	31 0	26	27	27	28	28	29	29	30	30	31	6	4
												7	5
												8	6
												9	7
30 20	0 51	0	1	1	2	2	3	3	4	4	5		
30 11	30 42	5	6	6	7	7	8	8	9	9	10		
30 2	30 33	10	11	11	12	12	13	13	14	14	15		
29 54	30 24	15	16	16	17	17	18	18	19	19	20		
29 45	0 15	20	21	21	22	22	23	23	24	24	25		
29 36	0 6	25	26	26	27	27	28	28	29	29	30		

TABLES OF NAUTICAL ASTRONOMY.

TABLE VIII.

Parallax of the Moon less Refraction.

Apparent altitude.	Horizontal parallax.														
	53'		54'		55'		56'		57'		58'		59'		
0	25	57	26	27	26	57	27	27	27	27	28	57	28	57	
10	25	49	26	19	26	49	27	19	27	19	28	48	28	48	
20	25	41	26	11	26	41	27	11	27	40	28	40	28	40	
30	25	34	26	3	26	33	27	2	27	32	28	31	28	31	
40	25	26	26	5	26	25	26	14	27	23	27	53	28	22	
50	25	18	25	47	25	16	26	46	27	15	27	44	28	13	
61	0	25	10	25	39	26	8	26	37	27	6	27	36	28	5
10	25	2	25	31	26	0	26	29	26	55	27	27	27	27	56
20	24	54	25	23	25	52	26	21	26	49	27	18	27	47	47
30	24	46	25	15	25	44	26	12	26	41	27	10	27	38	34
40	24	39	25	7	25	36	26	4	26	32	27	1	27	29	29
50	24	31	24	59	25	27	25	56	26	24	26	52	27	21	21
62	0	24	23	24	51	25	14	25	17	26	15	26	42	27	12
10	24	15	24	43	25	11	25	30	26	7	26	35	27	3	3
20	24	7	24	35	25	3	25	30	26	58	26	27	26	54	54
30	23	59	24	26	24	54	25	22	25	50	26	17	26	45	45
40	23	51	24	17	24	46	25	13	25	41	26	8	26	36	36
50	23	43	24	10	24	38	25	5	25	32	26	0	26	27	27
63	0	23	35	24	2	24	29	24	56	25	24	25	31	26	14
10	23	27	23	54	24	21	24	48	25	16	25	42	26	9	9
20	23	19	23	45	24	12	24	40	25	6	25	33	26	0	0
30	23	11	23	37	24	4	24	32	25	58	25	24	26	51	51
40	23	2	23	29	24	5	24	24	25	50	25	15	26	42	42
50	22	54	23	21	23	17	24	16	25	42	25	7	26	33	33
64	0	22	46	23	11	23	9	24	8	24	1	24	58	25	24
10	22	38	23	4	23	30	23	57	24	5	24	19	25	15	15
20	22	30	23	50	23	22	23	48	24	4	24	10	25	6	6
30	22	22	23	18	23	14	23	40	24	5	24	1	25	57	57
40	22	14	22	9	23	5	23	31	24	5	24	4	25	48	48
50	22	6	22	31	22	57	23	22	24	46	24	16	25	39	39
65	0	21	57	22	23	48	23	13	23	30	24	1	25	0	0
10	21	49	22	14	22	40	23	5	23	20	25	5	25	20	20
20	21	41	22	6	22	31	22	5	23	11	25	46	25	11	11
30	21	33	21	58	22	23	22	17	23	12	25	37	25	2	2
40	21	25	21	49	22	14	22	8	23	3	25	28	25	13	13
50	21	16	21	41	22	5	22	50	22	55	25	19	25	44	44

TABLE VIII.

Parallax of the Moon less Refraction.

		Proportional parts for										For the altitude.	
60'		0'	1"	2"	3"	4"	5"	6"	7"	8"	9"		—
29'	27"	29'	57"	0	5	1	1	2	3	4	4	1'	1"
29	18	29	48	5	6	6	7	7	8	9	9	2	2
29	9	29	39	10	10	11	11	12	12	13	14	3	3
29	1	29	30	15	15	16	16	17	17	18	19	4	4
28	1	29	21	20	20	21	21	22	22	23	24	5	5
28	43	29	12	25	25	26	26	27	27	28	29	6	6
												7	7
												8	8
28	34	29	3	0	0	1	1	2	3	4	4	9	9
25	2	28	54	5	5	6	6	7	8	9	9		
25	16	28	4	10	10	10	11	11	12	13	14		
25	7	28	5	14	14	15	15	16	17	18	19		
27	58	28	46	19	19	20	20	21	22	22	23		
27	49	28	17	24	24	25	25	26	27	27	28		
27	40	28	8	0	0	1	1	2	3	4	4	1	1
27	1	27	59	5	5	6	6	7	8	8	9	2	2
27	22	27	50	10	10	10	11	11	12	12	13	3	3
27	13	27	41	14	14	15	15	16	16	17	18	4	4
27	4	27	31	18	18	19	19	20	20	21	22	5	5
26	54	27	22	23	24	24	25	25	26	27	27	6	6
												7	7
												8	8
26	45	27	13	0	0	1	1	2	3	4	4	9	9
26	36	27	3	4	5	5	6	6	7	8	8		
26	27	26	54	10	10	10	11	11	12	12	13		
26	18	26	45	14	13	14	15	15	16	17	17		
26	9	26	35	18	18	19	19	20	20	21	21		
26	0	26	26	22	23	23	24	24	25	25	26		
25	50	26	17	0	0	1	1	2	3	3	4	1	1
25	41	26	7	4	5	5	6	6	7	8	8	2	2
25	32	25	58	9	9	9	10	10	11	11	12	3	3
25	23	25	49	13	13	14	14	15	15	16	16	4	4
25	15	25	39	17	18	18	19	19	20	20	21	5	5
25	4	25	30	22	22	22	23	23	24	24	25	6	6
												7	7
												8	8
24	55	25	20	0	0	1	1	2	3	3	4	9	9
24	46	25	11	4	5	5	6	6	7	7	8		
24	37	25	1	8	9	9	10	10	11	11	12		
24	27	24	52	12	13	13	14	14	15	15	16		
24	18	24	42	17	17	17	18	18	19	19	20		
24	8	24	33	21	21	22	22	23	23	24	24		

TABLES OF NAUTICAL ASTRONOMY.

TABLE VIII.

Parallax of the Moon less Refraction.

Apparent altitude.	Horizontal parallax.						
	53'	54'	55'	56'	57'	58'	59'
66° 0'	21' 9	21' 32	21' 5	22' 21	22' 46	22' 10	23' 34
10	21' 0	21' 24	21' 43	22' 13	22' 37	23' 1	23' 25
20	20' 52	21' 16	21' 40	22' 4	22' 28	22' 52	23' 16
30	20' 4	21' 7	21' 31	21' 13	22' 19	22' 43	23' 7
40	20' 35	20' 59	21' 23	21' 46	22' 10	22' 34	22' 56
50	20' 27	20' 50	21' 14	21' 37	22' 1	22' 25	22' 48
67° 0	20' 18	20' 41	21' 5	21' 29	21' 52	22' 16	22' 39
10	20' 10	20' 33	20' 57	21' 20	21' 43	22' 6	22' 30
20	20' 2	20' 25	20' 48	21' 11	21' 34	21' 57	22' 20
30	19' 53	20' 16	20' 39	21' 2	21' 25	21' 48	22' 11
40	19' 45	20' 8	20' 31	20' 53	21' 18	21' 40	22' 2
50	19' 37	19' 59	20' 22	20' 45	21' 10	21' 32	21' 52
68° 0	19' 28	19' 51	20' 13	20' 36	20' 58	21' 21	21' 43
10	19' 20	19' 42	20' 4	20' 27	20' 49	21' 11	21' 34
20	19' 11	19' 34	19' 56	20' 18	20' 40	21' 2	21' 24
30	19' 3	19' 25	19' 47	20' 9	20' 31	20' 53	21' 15
40	18' 55	19' 16	19' 38	20' 0	20' 22	20' 44	21' 6
50	18' 46	19' 8	19' 29	19' 51	20' 13	20' 34	20' 56
69° 0	18' 38	18' 9	19' 21	19' 42	20' 4	20' 25	20' 47
10	18' 29	18' 51	19' 12	19' 33	19' 55	20' 16	20' 37
20	18' 21	18' 42	19' 3	19' 24	19' 46	20' 7	20' 28
30	18' 12	18' 33	18' 54	19' 15	19' 36	19' 57	20' 18
40	18' 4	18' 25	18' 46	19' 7	19' 27	19' 48	20' 9
50	17' 54	18' 16	18' 37	18' 57	19' 18	19' 39	20' 0
70° 0	17' 47	18' 7	18' 28	18' 49	19' 9	19' 30	19' 50
10	17' 38	17' 59	18' 19	18' 39	19' 0	19' 20	19' 41
20	17' 30	17' 50	18' 10	18' 30	18' 51	19' 11	19' 31
30	17' 21	17' 41	18' 1	18' 21	18' 41	19' 1	19' 22
40	17' 13	17' 33	17' 53	18' 12	18' 32	18' 52	19' 12
50	17' 5	17' 24	17' 44	18' 3	18' 23	18' 43	19' 2
71° 0	16' 56	17' 13	17' 35	17' 54	18' 14	18' 33	18' 53
10	16' 47	17' 4	17' 26	17' 45	18' 5	18' 24	18' 43
20	16' 39	16' 55	17' 17	17' 36	17' 55	18' 13	18' 34
30	16' 30	16' 49	17' 8	17' 27	17' 46	18' 5	18' 24
40	16' 21	16' 40	16' 59	17' 18	17' 37	17' 56	18' 13
50	16' 13	16' 32	16' 50	17' 9	17' 28	17' 46	18' 5

TABLE VIII.

Parallax of the Moon less Refraction.

		Proportional parts for the parallax.											For the Altitude.	
60'	61'	0"	1"	2"	3"	4"	5"	6"	7"	8"	9"		—	
23' 59"	24' 23"	0	0	1	1	2	2	2	3	3	4	1'	1"	
23 50	24 14	4	4	5	5	6	6	6	7	7	8	2	2	
23 40	24 4	8	8	9	9	10	10	10	11	11	12	3	3	
23 31	23 55	12	12	13	13	14	14	14	15	15	16	4	4	
23 21	23 45	16	16	17	17	18	18	18	19	19	20	5	5	
23 12	23 35	20	20	21	21	21	22	22	23	23	23	6	5	
												7	6	
												8	7	
23 2	23 26	0	0	1	1	2	2	2	3	3	3	8	7	
22 53	23 16	4	4	5	5	5	6	6	6	7	7	9	8	
22 44	23 7	8	8	8	9	9	10	10	10	11	11			
22 34	22 57	11	12	12	13	13	13	14	14	14	15			
22 26	22 47	15	16	16	16	17	17	18	18	18	19			
22 15	22 38	19	19	20	20	21	21	21	22	22	22			
22 6	22 28	0	0	1	1	2	2	2	3	3	3	1	1	
21 56	22 18	4	4	4	5	5	5	6	6	7	7	2	2	
21 46	22 9	7	8	8	8	9	9	10	10	10	11	3	3	
21 37	21 59	11	11	12	12	12	13	13	14	14	14	4	4	
21 27	21 49	15	15	15	16	16	16	17	17	18	18	5	5	
21 18	21 40	18	19	19	19	20	20	20	21	21	22	6	5	
												7	6	
												8	7	
												9	8	
21 8	21 30	0	0	1	1	1	2	2	2	3	3			
20 59	21 20	4	4	4	5	5	5	6	6	6	7			
20 49	21 10	7	7	8	8	8	9	9	9	10	10			
20 39	21 0	11	11	11	12	12	12	13	13	13	14			
20 30	20 51	14	14	15	15	15	16	16	16	17	17			
20 20	20 41	18	18	18	19	19	19	20	20	20	21			
20 11	20 31	0	0	1	1	1	2	2	2	3	3	1	1	
20 1	20 21	3	4	4	4	5	5	5	6	6	6	2	2	
19 51	20 11	7	7	7	8	8	8	9	9	9	10	3	3	
19 42	20 2	10	10	11	11	11	12	12	12	13	13	4	4	
19 32	19 52	14	14	14	14	15	15	15	16	16	16	5	5	
19 22	19 42	17	17	17	18	18	18	19	19	19	20	6	6	
												7	7	
												8	8	
19 12	19 32	0	0	1	1	1	2	2	2	3	3			
19 3	19 22	3	3	4	4	4	5	5	5	6	6	9	8	
18 53	19 12	6	7	7	7	8	8	8	9	9	9			
18 43	19 2	10	10	10	10	11	11	11	12	12	12			
18 33	19 52	13	13	13	13	14	14	14	15	15	15			
18 24	19 42	16	16	16	17	17	17	18	18	18	19			

TABLE VIII.

Parallax of the Moon less Refraction.

Apparent altitude.	Horizontal parallax.						
	54'	55'	56'	57'	58'	59'	
72° 0'	16' 4"	16' 23"	16' 41"	17' 0"	17' 18"	17' 37"	17' 55"
10	15 56	16 14	16 32	16 51	17 9	17 27	17 46
20	15 47	16 5	16 23	16 42	17 0	17 18	17 36
30	15 38	15 56	16 14	16 32	16 50	17 9	17 27
40	15 30	15 47	16 5	16 23	16 41	16 59	17 17
50	15 21	15 39	15 56	16 14	16 32	16 50	17 7
73 0	15 12	15 30	15 47	16 5	16 22	16 40	16 58
10	15 4	15 21	15 38	15 56	16 13	16 31	16 48
20	14 56	15 12	15 29	15 47	16 4	16 21	16 38
30	14 46	15 3	15 20	15 37	15 54	16 11	16 29
40	14 38	14 56	15 11	15 28	15 45	16 9	16 19
50	14 29	14 46	15 2	15 19	15 36	15 52	16 9
74 0	14 20	14 37	14 53	15 10	15 26	15 43	15 59
10	14 11	14 27	14 43	15 0	15 16	15 33	15 49
20	14 3	14 19	14 35	14 51	15 6	15 21	15 40
30	13 54	14 10	14 26	14 42	14 58	15 14	15 30
40	13 45	14 1	14 17	14 33	14 49	15 5	15 20
50	13 37	13 52	14 8	14 24	14 39	14 55	15 11
75 0	13 28	13 43	13 59	14 14	14 30	14 45	15 1
10	13 19	13 34	13 50	14 5	14 20	14 36	14 51
20	13 10	13 25	13 41	13 56	14 11	14 26	14 41
30	13 1	13 16	13 31	13 47	14 2	14 17	14 31
40	12 53	13 8	13 22	13 37	13 52	14 7	14 22
50	12 44	12 59	13 13	13 28	13 43	13 57	14 12
76 0	12 35	12 49	13 4	13 19	13 34	13 48	14 2
10	12 26	12 41	12 55	13 9	13 24	13 38	13 52
20	12 17	12 32	12 46	13 0	13 14	13 28	13 43
30	12 9	12 23	12 37	12 51	13 5	13 19	13 33
40	12 0	12 13	12 27	12 41	12 19	13 9	13 22
50	11 51	12 4	12 18	12 31	12 45	12 59	13 12
77 0	11 42	11 56	12 9	12 23	12 36	12 50	13 5
10	11 3	11 46	11 59	12 12	12 26	12 39	12 53
20	11 25	11 37	11 50	12 3	12 17	12 30	12 43
30	11 16	11 29	11 42	11 55	12 8	12 21	12 34
40	11 7	11 20	11 32	11 45	11 58	12 11	12 24
50	10 58	11 11	11 23	11 36	11 48	12 1	12 14

TABLE VII.

Parallax of the Moon less Refraction.

		Proportional parts for the parallax											For the altitude.
60'		0"	1"	2"	3"	4"	5"	6"	7"	8"	9"		—
18' 14'	18 32'	0	0	1	1	1	2	2	2	3	3	1'	17
18 4	18 23	3	3	4	4	4	5	5	5	6	6	2	3
17 54	18 13	6	6	7	7	7	8	8	8	9	9	3	3
17 45	18 3	9	9	10	10	10	11	11	11	12	12	4	4
17 35	17 53	12	12	13	13	13	14	14	14	15	15	5	5
17 25	17 43	15	15	16	16	16	17	17	17	18	18	6	6
17 15	17 33	0	0	1	1	1	1	2	2	2	3	7	8
17 5	17 23	3	3	3	4	4	4	5	5	5	5	8	9
16 55	17 13	6	6	6	7	7	7	7	8	8	8	8	
16 46	17 3	9	9	9	9	9	10	10	11	11	11		
16 36	16 53	12	12	12	12	12	13	13	13	14	14		
16 26	16 43	15	15	15	15	15	16	16	16	16	17		
16 16	16 32	0	0	1	1	1	2	2	2	2	2	1	1
16 7	16 21	3	3	3	3	4	4	4	5	5	5	2	2
15 56	16 12	5	5	6	6	6	7	7	7	7	8	3	3
15 46	16 2	8	8	9	9	9	9	10	10	10	10	4	4
15 36	15 52	11	11	11	11	12	12	12	13	13	13	5	5
15 26	15 42	13	13	14	14	14	15	15	15	15	16	6	6
15 16	15 32	0	0	1	1	1	1	2	2	2	2	7	9
15 7	15 22	3	3	3	3	4	4	4	4	5	5	8	
14 57	15 12	5	5	6	6	6	6	7	7	7	7		
14 47	15 2	8	8	8	8	9	9	9	9	10	10		
14 37	14 52	10	10	11	11	11	11	12	12	12	12		
14 27	14 41	13	13	13	13	14	14	14	14	15	15		
14 17	14 31	0	0	0	1	1	1	1	2	2	2	1	1
14 7	14 21	2	3	3	3	3	3	4	4	4	4	2	2
13 57	14 11	5	5	5	5	6	6	6	6	7	7	3	3
13 47	14 1	7	7	7	7	8	8	8	9	9	9	4	4
13 36	13 50	9	10	10	10	10	10	11	11	11	11	5	5
13 26	13 40	12	12	12	12	13	13	13	13	14	14	6	6
13 17	13 30	0	0	0	1	1	1	1	2	2	2	7	9
13 7	13 19	2	2	3	3	3	3	4	4	4	4	8	
12 56	13 9	4	5	5	5	5	6	6	6	6	6		
12 47	13 0	7	7	7	7	7	8	8	8	8	8		
12 36	12 49	9	9	9	9	10	10	10	10	10	11		
12 26	12 39	11	11	11	12	12	12	12	13	13	13		

TABLES OF NAUTICAL ASTRONOMY.

TABLE VIII.

Parallax of the Moon, less Refraction.

Apparent altitude,	Horizontal parallax						
	53'	54'	55'	56'	57'	58'	59'
78° 0'	10 49'	11' 1"	11' 14'	11' 26'	11' 39'	11' 51'	12' 4'
10	10 40	10 52	11 5	11 17	11 29	11 42	11 54
20	11 31	10 43	10 55	11 8	11 20	11 32	11 44
30	10 22	10 34	10 45	10 58	11 10	11 22	11 34
40	10 14	10 25	10 37	10 49	11 1	11 12	11 24
50	10 5	10 16	10 28	10 39	10 51	11 3	11 14
79 0	9 56	10 7	10 19	10 30	10 41	10 52	11 4
10	9 47	9 58	10 9	10 21	10 32	10 43	10 54
20	9 38	9 49	10 0	10 11	10 22	10 33	10 44
30	9 29	9 40	9 51	10 2	10 13	10 24	10 35
40	9 20	9 31	9 42	9 52	10 3	10 14	10 25
50	9 11	9 22	9 32	9 43	9 53	10 4	10 15
80 0	9 2	9 13	9 23	9 33	9 44	9 54	10 5
10	8 53	9 3	9 14	9 24	9 34	9 44	9 55
20	8 44	8 54	9 4	9 14	9 25	9 35	9 45
30	8 35	8 45	8 55	9 5	9 15	9 25	9 35
40	8 26	8 36	8 46	8 56	9 6	9 16	9 26
50	8 17	8 27	8 37	8 46	8 5	9 5	9 15
81 0	8 8	8 1	8 27	8 37	8 47	8 57	9 5
10	7 59	8 9	8 15	8 27	8 36	8 47	8 57
20	7 51	8 0	8 9	8 16	8 27	8 37	8 47
30	7 43	7 50	7 59	8 5	8 17	8 27	8 37
40	7 35	7 41	7 50	7 59	8 7	8 17	8 27
50	7 26	7 32	7 41	7 49	7 58	8 8	8 18
82 0	7 17	7 21	7 3	7 40	7 45	7 55	8 5
10	7 8	7 14	7 22	7 30	7 39	7 49	7 59
20	6 57	7 5	7 13	7 21	7 30	7 37	7 47
30	6 48	6 15	7 3	7 11	7 19	7 27	7 35
40	6 39	6 46	6 54	7 2	7 9	7 17	7 25
50	6 30	6 37	6 45	6 52	6 59	7 7	7 14
83 0	6 21	6 28	6 35	6 42	6 50	6 57	7 4
10	6 12	6 19	6 26	6 33	6 40	6 47	6 54
20	6 3	6 10	6 16	6 23	6 30	6 37	6 44
30	5 53	6 0	6 7	6 14	6 21	6 27	6 34
40	5 44	5 51	5 58	6 4	6 11	6 18	6 24
50	5 35	5 42	5 49	5 55	6 1	6 8	6 14

Parallax of the Moon

			Proportional parts for the parallel										For the altitude.	
60'			0'	1'	2'	3'	4'	5'	6'	7'	8'	9'		
12	16	12 29'	0	0	0	1	1	1	1	1	2	2	2	2
12	6	12 19	2	2	2	3	3	3	3	3	4	4	4	4
11	56	12 8	4	4	4	5	5	5	5	5	6	6	6	6
11	46	11 58	6	6	6	7	7	7	7	7	8	8	8	8
11	36	11 48	8	8	8	9	9	9	9	9	10	10	10	10
11	26	11 38	10	10	10	11	11	11	11	11	12	12	12	12
11	16	11 27	0	0	0	1	1	1	1	1	2	2	2	2
11	6	11 17	2	2	2	3	3	3	3	3	4	4	4	4
10	56	11 7	4	4	4	5	5	5	5	5	6	6	6	6
10	46	10 56	6	6	6	7	7	7	7	7	8	8	8	8
10	36	10 46	8	8	8	9	9	9	9	9	10	10	10	10
10	26	10 36	10	10	10	11	11	11	11	11	12	12	12	12
10	15	10 26	0	0	0	1	1	1	1	1	2	2	2	2
10	5	10 15	2	2	2	3	3	3	3	3	4	4	4	4
9	56	10 5	4	4	4	5	5	5	5	5	6	6	6	6
9	46	9 55	6	6	6	7	7	7	7	7	8	8	8	8
9	36	9 44	8	8	8	9	9	9	9	9	10	10	10	10
9	26	9 34	10	10	10	11	11	11	11	11	12	12	12	12
9	14	9 24	0	0	0	1	1	1	1	1	2	2	2	2
9	4	9 13	1	1	1	2	2	2	2	2	3	3	3	3
8	56	9 3	3	3	3	4	4	4	4	4	5	5	5	5
8	46	8 52	5	5	5	6	6	6	6	6	7	7	7	7
8	36	8 42	7	7	7	8	8	8	8	8	9	9	9	9
8	26	8 32	9	9	9	10	10	10	10	10	11	11	11	11
8	13	8 21	0	0	0	1	1	1	1	1	2	2	2	2
8	3	8 11	1	1	1	2	2	2	2	2	3	3	3	3
7	56	8 1	3	3	3	4	4	4	4	4	5	5	5	5
7	46	7 50	5	5	5	6	6	6	6	6	7	7	7	7
7	36	7 40	7	7	7	8	8	8	8	8	9	9	9	9
7	26	7 29	9	9	9	10	10	10	10	10	11	11	11	11
7	12	7 19	0	0	0	1	1	1	1	1	2	2	2	2
7	2	7 9	1	1	1	2	2	2	2	2	3	3	3	3
6	56	6 58	3	3	3	4	4	4	4	4	5	5	5	5
6	46	6 48	5	5	5	6	6	6	6	6	7	7	7	7
6	36	6 37	7	7	7	8	8	8	8	8	9	9	9	9
6	26	6 27	9	9	9	10	10	10	10	10	11	11	11	11

TABLES OF MATHEMATICAL ASTRONOMY

TABLE VIII.

Parallax of the Moon less Refraction.

Apparent altitude.	Horizontal parallax.						
	53'	54'	55'	56'	57'	58'	59'
0	5' 26	5' 30	5' 34	5' 45'	5' 52	5' 58	6' 4"
10	5 17	5 23	5 30	5 36	5 43	5 48	5 54
20	5 8	5 14	5 21	5 26	5 32	5 38	5 44
30	4 59	5 5	5 11	5 17	5 22	5 28	5 34
40	4 50	4 56	5 1	5 7	5 13	5 18	5 24
50	4 41	4 47	5 5	4 57	5 3	5 8	5 14
85	4 32	4 37	4 43	4 48	4 53	4 58	5 4
10	4 23	4 28	4 33	4 38	4 43	4 48	4 53
20	4 14	4 19	4 24	4 29	4 34	4 38	4 43
30	4 5	4 10	4 14	4 19	4 24	4 29	4 33
40	3 56	4 1	4 5	4 10	4 14	4 19	4 23
50	3 47	3 51	3 56	4 0	4 4	4 9	4 13
86	3 38	3 42	3 46	3 50	3 55	3 59	4 3
10	3 29	3 33	3 37	3 41	3 45	3 49	3 53
20	3 20	3 24	3 28	3 32	3 36	3 40	3 43
30	3 11	3 14	3 18	3 22	3 26	3 29	3 33
40	3 2	3 5	3 9	3 12	3 16	3 19	3 23
50	2 53	2 56	2 59	3 2	3 6	3 9	3 12
87	2 43	2 47	2 50	2 53	2 56	2 59	3 2
10	2 34	2 37	2 40	2 43	2 46	2 49	2 52
20	2 25	2 28	2 31	2 34	2 37	2 39	2 42
30	2 16	2 19	2 21	2 24	2 27	2 29	2 32
40	2 7	2 10	2 12	2 14	2 17	2 19	2 22
50	1 58	2 0	2 2	2 5	2 7	2 9	2 12
88	1 49	1 51	1 53	1 55	1 57	1 59	2 2
10	1 39	1 42	1 44	1 46	1 48	1 50	1 51
20	1 31	1 33	1 35	1 36	1 38	1 40	1 41
30	1 22	1 23	1 25	1 26	1 28	1 30	1 31
40	1 13	1 14	1 15	1 17	1 18	1 20	1 21
50	1 4	1 5	1 6	1 7	1 8	1 10	1 11
89	0 55	0 56	0 57	0 58	0 59	1 0	1 1
10	0 45	0 46	0 47	0 48	0 49	0 50	0 51
20	0 36	0 37	0 38	0 38	0 39	0 40	0 41
30	0 27	0 27	0 28	0 28	0 29	0 30	0 30
40	0 18	0 19	0 19	0 19	0 20	0 20	0 20
50	0 9	0 9	0 9	0 10	0 10	0 10	0 10

TABLE. VII.

Parallax of the Moon less Refraction.

[illegible]

TABLE IX.

Change in Altitude during the last Minute which Precedes, and the first which follows the Sun's Passage over the Meridian.

Latitude	Declination of the same name as the latitude.												
	0°	2°	4°	6°	8°	10°	12°	14°	16°	18°	20°	22°	24°
0°	+	56 2	25 1	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
2	1 7	56 1	25 0	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
4	2 8	56 0	25 0	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
6	3 9	55 9	24 9	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
8	4 0	55 8	24 8	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
10	5 1	55 7	24 7	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
12	6 2	55 6	24 6	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
14	7 3	55 5	24 5	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
16	8 4	55 4	24 4	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
18	9 5	55 3	24 3	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
20	10 6	55 2	24 2	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
22	11 7	55 1	24 1	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
24	12 8	55 0	24 0	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
26	13 9	54 59	23 59	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
28	14 0	54 58	23 58	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
30	15 1	54 57	23 57	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
32	16 2	54 56	23 56	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
34	17 3	54 55	23 55	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
36	18 4	54 54	23 54	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
38	19 5	54 53	23 53	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
40	20 6	54 52	23 52	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
42	21 7	54 51	23 51	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
44	22 8	54 50	23 50	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
46	23 9	54 49	23 49	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
48	24 0	54 48	23 48	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
50	25 1	54 47	23 47	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
52	26 2	54 46	23 46	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
54	27 3	54 45	23 45	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
56	28 4	54 44	23 44	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
58	29 5	54 43	23 43	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
60	30 6	54 42	23 42	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
62	31 7	54 41	23 41	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
64	32 8	54 40	23 40	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
66	33 9	54 39	23 39	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
68	34 0	54 38	23 38	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
70	35 1	54 37	23 37	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
72	36 2	54 36	23 36	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
74	37 3	54 35	23 35	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
76	38 4	54 34	23 34	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
78	39 5	54 33	23 33	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
80	40 6	54 32	23 32	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
82	41 7	54 31	23 31	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
84	42 8	54 30	23 30	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
86	43 9	54 29	23 29	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
88	44 0	54 28	23 28	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4
90	45 1	54 27	23 27	1 7	1 1	1 1	9 7	9 9	8 1	7 0	5 4	4 9	4 4

TABLE IX.

Change in Altitude during the last Minute which precedes, and the first which follows the Sun's Passage over the Meridian.

Latitude	Declination of the same name as the Latitude											
	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°
0°	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
2	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
4	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
6	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
8	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
10	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
12	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
14	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
16	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
18	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
20	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
22	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
24	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
26	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
28	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
30	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
32	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
34	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
36	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
38	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
40	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
42	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
44	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
46	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
48	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
50	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
52	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
54	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
56	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
58	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
60	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
62	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
64	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
66	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
68	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
70	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
72	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
74	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
76	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
78	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1
80	56.2	56.1	56.0	55.9	55.8	55.7	55.6	55.5	55.4	55.3	55.2	55.1

TABLE X

Multipliers of the Numbers contained in TABLE IX.

Interval between noon and the time of observation.								
		1'	2'	3'	4'	5'	6'	8'
0	0.0	1.0	4.0	9.0	16.0	25.0	36.0	49.0
2	0.0	1.0	4.1	9.2	16.2	25.3	36.4	49.5
4	0.0	1.1	4.3	9.4	16.5	25.7	36.8	49.9
6	0.0	1.2	4.4	9.6	16.8	26.0	37.2	50.4
8	0.0	1.3	4.6	9.8	17.1	26.3	37.6	50.9
10	0.0	1.4	4.7	10.0	17.4	26.7	38.0	51.4
12	0.0	1.4	4.8	10.2	17.6	27.0	38.4	51.9
14	0.1	1.5	5.0	10.4	17.9	27.4	38.8	52.3
16	0.1	1.6	5.1	10.7	18.2	27.7	39.3	52.8
18	0.1	1.7	5.3	10.9	18.5	28.1	39.7	53.3
20	0.1	1.8	5.4	11.1	18.8	28.4	40.1	53.8
22	0.1	1.9	5.6	11.3	19.1	28.8	40.5	54.3
24	0.2	2.0	5.8	11.6	19.4	29.2	41.0	54.8
26	0.2	2.1	5.9	11.8	19.7	29.5	41.4	55.3
28	0.2	2.2	6.1	12.0	19.9	29.9	41.8	55.8
30	0.3	2.3	6.3	12.3	20.2	30.3	42.3	56.3
32	0.3	2.4	6.4	12.5	20.5	30.6	42.7	56.7
34	0.3	2.5	6.6	12.7	20.8	31.0	43.1	57.3
36	0.4	2.6	6.8	13.0	21.2	31.4	43.6	57.8
38	0.4	2.7	6.9	13.2	21.5	31.7	44.0	58.3
40	0.4	2.8	7.1	13.4	21.8	32.1	44.4	58.8
42	0.5	2.9	7.3	13.7	22.1	32.5	44.9	59.3
44	0.5	3.0	7.5	13.9	22.4	32.9	45.3	59.8
46	0.6	3.1	7.7	14.2	22.7	33.3	45.8	60.3
48	0.6	3.2	7.8	14.4	23.0	33.6	46.2	60.8
50	0.7	3.4	8.0	14.7	23.4	34.0	46.7	61.4
52	0.8	3.5	8.2	15.0	23.7	34.4	47.2	61.9
54	0.8	3.6	8.4	15.2	24.0	34.8	47.6	62.4
56	0.9	3.7	8.6	15.5	24.3	35.2	48.1	62.9
58	0.9	3.9	8.8	15.7	24.7	35.5	48.5	63.5

TABLE XI.

Numbers for finding the Corrections of Longitude by Marine Chronometers.

Days elapsed since the chronometer was regulated.	Multiples of the second difference.	Days elapsed since the chronometer was regulated.	Multiples of the second difference.	Days elapsed since the chronometer was regulated.	Multiples of the second difference.	Days elapsed since the chronometer was regulated.	Multiples of the second difference.
1	1	31	496	61	1891	91	4186
2	3	32	528	62	1953	92	4278
3	6	33	561	63	2016	93	4371
4	10	34	595	64	2080	94	4465
5	15	35	630	65	2145	95	4560
6	21	36	666	66	2211	96	4656
7	28	37	703	67	2278	97	4753
8	36	38	741	68	2346	98	4851
9	45	39	780	69	2415	99	4950
10	55	40	820	70	2485	100	5050
11	66	41	861	71	2556	101	5151
12	78	42	903	72	2628	102	5253
13	91	43	946	73	2701	103	5356
14	105	44	990	74	2775	104	5460
15	120	45	1035	75	2850	105	5565
16	136	46	1081	76	2926	106	5671
17	153	47	1128	77	3003	107	5778
18	171	48	1176	78	3081	108	5886
19	190	49	1225	79	3160	109	5995
20	210	50	1275	80	3240	110	6105
21	231	51	1326	81	3321	111	6216
22	253	52	1378	82	3403	112	6328
23	276	53	1431	83	3486	113	6441
24	300	54	1485	84	3570	114	6555
25	325	55	1540	85	3655	115	6670
26	351	56	1596	86	3741	116	6786
27	378	57	1653	87	3828	117	6903
28	406	58	1711	88	3916	118	7021
29	435	59	1770	89	4005	119	7140
30	465	60	1830	90	4095	120	7260

TABLE XII.

For finding the Correction of the loss of two Altitudes of the Sun taken out of the Meridian.

FIRST TERM.

Altitude.	Latitude.										
	0°	2°	4°	6°	8°	10°	12°	14°	16°	18°	20°
6°	1.00	1.00	1.01	1.01	1.02	1.02	1.02	1.03	1.03	1.03	1.04
8	1.00	1.01	1.01	1.02	1.02	1.03	1.03	1.04	1.04	1.05	1.05
10	1.00	1.01	1.01	1.02	1.03	1.03	1.04	1.04	1.05	1.06	1.06
12	1.00	1.01	1.02	1.02	1.03	1.04	1.05	1.05	1.06	1.07	1.08
14	1.00	1.01	1.02	1.03	1.04	1.04	1.05	1.06	1.07	1.08	1.09
16	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.10
18	1.00	1.01	1.02	1.03	1.05	1.06	1.07	1.08	1.09	1.11	1.12
20	1.00	1.01	1.03	1.04	1.05	1.06	1.08	1.09	1.10	1.12	1.13
22	1.00	1.01	1.03	1.04	1.06	1.07	1.09	1.10	1.12	1.13	1.15
24	1.00	1.02	1.03	1.05	1.06	1.08	1.10	1.11	1.13	1.15	1.16
26	1.00	1.02	1.03	1.05	1.07	1.09	1.10	1.12	1.14	1.16	1.18
28	1.00	1.02	1.04	1.06	1.08	1.09	1.11	1.13	1.15	1.17	1.19
30	1.00	1.02	1.04	1.06	1.08	1.10	1.12	1.14	1.17	1.19	1.21
32	1.00	1.02	1.04	1.07	1.09	1.11	1.13	1.16	1.18	1.20	1.23
34	1.00	1.02	1.05	1.07	1.10	1.12	1.14	1.17	1.19	1.22	1.25
36	1.00	1.03	1.05	1.08	1.10	1.13	1.15	1.18	1.21	1.24	1.26
38	1.00	1.03	1.06	1.08	1.11	1.14	1.17	1.20	1.22	1.25	1.28
40	1.00	1.03	1.06	1.09	1.12	1.15	1.18	1.21	1.24	1.27	1.31
42	1.00	1.03	1.06	1.10	1.13	1.16	1.19	1.23	1.26	1.29	1.33
44	1.00	1.03	1.07	1.10	1.14	1.17	1.21	1.24	1.28	1.31	1.35
46	1.00	1.04	1.07	1.11	1.15	1.18	1.22	1.26	1.30	1.34	1.38
48	1.00	1.04	1.08	1.12	1.16	1.20	1.24	1.28	1.32	1.36	1.40
50	1.00	1.04	1.08	1.13	1.17	1.21	1.25	1.30	1.34	1.39	1.43
52	1.00	1.05	1.09	1.14	1.18	1.23	1.27	1.32	1.37	1.42	1.47
54	1.00	1.05	1.10	1.15	1.19	1.24	1.29	1.34	1.40	1.45	1.50
56	1.00	1.05	1.10	1.16	1.21	1.26	1.32	1.37	1.43	1.48	1.54
58	1.00	1.06	1.11	1.17	1.22	1.28	1.34	1.40	1.46	1.52	1.58
60	1.00	1.06	1.12	1.18	1.24	1.31	1.37	1.43	1.50	1.56	1.63
62	1.00	1.07	1.13	1.20	1.26	1.33	1.40	1.47	1.54	1.61	1.69
64	1.00	1.07	1.14	1.22	1.29	1.36	1.44	1.51	1.59	1.67	1.75
66	1.00	1.08	1.16	1.24	1.32	1.40	1.48	1.56	1.64	1.73	1.82
68	1.00	1.09	1.17	1.26	1.35	1.44	1.53	1.62	1.71	1.80	1.90
70	1.00	1.10	1.19	1.29	1.39	1.49	1.58	1.69	1.79	1.89	2.00
72	1.00	1.11	1.22	1.32	1.43	1.54	1.65	1.77	1.88	2.00	2.12
74	1.00	1.12	1.24	1.37	1.49	1.62	1.74	1.87	2.00	2.13	2.27
76	1.00	1.14	1.28	1.42	1.56	1.71	1.85	2.00	2.15	2.30	2.46
78	1.00	1.16	1.33	1.50	1.66	1.83	2.00	2.17	2.36	2.53	2.71
80	1.00	1.20	1.40	1.60	1.80	2.00	2.21	2.41	2.63	2.84	3.06
82	1.00	1.25	1.50	1.75	2.00	2.26	2.51	2.77	3.04	3.31	3.59
84	1.00	1.31	1.67	2.00	2.34	2.68	3.02	3.37	3.73	4.09	4.46

TABLE XII.

For finding the Correction of the less of two Altitudes of the Sun taken out of the Meridian.

ARGUMENT.

Altitude.	Latitude.										
	0°	2°	4°	6°	8°	10°	12°	14°	16°	18°	20°
6°	1.01	1.01	1.01	1.01	1.02	1.02	1.03	1.03	1.05	1.06	1.07
8	1.01	1.01	1.01	1.02	1.02	1.03	1.03	1.04	1.05	1.06	1.08
10	1.02	1.02	1.02	1.02	1.03	1.03	1.04	1.05	1.06	1.07	1.08
12	1.02	1.02	1.03	1.03	1.03	1.04	1.05	1.05	1.06	1.08	1.09
14	1.03	1.03	1.03	1.04	1.04	1.05	1.05	1.06	1.07	1.08	1.10
16	1.04	1.04	1.04	1.05	1.05	1.06	1.06	1.07	1.08	1.09	1.11
18	1.05	1.05	1.05	1.06	1.06	1.07	1.08	1.08	1.09	1.11	1.12
20	1.06	1.07	1.07	1.07	1.08	1.08	1.09	1.10	1.11	1.12	1.13
22	1.08	1.08	1.08	1.09	1.09	1.10	1.10	1.11	1.12	1.13	1.15
24	1.10	1.10	1.10	1.10	1.11	1.11	1.12	1.12	1.14	1.15	1.17
26	1.11	1.11	1.12	1.12	1.12	1.13	1.14	1.15	1.16	1.17	1.18
28	1.13	1.13	1.14	1.14	1.14	1.15	1.16	1.17	1.18	1.19	1.21
30	1.16	1.16	1.16	1.16	1.17	1.17	1.18	1.19	1.20	1.21	1.23
32	1.18	1.18	1.18	1.19	1.19	1.20	1.21	1.22	1.23	1.24	1.26
34	1.21	1.21	1.21	1.21	1.22	1.23	1.23	1.24	1.26	1.27	1.28
36	1.24	1.24	1.24	1.24	1.25	1.26	1.26	1.27	1.29	1.30	1.32
38	1.27	1.27	1.27	1.28	1.28	1.29	1.30	1.31	1.32	1.33	1.35
40	1.31	1.31	1.31	1.31	1.32	1.33	1.34	1.35	1.36	1.37	1.39
42	1.35	1.35	1.35	1.35	1.36	1.37	1.38	1.39	1.40	1.42	1.43
44	1.39	1.39	1.39	1.40	1.40	1.41	1.42	1.43	1.45	1.46	1.48
46	1.44	1.44	1.44	1.45	1.45	1.46	1.47	1.48	1.50	1.51	1.53
48	1.50	1.50	1.50	1.50	1.51	1.52	1.53	1.54	1.56	1.57	1.59
50	1.56	1.56	1.56	1.56	1.57	1.58	1.59	1.60	1.62	1.64	1.66
52	1.62	1.63	1.63	1.63	1.64	1.65	1.66	1.67	1.69	1.71	1.73
54	1.70	1.70	1.71	1.71	1.72	1.73	1.74	1.75	1.77	1.79	1.81
56	1.79	1.79	1.79	1.80	1.81	1.82	1.83	1.84	1.86	1.88	1.90
58	1.89	1.89	1.89	1.90	1.91	1.92	1.93	1.95	1.96	1.98	2.01
60	2.00	2.00	2.01	2.01	2.02	2.03	2.05	2.06	2.08	2.10	2.13
62	2.13	2.13	2.13	2.14	2.15	2.16	2.18	2.20	2.22	2.24	2.27
64	2.28	2.28	2.29	2.29	2.30	2.32	2.33	2.35	2.37	2.40	2.43
66	2.46	2.46	2.47	2.47	2.48	2.50	2.51	2.53	2.56	2.59	2.62
68	2.67	2.67	2.68	2.68	2.70	2.71	2.73	2.75	2.78	2.81	2.84
70	2.92	2.93	2.93	2.94	2.95	2.97	2.99	3.01	3.04	3.07	3.11
72	3.24	3.24	3.24	3.25	3.27	3.29	3.31	3.34	3.37	3.40	3.44
74	3.63	3.63	3.64	3.65	3.66	3.68	3.71	3.74	3.77	3.82	3.86
76	4.13	4.14	4.14	4.16	4.17	4.20	4.23	4.26	4.30	4.35	4.40
78	4.81	4.81	4.82	4.84	4.86	4.88	4.92	4.96	5.00	5.06	5.12
80	5.76	5.76	5.77	5.79	5.82	5.85	5.89	5.94	5.99	6.06	6.13
82	7.19	7.19	7.20	7.23	7.26	7.30	7.35	7.41	7.48	7.56	7.65
84	9.57	9.57	9.59	9.62	9.66	9.72	9.78	9.86	9.95	10.06	10.18

TABLE XII:

For finding the Correction of the less of two Altitudes of the Sun taken out of the Meridian.

FIRST TERM.

Altitude.	Latitude.										
	20°	22°	24°	26°	28°	30°	32°	34°	36°	38°	40°
6°	1.04	1.04	1.05	1.05	1.06	1.07	1.07	1.07	1.08	1.08	1.09
8	1.05	1.06	1.06	1.07	1.08	1.08	1.09	1.10	1.10	1.11	1.12
10	1.06	1.07	1.08	1.08	1.09	1.10	1.11	1.12	1.13	1.13	1.15
12	1.08	1.09	1.10	1.10	1.11	1.12	1.13	1.14	1.15	1.17	1.18
14	1.09	1.10	1.11	1.12	1.13	1.14	1.16	1.17	1.18	1.20	1.21
16	1.10	1.12	1.13	1.14	1.15	1.17	1.18	1.19	1.21	1.23	1.24
18	1.12	1.13	1.15	1.16	1.17	1.19	1.20	1.22	1.24	1.25	1.27
20	1.13	1.15	1.16	1.18	1.19	1.21	1.23	1.25	1.26	1.28	1.31
22	1.15	1.16	1.18	1.20	1.22	1.23	1.25	1.27	1.29	1.32	1.34
24	1.16	1.18	1.20	1.22	1.24	1.26	1.28	1.30	1.32	1.35	1.37
26	1.18	1.20	1.22	1.24	1.26	1.28	1.31	1.33	1.35	1.38	1.41
28	1.19	1.22	1.24	1.26	1.28	1.31	1.33	1.36	1.39	1.42	1.45
30	1.21	1.23	1.26	1.28	1.31	1.33	1.36	1.39	1.42	1.45	1.49
32	1.23	1.25	1.28	1.31	1.33	1.36	1.39	1.42	1.45	1.49	1.52
34	1.25	1.27	1.30	1.33	1.36	1.39	1.42	1.46	1.49	1.53	1.57
36	1.26	1.29	1.32	1.35	1.39	1.42	1.45	1.49	1.53	1.57	1.61
38	1.28	1.32	1.35	1.38	1.42	1.45	1.49	1.53	1.57	1.61	1.66
40	1.31	1.34	1.37	1.41	1.45	1.48	1.52	1.57	1.61	1.66	1.70
42	1.33	1.36	1.40	1.44	1.48	1.52	1.56	1.61	1.65	1.70	1.76
44	1.35	1.39	1.43	1.47	1.51	1.56	1.60	1.65	1.70	1.76	1.81
46	1.38	1.42	1.46	1.51	1.55	1.60	1.65	1.70	1.75	1.81	1.87
48	1.40	1.45	1.50	1.54	1.59	1.64	1.69	1.75	1.81	1.87	1.93
50	1.43	1.48	1.53	1.58	1.63	1.69	1.75	1.80	1.87	1.93	2.00
52	1.47	1.52	1.57	1.62	1.68	1.74	1.80	1.86	1.93	2.00	2.07
54	1.50	1.56	1.61	1.67	1.73	1.80	1.86	1.93	2.00	2.08	2.16
56	1.54	1.60	1.66	1.72	1.79	1.86	1.93	2.00	2.08	2.16	2.24
58	1.58	1.65	1.71	1.78	1.85	1.92	2.00	2.08	2.16	2.25	2.34
60	1.63	1.70	1.77	1.85	1.92	2.00	2.08	2.17	2.26	2.35	2.45
62	1.69	1.76	1.84	1.92	2.00	2.09	2.18	2.27	2.37	2.47	2.58
64	1.75	1.83	1.91	2.00	2.09	2.18	2.28	2.38	2.49	2.60	2.72
66	1.82	1.91	2.00	2.10	2.19	2.30	2.40	2.52	2.63	2.76	2.89
68	1.90	2.00	2.10	2.21	2.32	2.43	2.55	2.67	2.80	2.93	3.08
70	2.00	2.11	2.22	2.34	2.46	2.59	2.72	2.85	3.00	3.15	3.31
72	2.12	2.24	2.37	2.50	2.64	2.78	2.92	3.08	3.24	3.41	3.58
74	2.27	2.41	2.55	2.70	2.85	3.01	3.18	3.35	3.53	3.73	3.93
76	2.46	2.62	2.79	2.96	3.13	3.31	3.51	3.71	3.91	4.13	
78	2.71	2.90	3.10	3.30	3.50	3.72	3.94	4.17	4.42		
80	3.06	3.29	3.53	3.77	4.02	4.27	4.54	4.83			
82	3.59	3.88	4.17	4.47	4.78	5.11	5.45				
84	4.46	4.84	5.24	5.64	6.06	6.49					

TABLE XII.

For finding the Correction of the sum of two Altitudes of the Sun taken out of the Meridian.

ARGUMENT.

Altitude.	Latitude.										
	20°	22°	24°	26°	28°	30°	32°	34°	36°	38°	40°
6°	1.07	1.09	1.10	1.12	1.14	1.16	1.19	1.21	1.24	1.28	1.31
8	1.08	1.09	1.11	1.12	1.14	1.17	1.19	1.22	1.25	1.28	1.32
10	1.08	1.10	1.11	1.13	1.15	1.17	1.20	1.23	1.26	1.29	1.33
12	1.09	1.10	1.12	1.14	1.16	1.18	1.21	1.23	1.26	1.30	1.34
14	1.10	1.11	1.13	1.15	1.17	1.19	1.22	1.24	1.27	1.31	1.35
16	1.11	1.12	1.14	1.16	1.18	1.20	1.23	1.26	1.29	1.32	1.36
18	1.12	1.13	1.15	1.17	1.19	1.21	1.24	1.27	1.30	1.33	1.37
20	1.13	1.15	1.17	1.18	1.21	1.23	1.26	1.28	1.31	1.35	1.39
22	1.15	1.16	1.18	1.20	1.22	1.25	1.27	1.30	1.33	1.37	1.41
24	1.17	1.18	1.20	1.22	1.24	1.26	1.29	1.32	1.35	1.39	1.43
26	1.18	1.20	1.22	1.24	1.26	1.29	1.31	1.34	1.38	1.41	1.45
28	1.21	1.22	1.24	1.26	1.28	1.31	1.34	1.37	1.40	1.44	1.48
30	1.23	1.25	1.26	1.29	1.31	1.33	1.36	1.39	1.42	1.46	1.51
32	1.26	1.27	1.29	1.31	1.34	1.36	1.39	1.42	1.45	1.50	1.54
34	1.28	1.30	1.32	1.34	1.37	1.39	1.42	1.46	1.49	1.53	1.58
36	1.32	1.33	1.35	1.38	1.40	1.43	1.46	1.49	1.53	1.57	1.61
38	1.35	1.37	1.39	1.41	1.44	1.47	1.50	1.53	1.57	1.61	1.66
40	1.39	1.41	1.43	1.45	1.48	1.51	1.54	1.58	1.61	1.66	1.70
42	1.43	1.45	1.47	1.50	1.52	1.55	1.59	1.62	1.66	1.71	1.76
44	1.48	1.50	1.52	1.55	1.58	1.61	1.64	1.68	1.72	1.76	1.82
46	1.53	1.55	1.58	1.60	1.63	1.66	1.70	1.74	1.78	1.83	1.88
48	1.59	1.61	1.64	1.66	1.69	1.73	1.76	1.80	1.85	1.90	1.95
50	1.66	1.68	1.70	1.73	1.76	1.80	1.83	1.88	1.92	1.97	2.03
52	1.73	1.75	1.78	1.81	1.84	1.88	1.92	1.96	2.01	2.06	2.12
54	1.81	1.84	1.86	1.89	1.93	1.97	2.01	2.05	2.10	2.16	2.22
56	1.90	1.93	1.96	1.99	2.03	2.07	2.11	2.16	2.21	2.27	2.34
58	2.01	2.04	2.07	2.10	2.14	2.18	2.23	2.28	2.33	2.40	2.46
60	2.13	2.16	2.19	2.22	2.27	2.31	2.36	2.41	2.47	2.54	2.61
62	2.27	2.30	2.33	2.37	2.41	2.46	2.51	2.57	2.63	2.70	2.78
64	2.43	2.46	2.50	2.54	2.58	2.63	2.69	2.75	2.82	2.90	2.98
66	2.62	2.65	2.69	2.73	2.79	2.84	2.90	2.97	3.04	3.12	3.21
68	2.84	2.88	2.92	2.97	3.02	3.08	3.15	3.22	3.30	3.39	3.49
70	3.11	3.15	3.20	3.25	3.31	3.38	3.45	3.53	3.61	3.71	3.82
72	3.44	3.49	3.54	3.60	3.67	3.74	3.81	3.90	4.00	4.11	4.22
74	3.86	3.91	3.97	4.04	4.11	4.19	4.28	4.38	4.48	4.60	4.74
76	4.40	4.46	4.53	4.60	4.68	4.77	4.87	4.99	5.11	5.25	
78	5.12	5.19	5.27	5.36	5.45	5.55	5.67	5.80	5.95		
80	6.13	6.21	6.30	6.41	6.52	6.65	6.79	6.95			
82	7.65	7.75	7.87	8.00	8.14	8.30	8.47				
84	10.18	10.32	10.47	10.64	10.84	11.05					

For finding the Correction of the \log of two Altitudes of the Sun taken out of the Meridian.

ARGUMENT.

[illegible]

TABLE XII.

For finding the Correction of the loss of two Altitudes of the Sun taken out of the Meridian.

FIRST TERM.

[illegible]

TABLE XIII.

For finding the Correction of the Less of two Altitudes of the Sun taken out of the Meridian.

SECOND TERM.

Argument.	Declination of the same name as the latitude.												
	0°	2°	4°	6°	8°	10°	12°	14°	16°	18°	20°	22°	24°
1-00	0-00	0-04	0-07	0-11	0-14	0-17	0-21	0-24	0-28	0-31	0-34	0-38	0-41
1-10	0-00	0-04	0-08	0-12	0-15	0-19	0-23	0-27	0-30	0-34	0-38	0-41	0-45
1-20	0-00	0-05	0-08	0-13	0-17	0-21	0-25	0-29	0-33	0-37	0-41	0-45	0-49
1-30	0-00	0-05	0-09	0-14	0-18	0-23	0-27	0-32	0-36	0-40	0-45	0-49	0-53
1-40	0-00	0-05	0-10	0-15	0-20	0-24	0-29	0-34	0-39	0-43	0-48	0-53	0-57
1-50	0-00	0-05	0-11	0-16	0-21	0-26	0-31	0-36	0-41	0-46	0-51	0-56	0-61
1-60	0-00	0-06	0-11	0-17	0-22	0-28	0-33	0-39	0-44	0-49	0-55	0-60	0-65
1-70	0-00	0-06	0-12	0-18	0-24	0-30	0-35	0-41	0-47	0-53	0-58	0-64	0-69
1-80	0-00	0-06	0-13	0-19	0-25	0-31	0-37	0-44	0-50	0-56	0-62	0-67	0-73
1-90	0-00	0-07	0-13	0-20	0-26	0-33	0-40	0-46	0-52	0-59	0-65	0-71	0-77
2-00	0-00	0-07	0-14	0-21	0-28	0-35	0-42	0-48	0-55	0-62	0-68	0-75	0-81
2-10	0-00	0-07	0-15	0-22	0-29	0-37	0-44	0-51	0-58	0-65	0-72	0-79	0-85
2-20	0-00	0-08	0-15	0-23	0-31	0-38	0-46	0-53	0-61	0-68	0-75	0-82	0-90
2-30	0-00	0-08	0-16	0-24	0-32	0-40	0-48	0-56	0-63	0-71	0-79	0-86	0-94
2-40	0-00	0-08	0-17	0-25	0-33	0-42	0-50	0-58	0-66	0-74	0-82	0-90	0-98
2-50	0-00	0-09	0-17	0-26	0-35	0-43	0-52	0-61	0-69	0-77	0-86	0-94	1-02
2-60	0-00	0-09	0-18	0-27	0-36	0-45	0-54	0-63	0-72	0-80	0-89	0-97	1-06
2-70	0-00	0-09	0-19	0-28	0-38	0-47	0-56	0-65	0-74	0-83	0-92	1-01	1-10
2-80	0-00	0-10	0-20	0-29	0-39	0-49	0-58	0-68	0-77	0-87	0-96	1-05	1-14
2-90	0-00	0-10	0-20	0-30	0-40	0-50	0-60	0-70	0-80	0-90	0-99	1-09	1-18
3-00	0-00	0-11	0-21	0-31	0-42	0-52	0-62	0-73	0-83	0-93	1-03	1-12	1-22
3-10	0-00	0-11	0-22	0-32	0-43	0-54	0-65	0-75	0-85	0-96	1-06	1-16	1-26
3-20	0-00	0-11	0-22	0-34	0-45	0-56	0-67	0-77	0-88	0-99	1-10	1-20	1-30
3-30	0-00	0-12	0-23	0-35	0-46	0-57	0-69	0-80	0-91	1-02	1-13	1-24	1-34
3-40	0-00	0-12	0-24	0-36	0-47	0-59	0-71	0-82	0-94	1-05	1-16	1-27	1-38
3-50	0-00	0-12	0-24	0-37	0-49	0-61	0-73	0-85	0-97	1-08	1-20	1-31	1-42
3-60	0-00	0-13	0-25	0-38	0-50	0-63	0-75	0-87	0-99	1-11	1-23	1-35	1-46
3-70	0-00	0-13	0-26	0-39	0-52	0-64	0-77	0-90	1-02	1-14	1-27	1-39	1-51
3-80	0-00	0-13	0-27	0-40	0-53	0-66	0-79	0-92	1-05	1-17	1-30	1-42	1-55
3-90	0-00	0-14	0-27	0-41	0-54	0-68	0-81	0-94	1-08	1-21	1-33	1-46	1-59
4-00	0-00	0-14	0-28	0-42	0-56	0-70	0-83	0-97	1-10	1-24	1-37	1-50	1-63
4-10	0-00	0-14	0-29	0-43	0-57	0-71	0-85	0-99	1-13	1-27	1-40	1-54	1-67
4-20	0-00	0-15	0-29	0-44	0-59	0-73	0-87	1-02	1-16	1-30	1-44	1-57	1-71
4-30	0-00	0-15	0-30	0-45	0-60	0-75	0-89	1-04	1-19	1-33	1-47	1-61	1-75
4-40	0-00	0-15	0-31	0-46	0-61	0-76	0-92	1-07	1-21	1-36	1-51	1-65	1-79
4-50	0-00	0-16	0-31	0-47	0-63	0-78	0-94	1-09	1-24	1-39	1-54	1-69	1-83

TABLE XIII.

For finding the Correction of the less of two Altitudes of the Sun taken out of the Meridian.

SECOND TERM.

Argument.	Declination of the same name as the latitude.												
	0°	2°	4°	6°	8°	10°	12°	14°	16°	18°	20°	22°	24°
4-50	0-00	0-16	0-31	0-47	0-63	0-78	0-94	1-09	1-24	1-39	1-54	1-69	1-83
4-60	0-00	0-16	0-32	0-48	0-64	0-80	0-96	1-11	1-27	1-42	1-57	1-72	1-87
4-70	0-00	0-16	0-33	0-49	0-65	0-82	0-98	1-14	1-30	1-45	1-61	1-76	1-91
4-80	0-00	0-17	0-34	0-50	0-67	0-83	1-00	1-16	1-32	1-48	1-64	1-80	1-95
4-90	0-00	0-17	0-34	0-51	0-68	0-85	1-02	1-19	1-35	1-51	1-68	1-84	1-99
5-00	0-00	0-18	0-35	0-52	0-70	0-87	1-04	1-21	1-38	1-55	1-71	1-87	2-03
5-10	0-00	0-18	0-36	0-53	0-71	0-89	1-06	1-23	1-41	1-58	1-74	1-91	2-07
5-20	0-00	0-18	0-36	0-54	0-72	0-90	1-08	1-26	1-43	1-61	1-78	1-95	2-12
5-30	0-00	0-19	0-37	0-55	0-74	0-92	1-10	1-28	1-46	1-64	1-81	1-99	2-16
5-40	0-00	0-19	0-38	0-56	0-75	0-94	1-12	1-31	1-49	1-67	1-85	2-02	2-20
5-50	0-00	0-19	0-38	0-58	0-77	0-96	1-14	1-33	1-52	1-70	1-88	2-06	2-24
5-60	0-00	0-20	0-39	0-59	0-78	0-97	1-16	1-36	1-54	1-73	1-92	2-10	2-28
5-70	0-00	0-20	0-40	0-60	0-79	0-99	1-19	1-38	1-57	1-76	1-95	2-14	2-32
5-80	0-00	0-20	0-41	0-61	0-81	1-01	1-21	1-40	1-60	1-79	1-98	2-17	2-36
5-90	0-00	0-21	0-41	0-62	0-82	1-02	1-23	1-43	1-63	1-82	2-02	2-21	2-40
6-00	0-00	0-21	0-42	0-63	0-84	1-04	1-25	1-45	1-65	1-85	2-05	2-25	2-44
6-10	0-00	0-21	0-43	0-64	0-85	1-06	1-27	1-48	1-68	1-89	2-09	2-29	2-48
6-20	0-00	0-22	0-43	0-65	0-86	1-08	1-29	1-50	1-71	1-92	2-12	2-32	2-52
6-30	0-00	0-22	0-44	0-66	0-88	1-09	1-31	1-52	1-74	1-95	2-16	2-36	2-56
6-40	0-00	0-22	0-45	0-67	0-89	1-11	1-33	1-55	1-76	1-98	2-19	2-40	2-60
6-50	0-00	0-23	0-45	0-68	0-91	1-13	1-35	1-57	1-79	2-01	2-22	2-44	2-64
6-60	0-00	0-23	0-46	0-69	0-92	1-15	1-37	1-60	1-82	2-04	2-26	2-47	2-68
6-70	0-00	0-23	0-47	0-70	0-93	1-16	1-39	1-62	1-85	2-07	2-29	2-51	2-73
6-80	0-00	0-24	0-47	0-71	0-95	1-18	1-41	1-65	1-87	2-10	2-33	2-55	2-77
6-90	0-00	0-24	0-48	0-72	0-96	1-20	1-44	1-67	1-90	2-13	2-36	2-59	2-81
7-00	0-00	0-24	0-49	0-73	0-97	1-22	1-46	1-69	1-93	2-16	2-39	2-62	2-85
7-10	0-00	0-25	0-50	0-74	0-99	1-23	1-48	1-72	1-96	2-19	2-43	2-66	2-89
7-20	0-00	0-25	0-50	0-75	1-00	1-25	1-50	1-74	1-99	2-23	2-46	2-70	2-93
7-30	0-00	0-26	0-51	0-76	1-02	1-27	1-52	1-77	2-01	2-26	2-50	2-74	2-97
7-40	0-00	0-26	0-52	0-77	1-03	1-29	1-54	1-79	2-04	2-29	2-53	2-77	3-01
7-50	0-00	0-26	0-52	0-78	1-04	1-30	1-56	1-81	2-07	2-32	2-57	2-81	3-05
7-60	0-00	0-27	0-53	0-79	1-06	1-32	1-58	1-84	2-10	2-35	2-60	2-85	3-09
7-70	0-00	0-27	0-54	0-81	1-07	1-34	1-60	1-86	2-12	2-38	2-63	2-88	3-13
7-80	0-00	0-27	0-54	0-82	1-09	1-35	1-62	1-89	2-15	2-41	2-67	2-92	3-17
7-90	0-00	0-28	0-55	0-83	1-10	1-37	1-64	1-91	2-18	2-44	2-70	2-96	3-21
8-00	0-00	0-28	0-56	0-84	1-11	1-39	1-66	1-94	2-21	2-47	2-74	3-00	3-25

TABLE XIII.

For finding the Correction of the less of two Altitudes of the Sun taken out of the Meridian.

SECOND TERM.

Argument.	Declination of a different name from the latitude.												
	0°	2°	4°	6°	8°	10°	12°	14°	16°	18°	20°	22°	24°
1-00	2-00	1-97	1-93	1-90	1-86	1-83	1-79	1-76	1-72	1-69	1-66	1-63	1-59
1-10	2-00	1-96	1-92	1-89	1-85	1-81	1-77	1-73	1-70	1-66	1-62	1-59	1-55
1-20	2-00	1-96	1-92	1-88	1-83	1-79	1-75	1-71	1-67	1-63	1-59	1-55	1-51
1-30	2-00	1-96	1-91	1-86	1-82	1-77	1-73	1-69	1-64	1-60	1-56	1-51	1-47
1-40	2-00	1-95	1-90	1-85	1-81	1-76	1-71	1-66	1-61	1-57	1-52	1-48	1-43
1-50	2-00	1-95	1-90	1-84	1-79	1-74	1-69	1-64	1-59	1-54	1-49	1-44	1-39
1-60	2-00	1-94	1-89	1-83	1-78	1-72	1-67	1-61	1-56	1-51	1-45	1-40	1-35
1-70	2-00	1-94	1-88	1-82	1-76	1-71	1-65	1-59	1-53	1-48	1-42	1-36	1-31
1-80	2-00	1-94	1-87	1-82	1-75	1-69	1-63	1-56	1-50	1-44	1-38	1-32	1-27
1-90	2-00	1-93	1-87	1-80	1-74	1-67	1-61	1-54	1-48	1-41	1-35	1-29	1-23
2-00	2-00	1-93	1-86	1-79	1-72	1-65	1-58	1-52	1-45	1-38	1-32	1-25	1-19
2-10	2-00	1-93	1-85	1-78	1-71	1-64	1-56	1-49	1-42	1-35	1-28	1-21	1-15
2-20	2-00	1-92	1-85	1-77	1-69	1-62	1-54	1-47	1-39	1-32	1-25	1-18	1-11
2-30	2-00	1-92	1-84	1-76	1-68	1-60	1-52	1-44	1-37	1-29	1-21	1-14	1-07
2-40	2-00	1-92	1-83	1-75	1-67	1-58	1-50	1-42	1-34	1-26	1-18	1-10	1-02
2-50	2-00	1-91	1-83	1-74	1-65	1-57	1-48	1-40	1-31	1-23	1-15	1-06	0-98
2-60	2-00	1-91	1-82	1-73	1-64	1-55	1-46	1-37	1-28	1-20	1-11	1-03	
2-70	2-00	1-91	1-81	1-72	1-62	1-53	1-44	1-35	1-26	1-17	1-08	0-99	
2-80	2-00	1-90	1-81	1-71	1-61	1-51	1-42	1-32	1-23	1-14	1-04		
2-90	2-00	1-90	1-80	1-70	1-60	1-50	1-40	1-30	1-20	1-10	1-01		
3-00	2-00	1-90	1-79	1-69	1-58	1-48	1-38	1-27	1-17	1-07	0-97		
3-10	2-00	1-89	1-78	1-68	1-57	1-46	1-36	1-25	1-15	1-04			
3-20	2-00	1-89	1-78	1-67	1-56	1-44	1-34	1-23	1-12	1-01			
3-30	2-00	1-89	1-77	1-66	1-55	1-43	1-31	1-20	1-09	0-98			
3-40	2-00	1-88	1-76	1-65	1-53	1-41	1-29	1-18	1-06				
3-50	2-00	1-88	1-76	1-63	1-51	1-39	1-27	1-15	1-03				
3-60	2-00	1-87	1-75	1-62	1-50	1-38	1-25	1-13	1-01				
3-70	2-00	1-87	1-74	1-61	1-49	1-36	1-23	1-11	0-98				
3-80	2-00	1-87	1-74	1-60	1-47	1-34	1-21	1-08					
3-90	2-00	1-86	1-73	1-59	1-46	1-32	1-19	1-06					
4-00	2-00	1-86	1-72	1-58	1-44	1-31	1-17	1-03					
4-10	2-00	1-86	1-71	1-57	1-43	1-29	1-15	1-01					
4-20	2-00	1-85	1-71	1-56	1-42	1-27	1-13	0-98					
4-30	2-00	1-85	1-70	1-55	1-40	1-25	1-11						
4-40	2-00	1-85	1-69	1-54	1-39	1-24	1-09						
4-50	2-00	1-84	1-69	1-53	1-37	1-22	1-06						

For finding the Correction of the loss of two Altitudes of the Sun taken out of the Meridian.

SECOND TERM.

[illegible]

TABLE XIV.

Azimuth corresponding to the Way made in Latitude.

Azimuth.	Multiplier.	Azimuth.	Multiplier.	Azimuth.	Multiplier.	Azimuth.	Multiplier.	Azimuth.	Multiplier.
0°	0.00	30°	0.13	60°	0.50	90°	1.00	120°	1.50
1	0.00	31	0.14	61	0.52	91	1.02	121	1.52
2	0.00	32	0.15	62	0.53	92	1.04	122	1.53
3	0.00	33	0.16	63	0.55	93	1.05	123	1.55
4	0.00	34	0.17	64	0.56	94	1.07	124	1.56
5	0.00	35	0.18	65	0.58	95	1.09	125	1.57
6	0.01	36	0.19	66	0.59	96	1.11	126	1.59
7	0.01	37	0.20	67	0.61	97	1.12	127	1.60
8	0.01	38	0.21	68	0.63	98	1.14	128	1.62
9	0.01	39	0.22	69	0.64	99	1.16	129	1.63
10	0.02	40	0.23	70	0.66	100	1.17	130	1.64
11	0.02	41	0.25	71	0.67	101	1.19	131	1.66
12	0.02	42	0.26	72	0.69	102	1.21	132	1.67
13	0.03	43	0.27	73	0.71	103	1.23	133	1.68
14	0.03	44	0.28	74	0.72	104	1.24	134	1.70
15	0.03	45	0.29	75	0.74	105	1.26	135	1.71
16	0.04	46	0.31	76	0.76	106	1.28	136	1.72
17	0.04	47	0.32	77	0.78	107	1.29	137	1.73
18	0.05	48	0.33	78	0.79	108	1.31	138	1.74
19	0.06	49	0.34	79	0.81	109	1.33	139	1.76
20	0.06	50	0.36	80	0.83	110	1.34	140	1.77
21	0.07	51	0.37	81	0.84	111	1.36	141	1.78
22	0.07	52	0.38	82	0.86	112	1.38	142	1.79
23	0.08	53	0.40	83	0.88	113	1.39	143	1.80
24	0.09	54	0.41	84	0.90	114	1.41	144	1.81
25	0.09	55	0.43	85	0.91	115	1.42	145	1.82
26	0.10	56	0.44	86	0.92	116	1.44	146	1.83
27	0.11	57	0.46	87	0.95	117	1.45	147	1.84
28	0.12	58	0.47	88	0.97	118	1.47	148	1.85
29	0.13	59	0.49	89	0.99	119	1.49	149	1.86
30	0.13	60	0.50	90	1.00	120	1.50	150	1.87

TABLE XV.

*Altitude of the Sun at the Instant of his passing the prime vertical, o
at that of his greatest Azimuth.*

Latitude.	Declination of the same name as the latitude.							
	0°	2°	4°	6°	8°	10°	12°	
0°	0° 0'	0° 0'	0° 0'	0° 0'	0° 0'	0° 0'	0° 0'	
2	0 0	30 0	30 1	30 3	31 1	11 36	9 40	
4	0 0	30 1	30 2	30 4	30 5	23 41	19 36	
6	0 0	19 30	41 3	40 0	48 40	40 9	30 11	
8	0 0	14 31	30 5	48 41	90 0	53 12	42 1	
10	0 0	11 35	23 41	37 0	53 17	90 0	56 38	
12	0 0	9 40	19 36	30 11	42 2	56 39	90 0	
14	0 0	8 18	16 45	25 36	35 7	45 53	59 15	
16	0 0	7 16	14 40	22 17	30 20	39 4	48 59	
18	0 0	6 29	13 3	19 46	26 46	34 11	42 17	
20	0 0	5 51	11 46	17 48	24 1	30 31	37 27	
22	0 0	5 21	10 44	16 12	21 49	27 37	33 43	
24	0 0	4 55	9 53	14 53	20 1	25 17	30 48	
26	0 0	4 34	9 9	13 48	18 31	23 21	28 19	
28	0 0	4 16	8 33	12 52	17 15	21 43	26 17	
30	0 0	4 0	8 1	12 4	16 10	20 19	24 34	
32	0 0	3 52	7 34	11 23	15 14	19 8	23 6	
34	0 0	3 35	7 10	10 46	14 25	18 8	21 50	
36	0 0	3 24	6 49	10 15	13 42	17 11	20 43	
38	0 0	3 15	6 31	9 47	13 4	16 23	19 44	
40	0 0	3 7	6 14	9 21	12 30	15 40	18 52	
42	0 0	2 59	5 59	8 59	11 59	15 3	18 6	
44	0 0	2 53	5 46	8 39	11 33	14 29	17 25	
46	0 0	2 47	5 34	8 21	11 10	13 58	16 48	
48	0 0	2 41	5 23	8 5	10 48	13 31	16 15	
50	0 0	2 36	5 13	7 50	10 28	13 6	15 45	
52	0 0	2 32	5 5	7 37	10 11	12 44	15 18	
54	0 0	2 28	4 57	7 25	9 54	12 34	14 53	
56	0 0	2 25	4 50	7 14	9 40	12 6	14 32	
58	0 0	2 22	4 43	7 4	9 27	11 49	14 12	
60	0 0	2 19	4 37	6 56	9 15	11 34	13 54	
62	0 0	2 16	4 31	6 48	9 4	11 21	13 37	
64	0 0	2 14	4 27	6 41	8 53	11 9	13 25	
66	0 0	2 12	4 22	6 34	8 44	10 58	13 10	
68	0 0	2 9	4 19	6 28	8 37	10 48	12 58	
70	0 0	2 8	4 13	6 22	8 31	10 39	12 45	
72	0 0	2 6	4 12	6 19	8 25	10 31	12 34	
74	0 0	2 5	4 9	6 15	8 20	10 25	12 30	
76	0 0	2 4	4 7	6 11	8 15	10 19	12 23	
80	0 0	2 2	4 4	6 5	8 8	10 10	12 11	

TABLE XV.

Altitude of the Sun at the Instant of his passing the prime vertical, or at that of his greatest Azimuth.

Latitude.	Declination of the same name as the latitude.						
	12°	14°	16°	18°	20°	22°	24°
0°	0° 0'	0° 0'	0° 0'	0° 0'	0° 0'	0° 0'	0° 0'
2	9 40	8 18	7 15	6 29	5 52	5 22	4 55
4	19 36	16 45	14 15	12 3	9 20	10 43	9 53
6	30 11	25 36	22 17	19 46	17 48	16 12	14 38
8	41 1	35 7	30 20	26 46	24 7	21 48	20 1
10	56 38	45 52	39 8	34 11	30 30	27 37	25 16
12	70 0	59 15	48 58	42 17	37 26	33 42	30 44
14	89 15	90 0	61 22	51 36	45 1	40 13	36 30
16	108 59	61 23	90 0	63 6	50 12	47 22	42 39
18	124 17	51 32	63 8	90 0	64 37	53 11	49 26
20	137 27	45 2	53 43	64 40	90 0	65 11	57 13
22	148 43	40 14	47 22	53 35	65 56	90 0	67 55
24	158 45	36 30	42 40	49 27	57 15	67 6	90 0
26	168 19	33 30	38 58	44 50	51 18	58 44	68 7
28	176 17	31 1	35 57	41 10	46 47	52 57	60 3
30	184 34	28 56	33 57	38 11	43 10	48 32	54 26
32	193 6	27 10	31 20	35 41	40 12	45 0	50 8
34	201 50	25 38	29 32	33 33	37 43	42 11	46 40
36	209 43	24 18	27 58	31 43	35 35	39 15	43 48
38	217 44	23 9	26 30	30 8	33 45	37 29	41 22
40	225 52	22 7	25 24	28 44	32 9	35 39	39 15
42	234 6	21 12	24 20	27 30	30 45	34 3	37 26
44	242 25	20 23	23 23	26 25	29 30	32 38	35 50
46	250 48	19 39	22 32	25 27	28 24	31 23	34 26
48	258 15	19 0	21 46	24 34	27 27	30 16	33 11
50	265 45	18 25	21 5	23 48	26 32	29 17	32 4
52	273 18	17 53	20 29	23 6	25 44	28 23	31 5
54	281 33	17 24	19 45	22 28	25 1	27 33	30 11
56	289 32	16 58	19 25	21 53	24 22	26 52	29 23
58	297 12	16 35	18 58	21 22	23 47	26 13	28 40
60	305 54	16 13	18 33	20 54	23 16	25 38	28 1
62	313 37	15 54	18 12	20 30	22 48	25 7	27 26
64	321 23	15 37	17 53	20 7	22 22	24 38	26 54
66	329 10	15 22	17 34	19 46	22 0	24 13	26 26
68	337 58	15 8	17 18	19 28	21 39	23 50	26 1
70	346 47	14 45	17 3	19 12	21 21	23 30	25 39
72	355 38	14 44	16 51	18 54	21 5	23 12	25 19
74	364 30	14 35	16 40	18 45	20 51	22 56	25 2
76	373 23	14 27	16 30	18 34	20 39	22 43	24 47
80	382 11	14 14	16 15	18 17	20 20	22 22	24 24

TABLE XVI.

Right Ascensions and Declinations of thirty-six of the principal fixed Stars, for the 1st of January, 1815, with their Annual Variations.

Names and characters.	Right ascension in sidereal time.	Annual variation.	Declination.	Annual variation.
Mag.	h. m. s.	+	° ' "	+
γ Pegasi 2	0 3 43.02	3.069	14 9 26.70 N.	+ 20.20
α Arietis 2.3	1 56 45.78	3.347	22 35 1.91 N.	+ 17.47
α Ceti 2	2 53 36.91	3.115	3 21 35.15 N.	+ 14.75
Aldebaran .. 1	4 25 18.84	3.426	16 7 43.40 N.	+ 8.00
Capella 1	5 3 2.33	4.415	45 47 47.01 N.	+ 4.57
Rigel 1	5 55 39.00	2.876	8 25 15.04 S.	- 4.92
β Tauri 2	5 14 36.20	3.781	28 36 27.73 N.	+ 3.91
α Orionis 1	5 45 9.42	3.243	7 21 51.57 N.	+ 1.49
Sirius 1	6 36 59.94	2.653	16 27 59.43 S.	+ 4.21
Castor 2	7 22 46.60	3.853	32 17 0.22 N.	- 7.06
Procyon 1.2	7 29 36.45	3.142	5 41 34.71 N.	- 8.53
Pollux 2	7 33 58.62	3.688	28 27 51.21 N.	- 7.93
α Hydra 2	9 18 29.60	2.946	7 51 35.60 S.	- 15.10
Regulus 1	9 58 30.56	3.212	12 52 7.03 N.	- 17.19
β Leonis 1.2	11 39 36.74	3.067	15 25 24.88 N.	- 20.04
β Virginis 3	11 41 3.39	3.125	2 33 30.44 N.	- 20.22
α Virginis 1	13 15 27.61	3.147	10 11 19.00 S.	+ 18.80
Arcturus 1	14 7 13.38	2.728	20 9 10.39 N.	- 18.74
α Libra 2	14 40 28.33	3.296	15 12 59.37 S.	+ 15.19
α Libra 2	14 40 39.66	3.297	15 15 43.73 S.	+ 15.21
α Corona 2.3	15 26 51.53	2.545	27 20 42.13 N.	- 12.49
α Serpentis 2	15 35 9.67	2.945	7 1 3.30 N.	- 11.70
Antares 1	16 18 4.98	3.658	26 0 20.79 S.	+ 8.43
α Hercules 2.3	17 6 12.91	2.781	14 36 46.16 N.	- 4.48
α Ophiuchi 2	17 26 20.90	2.716	12 42 20.61 N.	- 3.03
α Lyra 1	18 30 40.32	2.027	39 37 2.53 N.	+ 7.91
γ Aquilæ 3	19 37 27.53	2.646	10 10 22.14 N.	+ 8.38
α Aquilæ 1.2	19 41 45.15	2.925	8 23 20.63 N.	+ 9.11
β Aquilæ 3.4	19 46 13.34	2.944	5 57 18.41 N.	+ 8.57
α Capricorni 4	20 7 23.05	3.336	13 4 2.50 S.	- 10.80
α Capricorni 3	20 7 46.89	3.339	13 6 21.07 S.	- 10.81
α Crani 1.2	20 35 7.40	2.038	44 37 26.88 N.	+ 12.56
α Aquarii 3	21 56 16.48	3.081	1 12 39.42 S.	- 17.36
Fomalhaut 1.2	22 47 24.27	3.343	50 35 44.40 S.	- 19.42
α Pegasi 2	22 55 32.88	2.713	12 12 54.39 N.	+ 19.43
α Andromedæ 2	23 58 50.52	3.070	28 4 14.07 N.	+ 19.99

TABLE XVII.

Logarithms of Numbers and their Complements from 1 to 3500.

When the given number contains integer places, let the number of those places be denoted by n , then the

Index of the log. = $n - 1$; and, n being less than 1 (which it is in all common cases), the index of the comp. log. = $10 - n$; except when the given number is 10, 100, 1000, &c. and then it is $11 - n$.

And when the given number consists wholly of decimals, let d denote the number of places which the first efficient figure is from the decimal point, then the

Index of the log. = $-d$ and the index of comp. log. = $9 + d$.

Note.—From the places where the points occur in the logarithms, and the first figures of the numbers change from 0 to 1, in the complements, the two common figures in the next line are to be taken.

N.	Log.	Comp. log.	N.	Log.	Comp. log.	N.	Log.	Comp. log.
1	0.00000	10.00000	34	1.53138	8.46852	67	1.82607	8.17393
2	0.30103	9.69897	35	1.54407	8.45593	68	1.83251	8.16749
3	0.47712	9.52288	36	1.55630	8.44370	69	1.83885	8.16115
4	0.60206	9.39794	37	1.56820	8.43180	70	1.84510	8.15490
5	0.69897	9.30103	38	1.57978	8.42022	71	1.85126	8.14874
6	0.77815	9.22185	39	1.59106	8.40894	72	1.85733	8.14267
7	0.84510	9.15490	40	1.60206	8.39794	73	1.86332	8.13668
8	0.90309	9.09691	41	1.61278	8.38722	74	1.86923	8.13077
9	0.95424	9.04576	42	1.62325	8.37675	75	1.87506	8.12494
10	1.00000	9.00000	43	1.63347	8.36653	76	1.88081	8.11919
11	1.04139	8.95861	44	1.64345	8.35655	77	1.88649	8.11351
12	1.07918	8.92082	45	1.65321	8.34679	78	1.89209	8.10791
13	1.11394	8.88605	46	1.66276	8.33724	79	1.89763	8.10237
14	1.14613	8.85387	47	1.67210	8.32790	80	1.90309	8.09691
15	1.17609	8.82391	48	1.68124	8.31876	81	1.90848	8.09152
16	1.20412	8.79588	49	1.69020	8.30980	82	1.91381	8.08619
17	1.23042	8.76955	50	1.69897	8.30103	83	1.91908	8.08092
18	1.25527	8.74473	51	1.70757	8.29243	84	1.92428	8.07572
19	1.27875	8.72125	52	1.71600	8.28400	85	1.92942	8.07058
20	1.30103	8.69897	53	1.72428	8.27572	86	1.93450	8.06550
21	1.32222	8.67778	54	1.73239	8.26761	87	1.93952	8.06048
22	1.34240	8.65758	55	1.74036	8.25964	88	1.94448	8.05552
23	1.36173	8.63827	56	1.74819	8.25181	89	1.94939	8.05061
24	1.38021	8.61979	57	1.75587	8.24413	90	1.95424	8.04576
25	1.39794	8.60206	58	1.76343	8.23657	91	1.95904	8.04096
26	1.41497	8.58503	59	1.77085	8.22915	92	1.96379	8.03621
27	1.43136	8.56864	60	1.77815	8.22185	93	1.96848	8.03152
28	1.44716	8.55284	61	1.78533	8.21467	94	1.97313	8.02687
29	1.46240	8.53760	62	1.79239	8.20761	95	1.97772	8.02228
30	1.47712	8.52288	63	1.79934	8.20066	96	1.98227	8.01773
31	1.49136	8.50864	64	1.80618	8.19382	97	1.98677	8.01323
32	1.50515	8.49485	65	1.81291	8.18709	98	1.99123	8.00877
33	1.51854	8.48149	66	1.81954	8.18046	99	1.99563	8.00437

										Complements.										
N	0	1	2	3	4	5	6	7	8	9	C	1	2	3	4	5	6	7	8	9
100	00	043	087	130	173	216	260	303	346	389	00000	957	913	870	827	783	740	697	654	611
101	432	475	518	561	604	647	689	732	775	817	99568	525	482	438	396	353	311	268	225	183
102	860	903	945	988	1031	1074	1117	1159	1202	1245	140097	035	012	070	928	885	843	801	759	717
103	01284	326	368	410	452	494	536	578	620	662	98716	674	632	590	548	506	464	422	380	338
104	703	745	787	828	870	912	953	995	1037	1079	297	255	213	172	130	088	047	005	964	922
105	02119	160	202	244	285	327	368	409	450	491	97881	840	798	757	716	675	634	593	551	510
106	531	571	612	653	694	735	776	816	857	898	469	428	388	347	306	265	224	184	143	103
107	938	979	1020	1061	1101	1141	1181	1222	1262	1302	062	021	981	940	900	859	819	778	738	698
108	03342	383	423	463	503	543	583	623	663	703	266	226	187	147	107	067	027	007	967	927
109	743	782	822	862	902	941	981	1021	1061	1101	857	816	776	736	696	656	616	576	536	496
110	01139	179	218	258	297	336	375	415	454	493	95361	821	782	742	703	664	625	585	546	507
111	532	571	610	649	688	727	766	805	844	883	478	437	396	355	314	273	234	195	156	117
112	922	961	999	1038	1077	1115	1154	1192	1231	1269	078	039	001	962	923	885	846	806	769	731
113	03308	346	385	423	461	500	538	576	614	652	94692	654	615	577	539	500	461	422	383	344
114	690	729	767	805	843	880	918	956	994	1032	310	271	233	195	157	120	082	044	006	968
115	06070	107	145	183	221	258	296	333	371	408	93930	892	855	817	779	742	704	667	629	592
116	448	483	521	558	595	633	670	707	744	781	554	517	479	442	405	367	330	293	256	219
117	819	856	893	930	967	1004	1041	1078	1114	1151	181	144	107	070	033	996	959	922	886	849
118	07188	225	262	299	335	372	408	444	482	518	92812	775	736	696	656	618	582	545	518	482
119	535	591	628	664	700	737	773	809	846	882	445	409	372	336	300	263	227	191	154	118
120	918	954	990	1027	1063	1099	1135	1171	1207	1243	082	046	010	973	937	901	865	830	793	757
121	08276	314	350	386	422	458	493	529	565	600	91722	686	650	614	578	542	507	471	435	400
122	636	672	707	743	778	814	849	884	920	955	364	328	293	257	222	186	151	116	080	045
123	990	1026	1061	1096	1131	1167	1202	1237	1272	1307	010	9	939	904	869	833	798	763	728	693
124	09342	377	412	447	482	517	552	587	621	656	90658	620	585	553	518	483	448	413	379	344
125	691	728	765	795	830	864	899	933	968	1003	309	274	239	205	170	136	101	067	032	997
126	10037	071	108	144	175	209	243	278	312	346	89963	929	894	860	825	791	757	721	688	654
127	380	415	449	483	517	551	585	619	653	687	620	585	551	517	483	449	415	381	347	313
128	721	755	788	823	856	890	924	958	992	1025	279	245	211	177	144	110	076	042	008	975
129	11059	099	126	160	193	227	260	294	327	361	68941	907	874	840	807	773	740	706	673	639
130	394	428	461	494	528	561	594	628	661	694	606	572	539	506	472	439	406	372	339	306
131	727	760	793	826	859	893	926	959	991	1024	273	240	207	174	141	107	074	041	009	976
132	12057	090	123	156	189	222	254	287	320	352	87943	910	877	844	811	778	746	713	680	646
133	383	418	450	483	516	548	581	613	646	678	615	582	550	517	484	452	419	387	354	322
134	710	743	775	808	840	872	904	937	969	1001	290	257	225	192	160	128	096	063	031	999
135	13033	065	096	130	162	194	226	258	290	322	88567	935	902	870	838	806	774	742	710	678
136	354	386	418	450	481	513	545	577	609	640	646	614	582	550	519	487	455	423	391	359
137	672	704	736	767	799	830	862	893	925	956	466	436	406	375	345	315	285	254	224	194
138	988	1019	1051	1082	1113	1144	1175	1206	1237	1268	912	881	849	818	786	755	724	693	662	631
139	14301	353	384	415	446	477	508	539	570	601	85699	667	636	605	574	543	512	480	449	418
140	613	644	675	706	737	768	798	829	860	891	387	356	325	294	263	232	202	171	140	109
141	922	953	983	1014	1045	1076	1106	1137	1168	1198	075	047	017	986	955	924	894	863	832	802
142	52229	259	290	320	351	381	412	442	473	503	84771	741	710	680	649	618	588	558	527	497
143	534	564	594	625	655	685	715	746	776	806	466	436	406	375	345	315	285	254	224	194
144	836	866	896	927	957	987	1017	1047	1077	1107	164	134	104	073	043	013	983	953	923	893
145	16137	167	197	227	258	288	318	348	378	408	83863	833	803	773	743	713	684	654	624	595
146	486	465	495	524	554	584	613	643	673	702	505	475	445	415	385	355	325	295	265	235
147	732	761	791	820	850	879	909	938	967	997	268	239	209	180	150	121	091	062	033	005
148	17024	055	085	114	143	173	202	231	260	289	82974	943	913	883	853	823	793	763	733	703
149	519	548	577	606	635	664	693	722	751	780	681	652	623	594	565	535	507	477	447	417

Logarithms.											Complements.										
N	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	
150	17609	638	667	696	725	754	782	811	840	869	82391	362	333	304	275	246	218	189	160	131	
151	898	926	955	984	13	41	70	99	127	156	102	074	045	016	987	959	930	901	873	844	
152	18184	213	241	270	298	327	355	384	412	441	81810	787	759	730	702	673	644	616	588	559	
153	469	497	526	554	582	611	639	667	696	724	581	503	474	446	418	389	361	333	304	276	
154	752	780	808	837	865	893	921	949	977	1000	248	220	192	163	135	107	079	051	023	995	
155	19033	061	089	117	145	173	201	229	257	285	80967	939	911	883	855	827	799	771	743	715	
156	312	340	368	396	424	451	479	507	535	562	688	660	632	604	576	549	521	493	465	438	
157	590	618	645	673	700	728	756	783	811	838	410	382	353	327	300	272	244	217	189	162	
158	866	893	921	948	975	1000	30	58	85	112	194	107	079	052	025	997	970	942	915	888	
159	20140	167	194	222	249	276	303	330	358	385	79860	833	806	778	751	724	697	670	642	615	
160	412	439	466	493	520	547	575	602	629	656	588	561	534	507	480	453	425	398	371	344	
161	689	710	736	763	790	817	844	871	898	925	317	290	264	237	210	183	156	129	102	075	
162	951	972	993	1000	1000	1000	1000	1000	1000	1000	049	022	995	968	941	915	888	861	835	808	
163	21219	245	272	299	325	352	378	405	431	458	78781	755	728	701	675	648	622	595	569	542	
164	494	511	537	564	590	617	643	669	696	722	516	489	463	436	410	383	357	331	304	278	
165	748	775	801	827	854	880	906	932	958	985	252	225	199	173	146	120	094	068	042	015	
166	22011	037	063	089	115	141	167	194	220	246	77989	968	937	911	885	859	833	807	780	754	
167	272	298	324	350	375	401	427	453	479	505	728	702	676	650	625	599	573	547	521	495	
168	581	607	633	658	684	710	736	762	788	814	469	443	417	392	366	340	314	289	263	237	
169	789	814	840	866	891	917	943	968	994	1000	211	186	160	134	109	083	057	032	006	981	
170	23045	070	096	121	147	172	198	223	249	274	76955	930	904	879	853	828	802	777	751	726	
171	300	325	350	376	401	426	452	477	502	528	700	675	650	624	599	574	548	523	498	472	
172	553	578	603	628	654	679	704	729	754	779	447	422	397	372	346	321	296	271	246	221	
173	805	830	855	880	905	930	955	980	1000	1000	195	170	145	120	095	070	045	020	995	970	
174	24055	080	105	130	155	179	204	229	254	279	75945	920	895	870	845	821	796	771	746	721	
175	304	329	353	378	403	428	452	477	502	527	696	671	647	622	597	572	548	523	498	473	
176	551	576	601	625	650	674	699	724	748	773	449	424	399	375	350	326	301	276	252	227	
177	797	822	846	871	895	920	944	969	993	1000	203	178	154	129	105	080	056	031	007	982	
178	25049	066	091	115	139	164	188	212	237	261	74058	934	909	885	861	836	812	788	763	739	
179	285	310	334	358	382	406	431	455	479	503	715	690	666	642	618	594	569	545	521	497	
180	527	551	575	600	624	648	672	696	720	744	473	449	425	400	376	352	328	304	280	256	
181	762	792	816	840	864	888	912	935	959	983	232	208	184	160	136	112	088	065	041	017	
182	26007	031	055	079	102	126	150	174	198	221	73993	969	945	921	898	874	850	826	802	779	
183	245	269	292	316	340	364	387	411	435	458	755	731	708	684	660	636	613	589	565	542	
184	489	505	529	552	576	600	623	647	670	694	518	495	471	448	424	400	377	353	330	306	
185	717	741	764	787	811	834	858	881	905	928	283	259	236	213	189	166	142	119	095	072	
186	951	975	998	1000	1000	1000	1000	1000	1000	1000	409	025	002	979	955	932	909	886	862	839	
187	27184	207	231	254	277	300	323	346	370	393	7216	793	769	746	723	700	677	654	630	607	
188	416	440	462	485	508	531	554	577	600	623	584	560	538	515	497	469	446	423	400	377	
189	646	669	692	715	738	761	784	807	830	853	354	331	308	285	262	239	216	193	170	148	
190	875	898	921	944	967	989	1000	1000	1000	1000	125	102	079	056	033	011	988	965	942	919	
191	28103	126	149	171	194	217	240	262	285	307	71897	874	851	828	806	783	760	738	715	693	
192	330	353	375	398	420	443	466	488	511	533	670	647	623	602	580	557	534	512	489	467	
193	556	578	601	623	646	668	690	713	735	758	444	420	396	377	354	332	310	287	265	242	
194	780	803	825	847	870	892	914	937	959	981	220	197	173	153	130	108	086	063	041	019	
195	29003	026	048	070	092	115	137	159	181	203	70997	974	952	930	908	885	863	841	819	797	
196	226	248	270	292	314	336	358	380	402	425	774	752	730	708	686	664	642	620	598	575	
197	447	469	491	513	535	557	579	601	623	645	553	531	509	487	465	443	421	399	377	355	
198	666	688	710	732	754	776	798	820	842	865	334	312	290	268	246	224	202	180	158	137	
199	885	907	929	951	972	994	1000	1000	1000	1000	115	093	071	049	028	006	984	962	941	919	

Logarithms.											Complements.										
N	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	
200	36403	125	146	168	190	211	233	255	276	298	69897	875	854	832	810	789	767	745	724	702	
201	320	341	363	384	406	427	449	471	492	514	680	659	637	616	594	573	551	529	508	486	
202	525	547	570	600	621	642	664	685	707	728	485	443	422	400	379	358	336	315	293	272	
203	750	771	792	814	835	856	878	899	920	942	250	229	208	188	165	144	122	101	080	068	
204	963	984	..	6.	27.	48.	69.	91.	112	133	037	016	994	973	952	931	909	888	867	846	
205	31175	197	218	239	260	281	302	323	344	366	68825	803	782	761	740	719	698	677	656	634	
206	387	408	429	440	471	492	513	534	555	576	613	592	571	550	529	508	487	466	445	424	
207	597	618	639	660	681	702	723	744	765	785	405	382	361	340	319	298	277	256	235	213	
208	806	827	848	869	890	911	931	952	973	994	194	173	152	131	110	089	069	048	027	006	
209	32045	035	056	077	098	118	139	160	180	201	67985	965	944	923	902	882	861	840	820	799	
210	222	243	263	284	305	325	346	366	387	408	778	757	737	716	695	675	654	634	613	592	
211	428	449	469	490	510	531	552	572	593	613	572	551	531	510	490	469	448	428	407	387	
212	634	654	674	695	715	736	756	777	797	818	366	346	326	305	285	264	244	223	203	182	
213	838	858	879	899	919	940	960	980	..	1. 21	162	142	121	101	081	060	040	020	999	979	
214	33041	062	082	102	122	143	163	183	203	224	66950	938	918	898	878	857	837	827	797	776	
215	244	264	284	304	325	345	365	385	405	425	756	736	716	696	675	655	635	615	595	575	
216	445	465	486	506	526	546	566	586	606	626	555	535	514	494	474	454	434	414	394	374	
217	646	666	686	706	726	746	766	786	806	826	354	334	314	294	274	254	234	214	194	174	
218	846	866	886	906	925	945	965	985	..	5. 23	154	134	115	095	075	055	035	015	995	975	
219	34044	064	084	104	124	143	163	183	203	222	65956	936	916	896	876	857	837	817	797	778	
220	242	262	282	301	321	341	361	380	400	420	758	738	718	699	679	659	639	620	600	580	
221	439	459	478	498	518	537	557	577	596	616	561	541	522	502	482	463	443	423	404	384	
222	635	655	674	694	713	733	752	772	791	811	365	345	326	306	287	267	248	228	209	189	
223	830	850	869	889	908	928	947	967	986	..	5	170	150	131	111	092	072	053	033	014	
224	35025	044	064	083	102	122	141	160	180	199	64975	956	936	917	898	878	859	840	820	801	
225	218	237	257	276	295	315	334	353	372	392	782	762	743	724	705	685	666	647	628	608	
226	411	430	449	468	488	507	526	545	564	583	589	570	551	532	512	493	474	455	436	417	
227	603	622	641	660	679	698	717	736	755	774	397	378	359	340	321	302	283	264	245	226	
228	793	812	832	851	870	889	908	927	946	965	207	188	168	149	130	111	092	073	054	035	
229	983	..	2. 21	40	59	78	97	116	135	154	017	998	979	960	941	922	903	884	865	846	
230	36173	192	210	229	248	267	286	305	324	342	63827	308	290	271	252	233	214	195	176	158	
231	361	380	399	418	436	455	474	493	511	530	639	620	601	582	564	545	526	507	489	470	
232	549	567	586	605	624	642	661	680	698	717	451	433	414	395	376	358	339	320	302	283	
233	736	754	773	791	810	829	847	866	884	903	264	246	227	209	190	171	153	134	116	097	
234	932	940	959	977	996	..	14.	33.	51.	70.	078	060	041	023	004	986	967	949	930	912	
235	37107	125	144	162	181	199	217	236	254	273	62893	875	856	838	829	811	793	764	746	727	
236	291	310	328	346	365	383	401	420	438	456	709	690	672	654	635	617	599	580	562	544	
237	475	495	511	530	548	566	585	603	621	639	525	507	489	470	452	434	415	397	379	361	
238	658	676	694	712	731	749	767	785	803	822	342	324	306	288	269	251	233	215	197	178	
239	840	858	876	894	912	931	949	967	985	..	3	160	142	124	106	088	069	051	033	015	
240	38021	039	057	075	093	111	130	148	166	184	61979	961	943	925	907	889	870	852	834	826	
241	202	220	238	256	274	292	310	328	346	364	798	780	762	744	726	708	690	672	654	636	
242	381	399	417	435	453	471	489	507	525	543	619	601	583	565	547	529	511	493	475	457	
243	561	578	596	614	632	650	668	686	703	721	439	422	404	386	368	350	332	314	297	279	
244	739	757	775	792	810	828	846	863	881	899	261	243	225	208	190	172	154	137	119	101	
245	917	934	952	970	987	..	3. 23.	40.	58.	76.	083	066	048	030	013	995	977	960	942	924	
246	39093	111	129	146	164	182	199	217	234	252	60907	889	871	854	836	818	801	783	766	748	
247	270	287	305	322	340	357	375	393	410	428	730	713	695	678	660	643	625	607	590	572	
248	446	463	480	498	515	533	550	568	585	602	555	537	520	502	485	467	450	432	415	398	
249	630	637	655	672	690	707	724	742	759	776	380	363	345	328	310	293	276	258	241	224	

Logarithms.											Complements.										
N	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	
250	99794	811	829	846	853	881	898	915	933	950	60200	189	171	154	137	119	102	885	797	650	
251	967	985	2	19	36	54	71	88	106	123	033	015	998	981	964	946	929	912	894	877	
252	46140	145	174	192	209	226	243	260	278	296	59860	843	826	808	791	774	757	740	722	705	
253	912	829	846	853	881	898	915	932	449	466	698	671	654	637	619	602	585	568	551	534	
254	483	500	518	535	552	569	586	603	620	637	517	500	482	465	448	431	414	397	380	363	
255	654	671	688	705	722	739	756	773	790	807	546	529	512	495	478	461	444	427	410	393	
256	824	841	858	875	892	909	926	943	959	976	176	159	142	125	108	91	74	57	41	24	
257	993	10	27	44	61	78	95	111	128	145	007	990	973	956	939	922	905	889	872	855	
258	41169	179	196	212	229	246	263	280	297	313	59838	821	804	788	771	754	737	720	704	687	
259	330	347	363	380	397	414	430	447	464	481	670	653	637	620	603	586	570	553	536	519	
260	497	514	531	547	564	581	597	614	631	647	503	486	469	453	436	419	403	386	369	353	
261	664	681	697	714	731	747	764	780	797	813	336	319	303	286	269	253	236	220	203	187	
262	830	847	863	880	896	913	929	946	962	979	170	153	137	120	104	87	71	54	38	21	
263	996	1	18	35	52	69	86	103	120	137	064	988	971	955	938	922	906	889	873	856	
264	42160	127	143	210	226	243	259	275	292	308	57840	823	807	790	774	757	741	725	708	692	
265	525	541	557	574	590	606	623	639	655	672	675	659	643	626	610	594	577	561	545	528	
266	488	504	521	537	553	570	586	602	619	635	512	496	479	463	447	430	414	398	381	365	
267	651	667	684	700	716	732	749	765	781	797	349	332	316	300	284	268	251	235	219	203	
268	813	830	846	862	878	894	911	927	943	959	187	170	154	138	122	106	89	73	57	41	
269	975	991	1	17	34	50	67	82	88	104	025	009	993	976	959	944	928	912	896	880	
270	43186	152	168	185	201	217	233	249	265	281	56864	858	839	815	799	783	767	751	735	719	
271	297	313	329	345	361	377	393	409	425	441	702	687	671	655	639	623	607	591	575	559	
272	457	473	489	505	521	537	553	568	584	600	543	527	511	495	479	463	447	432	416	400	
273	616	632	648	664	680	696	712	727	743	759	384	368	352	336	320	304	288	273	257	241	
274	775	791	807	823	838	854	870	886	902	917	225	209	193	177	162	146	130	114	98	83	
275	933	949	965	981	996	1	12	28	44	59	7	067	051	035	019	004	988	972	956	941	
276	44091	107	122	138	154	169	185	201	217	232	5909	893	878	862	846	831	815	799	783	768	
277	248	264	279	295	311	326	342	358	375	389	752	736	721	705	689	674	658	642	627	611	
278	404	420	436	451	467	482	498	514	529	545	596	580	564	548	533	518	502	486	471	455	
279	560	575	591	607	623	638	654	669	685	700	440	424	409	393	377	362	346	331	315	300	
280	716	731	747	762	778	793	809	824	840	855	924	909	893	878	862	846	831	815	799	783	
281	871	886	901	917	932	948	963	979	994	1	54129	114	109	089	078	052	037	021	006	991	
282	45025	040	056	071	086	102	117	133	148	163	975	960	944	929	914	898	883	867	852	837	
283	179	194	209	225	240	255	271	286	301	316	821	806	791	775	760	745	729	714	699	684	
284	332	347	362	378	393	408	423	439	454	469	668	653	638	622	607	592	577	561	546	531	
285	484	500	515	530	545	561	576	591	606	621	516	500	485	470	455	439	424	409	394	379	
286	637	652	667	682	697	712	728	743	758	773	363	348	333	318	303	288	272	257	242	227	
287	788	803	818	834	849	864	879	894	909	924	212	197	182	166	151	136	121	106	91	76	
288	939	954	969	984	999	1	15	30	45	60	061	046	031	016	001	985	970	955	940	925	
289	46090	105	120	135	150	165	180	195	210	225	5910	895	880	865	850	835	820	805	790	775	
290	240	255	270	285	300	315	330	344	359	374	760	745	730	715	700	685	670	655	641	626	
291	389	404	419	434	449	464	479	494	509	523	611	596	581	566	551	536	521	506	492	477	
292	538	553	568	583	598	613	627	642	657	672	462	447	432	417	402	387	373	358	343	328	
293	687	702	716	731	746	761	776	790	805	820	313	298	284	269	254	239	224	210	195	180	
294	825	849	864	879	894	908	923	938	953	967	165	151	136	121	106	92	77	62	47	33	
295	982	997	1	12	26	41	56	70	85	100	018	003	988	974	969	954	939	915	900	886	
296	47129	144	158	173	188	202	217	232	246	261	52871	856	842	827	812	798	783	768	754	739	
297	278	293	308	323	338	353	368	383	398	413	724	710	695	681	666	651	637	622	608	593	
298	422	436	451	465	480	494	509	523	538	553	572	564	549	535	520	506	491	477	462	447	
299	567	582	596	611	625	640	654	669	683	698	433	418	404	389	375	360	346	331	317	302	

Logarithms.											Complements.										
N	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	
300	712	727	741	755	770	784	799	813	828	842	5288	273	259	245	230	216	201	187	172	158	
301	857	871	885	900	914	929	943	957	972	986	143	129	115	100	085	071	057	043	028	014	
302	1001	015	029	044	058	072	087	101	116	130	5199	985	971	956	942	928	913	899	884	870	
303	144	159	172	187	202	216	230	244	259	273	656	841	827	812	796	784	770	756	741	727	
304	287	302	316	330	344	359	373	387	401	416	713	698	684	670	656	641	627	613	599	584	
305	459	473	488	503	517	531	546	560	574	589	570	556	542	527	513	499	485	471	456	442	
306	572	586	600	615	629	643	657	671	685	700	428	414	400	385	371	357	344	329	315	300	
307	714	728	742	756	770	784	799	813	827	842	286	272	258	244	230	216	201	187	173	159	
308	855	869	883	897	911	925	940	954	968	982	145	131	117	103	089	075	060	046	032	018	
309	996	10	24	38	52	66	80	94	108	122	004	990	976	962	948	934	920	906	892	878	
310	191	206	219	233	247	261	275	289	303	317	5086	485	471	458	444	431	417	404	390	377	
311	276	290	304	318	332	346	360	374	388	401	724	710	696	682	668	654	640	626	612	599	
312	415	429	443	457	471	485	499	513	527	541	585	571	557	543	529	515	501	487	473	459	
313	554	568	582	596	610	624	638	651	665	679	446	432	418	404	390	376	362	349	335	321	
314	693	707	721	734	748	762	776	790	805	817	307	293	279	265	252	238	224	210	197	183	
315	831	845	859	872	886	900	914	927	941	955	169	155	141	128	114	100	086	073	059	045	
316	969	982	996	10	24	37	51	65	78	92	031	018	004	990	976	963	949	935	922	908	
317	506	520	533	547	561	574	588	602	615	629	4989	480	467	453	439	426	412	398	385	371	
318	243	256	270	284	297	311	325	338	352	365	757	744	730	715	703	689	675	662	648	635	
319	379	393	406	420	434	447	461	474	488	501	621	607	594	580	567	553	539	526	512	499	
320	515	529	542	556	569	583	596	610	623	637	485	471	458	444	431	417	404	390	377	363	
321	650	664	678	691	705	718	732	745	759	772	350	336	322	309	295	282	268	255	241	228	
322	786	799	813	826	839	853	866	880	893	907	214	201	187	174	161	147	134	120	107	093	
323	920	934	947	961	974	987	10	14	28	41	080	066	053	039	026	013	999	986	972	959	
324	510	524	538	551	565	578	592	605	619	632	4894	932	919	905	892	879	865	851	838	825	
325	188	202	215	228	242	255	268	282	295	308	812	798	785	772	758	745	732	718	705	692	
326	322	335	348	362	375	388	402	415	428	441	678	665	652	638	625	612	598	585	572	559	
327	455	468	481	495	508	521	534	548	561	574	545	532	519	505	492	478	466	452	439	426	
328	587	601	614	627	640	653	667	680	693	706	413	399	386	373	360	347	333	320	307	294	
329	720	733	746	759	772	785	799	812	825	838	280	267	254	241	228	215	201	188	175	162	
330	851	865	878	891	904	917	930	943	957	970	149	135	122	109	096	083	070	057	043	030	
331	983	996	10	22	33	44	55	66	77	88	017	003	991	978	965	952	939	926	912	899	
332	114	127	140	153	166	179	192	205	218	231	4788	873	860	847	834	821	808	795	782	769	
333	244	257	270	283	297	310	323	336	349	362	756	743	730	717	703	690	677	664	651	638	
334	375	388	401	414	427	440	453	466	478	491	635	622	609	596	583	569	557	544	532	519	
335	504	517	530	543	556	569	582	595	608	621	496	483	470	457	444	431	418	405	392	379	
336	634	647	660	673	686	698	711	724	737	750	366	353	340	327	314	302	289	276	263	250	
337	763	776	789	802	814	827	840	853	866	879	237	224	211	198	186	173	160	147	134	121	
338	892	904	917	930	943	956	969	981	994	10	108	096	083	070	057	044	031	019	006	993	
339	530	543	556	569	582	595	608	621	634	647	4690	967	954	942	929	916	903	890	878	865	
340	148	161	173	186	199	212	224	237	250	263	852	839	827	814	801	788	776	763	750	737	
341	275	288	301	314	326	339	352	364	377	390	723	710	697	686	674	661	648	636	622	610	
342	403	415	428	441	453	466	479	491	504	517	597	585	572	559	547	534	521	509	496	483	
343	529	542	555	567	580	593	605	618	631	643	471	458	445	433	420	407	395	382	369	357	
344	656	668	681	694	706	719	731	744	757	769	344	332	319	306	294	281	269	256	242	231	
345	782	794	807	820	832	845	857	870	882	895	218	206	193	180	166	153	143	130	118	105	
346	908	920	933	945	958	970	983	995	10	120	092	080	067	055	042	030	017	005	992	980	
347	540	553	565	578	590	603	615	628	640	653	4596	955	942	930	917	905	892	880	867	855	
348	158	170	183	195	208	220	233	245	258	270	842	830	817	805	792	780	767	755	742	730	
349	282	295	307	320	332	345	357	370	382	394	718	705	693	680	668	655	643	630	618	606	

Logarithms.											Complements.										
N	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	
350	54497	419	332	444	156	469	481	494	506	518	455	381	568	558	548	538	518	506	494	482	
351	531	543	555	568	580	592	605	617	630	642	469	457	445	432	420	408	395	383	370	358	
352	634	650	679	691	704	716	728	740	753	765	346	333	321	309	296	284	272	260	247	235	
353	777	790	802	814	826	838	851	863	876	888	223	210	198	186	173	161	149	137	124	112	
354	900	913	925	937	949	962	974	986	998	111	100	988	975	963	951	938	926	914	902	889	
355	55023	035	047	059	072	084	096	108	121	133	449	777	965	953	941	928	916	904	892	879	
356	145	157	169	182	194	206	218	230	242	255	845	843	831	818	806	794	782	770	758	745	
357	267	279	291	303	315	328	340	352	364	376	753	721	709	697	685	672	660	648	636	624	
358	386	400	413	425	437	449	461	473	485	497	612	600	587	575	563	551	539	527	515	503	
359	509	521	534	546	558	570	582	594	606	618	491	479	466	454	442	430	418	406	394	382	
360	630	642	654	666	678	690	702	715	727	739	370	358	346	334	322	310	297	285	273	261	
361	751	763	775	787	799	811	823	835	847	859	249	237	225	213	201	189	177	165	153	141	
362	871	883	895	907	919	931	943	955	967	979	129	117	105	093	081	069	057	045	033	021	
363	991	103	115	126	138	150	162	174	186	198	099	987	985	974	962	950	938	926	914	902	
364	56140	129	134	146	158	170	182	194	205	217	498	878	866	854	842	830	818	806	795	783	
365	229	241	253	265	277	289	301	313	324	336	771	759	747	735	723	711	699	688	676	664	
366	348	360	372	384	395	407	419	431	443	455	654	640	628	616	605	593	581	569	557	545	
367	467	478	490	502	514	526	538	549	561	575	553	522	510	498	486	474	462	451	439	427	
368	585	597	608	620	632	644	655	667	679	691	415	403	392	380	368	356	345	333	321	309	
369	703	714	726	738	750	761	773	785	797	808	297	286	274	262	250	239	227	215	203	192	
370	820	832	844	855	867	879	890	902	914	926	180	168	156	145	133	121	110	098	086	074	
371	937	949	961	972	984	996	1008	1019	1031	1043	063	051	039	028	016	004	992	981	969	957	
372	57054	066	078	089	101	113	124	136	148	159	429	46	934	922	911	899	887	876	864	852	
373	171	182	194	206	217	229	241	252	264	275	829	818	806	794	782	771	759	748	736	725	
374	287	299	310	322	334	345	357	368	380	391	713	701	690	678	666	655	643	632	620	609	
375	403	415	426	438	449	461	473	484	496	507	597	585	574	562	551	539	527	516	504	493	
376	519	530	542	553	565	576	588	600	611	623	481	470	458	447	435	424	412	400	389	377	
377	634	646	657	669	680	692	703	715	726	738	366	354	343	331	320	308	297	285	274	262	
378	749	761	772	784	795	807	818	829	841	852	251	239	227	216	205	193	182	171	159	148	
379	864	875	887	898	910	921	933	944	955	967	136	125	113	102	090	079	067	056	045	033	
380	978	989	1000	1011	1022	1033	1044	1055	1066	1077	022	010	999	987	976	965	953	942	930	919	
381	58092	104	115	127	138	149	161	172	183	195	419	896	885	873	862	851	839	828	816	805	
382	206	218	229	240	252	263	274	286	297	308	794	782	771	760	748	737	726	714	703	692	
383	320	331	343	354	365	376	388	399	410	422	680	669	657	646	635	624	612	601	590	578	
384	433	444	456	467	478	490	501	512	523	535	567	556	544	533	522	510	499	488	477	465	
385	546	557	569	580	591	602	614	625	636	647	454	442	431	420	409	398	386	375	364	353	
386	659	670	681	692	704	715	726	737	749	760	341	330	319	308	296	285	274	263	251	240	
387	771	782	793	805	816	827	838	850	861	872	229	218	207	195	184	173	162	150	139	128	
388	883	894	906	917	928	939	950	961	973	984	117	106	094	083	072	061	050	039	027	016	
389	995	1006	1017	1028	1039	1050	1061	1072	1083	1094	005	994	983	972	960	949	938	927	916	905	
390	59106	118	129	140	151	162	173	184	195	207	408	892	881	870	859	848	838	827	816	805	
391	218	229	240	251	262	273	284	295	306	317	782	771	760	749	738	727	716	705	694	683	
392	329	340	351	362	373	384	395	406	417	428	660	649	638	627	616	605	594	583	572	561	
393	439	450	461	472	483	494	505	516	527	538	561	550	539	528	517	506	495	484	472	461	
394	550	561	572	583	594	605	616	627	638	649	460	449	438	427	416	405	395	384	373	362	
395	660	671	682	693	704	715	726	737	748	759	340	329	318	307	296	285	274	263	252	241	
396	769	780	791	802	813	824	835	846	857	868	231	220	209	198	187	176	165	154	143	132	
397	879	890	901	912	923	934	945	956	966	977	121	110	099	088	077	066	055	044	033	023	
398	988	999	1010	1021	1032	1043	1054	1065	1076	1087	012	001	990	979	968	957	946	935	925	914	
399	60097	108	119	130	141	152	163	174	185	196	399	892	881	870	859	848	838	827	816	805	

Logarithms.											Complements.										
N	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	
400	60206	217	928	239	249	260	271	282	293	304	39724	783	772	761	751	740	729	718	707	696	
401	314	825	336	847	357	369	379	390	401	412	686	675	664	653	643	631	621	610	599	588	
402	423	433	444	455	466	477	487	498	509	520	577	567	556	545	534	523	513	502	491	480	
403	530	541	552	563	574	585	595	606	617	627	470	459	448	437	426	416	405	394	383	373	
404	638	648	659	670	681	692	703	713	724	735	362	351	340	330	319	308	297	287	276	265	
405	745	756	767	778	788	799	810	820	831	842	253	244	233	222	212	201	190	179	168	158	
406	853	863	874	885	895	906	917	927	938	949	147	137	126	115	105	094	083	072	062	051	
407	959	970	981	991	..	2	13	23	34	44	041	030	019	008	..	987	977	966	955	945	
408	61066	077	087	098	109	119	130	140	151	162	38034	923	913	902	891	881	870	859	848	838	
409	172	183	194	204	215	225	236	246	257	269	826	817	806	795	785	775	764	753	743	731	
410	278	289	300	310	321	331	342	352	363	374	722	711	700	690	679	669	658	648	637	626	
411	384	395	405	416	426	437	447	458	469	479	616	605	595	584	574	563	553	543	531	521	
412	490	500	511	521	532	542	553	563	574	584	519	509	498	479	468	458	447	437	426	416	
413	595	605	616	626	637	648	658	669	679	689	405	395	384	374	363	353	344	331	321	311	
414	700	710	721	731	742	752	763	773	784	794	300	290	279	269	258	248	237	227	216	206	
415	805	815	826	836	847	857	868	878	888	899	195	185	174	164	153	143	132	122	111	101	
416	909	920	930	941	951	961	972	982	993	..	091	080	070	059	049	039	028	018	007	997	
417	62014	024	034	045	055	066	076	086	096	107	37986	976	966	955	945	934	924	913	903	893	
418	118	128	138	149	159	169	180	190	201	211	882	872	862	851	841	831	820	810	799	789	
419	221	232	242	252	263	273	283	294	304	315	779	768	758	747	737	727	717	706	696	685	
420	325	335	346	356	366	377	387	397	408	418	675	665	654	644	634	623	613	603	592	582	
421	428	438	449	459	469	480	490	500	511	521	572	562	551	541	531	520	510	500	489	479	
422	531	541	552	562	572	583	593	603	613	624	469	459	448	438	428	418	407	397	387	376	
423	634	644	655	665	675	685	696	706	716	726	366	356	345	335	325	315	304	294	284	274	
424	737	747	757	767	777	788	798	808	818	829	263	253	243	233	223	212	202	192	182	171	
425	839	849	859	869	880	890	900	910	921	931	161	151	141	131	121	110	100	090	079	069	
426	941	951	961	971	982	992	..	2	12	22	059	049	039	029	019	008	998	988	978	967	
427	63704	053	063	073	083	094	101	114	124	134	36957	947	937	927	917	906	896	886	876	866	
428	144	151	165	175	185	195	205	215	225	236	856	846	835	825	815	805	795	785	775	764	
429	246	256	266	276	286	296	306	316	327	337	754	744	734	724	714	704	694	684	673	663	
430	317	327	337	347	357	367	377	387	397	407	653	643	633	623	613	603	593	583	572	563	
431	448	458	468	478	488	498	508	518	528	538	552	542	532	522	512	502	492	482	472	462	
432	548	558	568	578	589	599	609	619	629	639	452	442	432	422	411	401	391	381	371	361	
433	649	659	669	679	689	699	709	719	729	739	351	341	331	321	311	301	291	281	271	261	
434	749	759	769	779	789	799	809	819	829	839	257	247	237	227	217	207	197	187	177	167	
435	849	859	869	879	889	899	909	919	929	939	151	141	131	121	111	101	091	081	071	061	
436	949	959	969	979	989	998	..	8	18	28	051	041	031	022	012	002	992	982	972	962	
437	64048	058	068	078	088	098	108	118	128	137	35952	942	932	922	912	902	892	882	872	863	
438	147	157	167	177	187	197	207	217	227	237	853	843	833	823	813	803	793	783	773	763	
439	246	256	266	276	286	296	306	316	325	335	754	744	734	724	714	704	694	684	675	665	
440	345	355	365	375	385	395	404	414	424	434	655	645	635	625	615	605	595	586	576	566	
441	444	454	464	474	483	493	503	513	523	539	556	546	536	526	517	507	497	487	477	468	
442	542	552	562	572	581	591	601	611	621	631	454	444	434	424	414	404	394	384	374	369	
443	640	650	660	670	680	689	699	709	719	728	360	350	340	330	320	311	301	291	281	273	
444	738	748	758	768	777	787	797	807	816	826	262	252	242	232	222	213	203	193	184	174	
445	836	846	855	865	875	885	894	904	914	924	164	154	145	135	125	115	105	096	086	076	
446	933	943	953	963	972	982	992	..	2	13	067	057	047	037	027	018	008	998	988	979	
447	65031	040	050	060	070	079	089	099	108	118	34965	960	950	940	930	921	911	901	892	882	
448	128	137	147	157	167	176	186	196	205	215	872	863	853	843	833	824	814	804	795	785	
449	223	231	241	254	263	273	283	293	302	312	775	766	756	746	737	727	717	708	699	689	

Logarithms.											Complements.										
N	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	
450	65321	391	340	350	360	369	379	389	398	408	346	79	689	660	630	640	631	621	611	602	592
451	418	427	437	446	456	466	475	485	495	504	336	73	563	534	544	534	524	515	505	496	
452	514	523	533	543	553	562	571	581	591	600	326	77	467	437	447	438	428	419	409	400	
453	610	619	629	639	648	658	667	677	686	696	316	37	351	355	355	355	355	355	355	355	
454	706	715	725	734	744	753	763	772	782	792	206	285	275	266	256	247	237	228	218	208	
455	802	811	820	830	839	849	858	868	877	887	196	189	180	170	161	151	142	132	123	113	
456	892	901	910	920	929	939	948	958	967	977	104	094	085	075	065	056	046	037	027	018	
457	982	991	1.11	1.12	1.13	1.14	1.15	1.16	1.17	1.18	008	009	009	009	009	009	009	009	009	009	
458	660	669	678	688	697	707	716	726	735	745	339	175	164	154	144	134	124	114	104	94	
459	181	191	200	210	219	229	238	248	257	267	276	809	800	790	781	771	762	752	743	734	
460	276	285	295	304	313	323	332	342	351	361	724	715	705	695	687	677	668	658	649	639	
461	370	379	389	398	408	417	427	436	446	455	620	621	611	602	592	583	573	564	555	545	
462	464	474	483	492	502	511	521	530	539	549	536	536	527	517	508	498	489	479	470	461	
463	558	567	577	586	596	605	615	624	633	642	442	433	423	414	404	395	386	376	367	358	
464	652	661	670	680	689	699	708	717	727	736	346	339	330	321	311	301	292	283	273	264	
465	745	755	764	773	783	792	801	811	820	829	245	245	236	227	217	208	198	189	180	171	
466	832	841	851	860	869	879	888	897	907	916	145	132	121	111	101	91	81	71	61	51	
467	924	933	943	952	962	971	981	990	1.00	1.01	058	059	050	040	031	021	011	001	991	981	
468	670	679	689	698	707	717	726	736	745	755	329	75	966	957	948	938	929	919	909	899	
469	117	126	136	145	155	164	173	182	191	200	853	874	864	855	846	836	827	818	809	800	
470	210	219	228	237	247	256	265	274	284	293	799	781	772	763	753	744	735	726	716	707	
471	302	311	320	329	338	348	357	367	376	385	698	689	680	670	661	652	641	633	624	615	
472	394	403	412	421	431	440	449	459	468	477	606	597	587	578	569	560	551	541	532	522	
473	486	495	504	514	523	532	541	550	559	569	514	506	496	487	477	468	459	450	441	431	
474	578	587	596	605	614	624	633	642	651	660	422	413	404	395	386	376	367	358	349	340	
475	669	678	688	697	707	716	726	735	744	752	331	322	313	303	294	285	276	267	258	248	
476	761	770	779	788	797	806	815	824	834	843	239	230	221	212	203	194	185	176	166	157	
477	842	851	860	869	878	888	897	906	915	924	148	139	130	121	112	103	94	85	75	66	
478	948	957	966	975	984	993	1.00	1.01	1.02	1.03	057	048	039	030	021	012	003	991	985	976	
479	680	689	698	708	717	727	736	746	755	765	319	66	957	948	939	930	921	912	903	894	
480	124	133	142	151	160	169	178	187	196	205	876	867	858	849	840	831	822	813	804	795	
481	214	223	232	241	251	260	269	278	287	296	767	757	747	737	727	717	707	697	687	677	
482	305	314	323	332	341	350	359	368	377	386	693	686	677	668	659	650	641	632	623	614	
483	395	404	413	422	431	440	449	458	467	476	605	596	587	578	569	560	551	542	533	524	
484	485	494	503	512	521	530	539	548	557	566	516	507	498	489	480	471	462	453	444	435	
485	574	583	592	601	610	619	628	637	646	655	426	417	408	399	390	381	372	363	354	345	
486	664	673	682	690	699	708	717	726	735	744	326	327	318	310	301	292	283	274	265	256	
487	753	762	771	780	789	797	806	815	824	833	247	238	229	220	211	203	194	185	176	167	
488	843	851	860	869	878	886	895	904	913	923	158	149	140	131	122	114	105	096	087	078	
489	931	940	949	957	966	975	984	993	1.00	1.01	069	060	051	043	034	025	016	007	998	989	
490	690	699	708	717	726	735	744	753	762	771	308	80	972	963	954	945	936	927	918	909	
491	106	115	126	135	144	153	161	170	179	188	892	883	874	865	857	848	839	830	821	812	
492	195	205	214	223	232	241	249	258	267	276	804	795	786	777	768	759	751	742	733	724	
493	285	293	302	311	320	329	337	346	355	364	715	707	698	689	680	671	663	654	645	636	
494	373	381	390	399	408	417	425	434	443	452	627	619	610	601	592	583	575	566	557	548	
495	460	469	478	487	496	505	513	522	531	539	530	521	512	503	494	485	477	468	460	451	
496	548	557	566	574	583	592	601	609	618	627	432	423	414	405	396	387	379	371	363	354	
497	636	644	653	662	671	679	688	697	706	715	364	356	347	338	329	321	312	303	295	286	
498	723	732	740	749	758	767	775	784	793	801	277	268	260	251	242	234	225	216	207	199	
499	810	819	827	836	845	853	862	871	880	888	190	181	173	164	155	147	138	129	120	112	

Logarithms.											Complements.										
N	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	
500	69897	90691	923	932	940	949	956	966	975		108	994	086	077	068	060	051	042	034	025	
501	988	992	1	10	18	27	36	44	53	62	016	008	999	990	982	975	964	955	947	938	
502	700	707	713	719	726	732	738	744	750	756	683	674	665	656	647	638	629	620	611	602	
503	157	163	169	175	181	187	193	199	205	211	843	835	826	817	809	800	791	782	774	766	
504	243	251	258	264	271	278	285	292	300	307	737	728	719	710	701	692	683	674	665	656	
505	329	338	346	355	363	372	381	389	398	406	677	669	661	652	643	634	625	616	607	598	
506	413	424	432	441	449	458	466	475	484	493	585	576	567	558	549	540	531	522	513	504	
507	501	509	518	526	535	544	553	561	569	578	499	491	482	474	465	456	447	438	429	420	
508	586	595	603	611	621	629	638	646	655	663	414	405	397	388	379	371	362	353	345	337	
509	672	680	689	697	706	714	723	731	740	748	328	320	311	302	294	286	277	269	260	251	
510	757	765	773	783	791	800	808	817	825	834	242	235	226	217	209	200	192	183	175	166	
511	842	851	859	868	876	885	893	901	910	918	159	149	141	132	124	115	107	098	090	082	
512	927	935	944	952	961	969	978	986	995	1	073	065	056	048	039	031	022	014	005	997	
513	7012	020	029	037	046	054	062	071	079	088	2898	280	271	263	254	246	238	229	221	212	
514	096	105	113	122	130	138	147	155	164	172	904	896	887	878	870	862	853	845	836	828	
515	181	189	198	206	214	223	231	240	248	257	819	811	802	794	786	777	769	760	752	743	
516	265	273	282	290	299	307	315	324	333	341	735	727	718	710	701	693	685	676	667	659	
517	349	357	367	375	383	391	399	408	416	425	631	623	614	605	597	589	580	572	564	555	
518	433	441	450	458	466	475	483	492	500	508	567	559	550	542	534	525	517	508	500	492	
519	517	525	533	542	550	559	567	575	584	592	482	475	467	458	450	441	433	425	416	408	
520	600	609	617	625	634	642	650	659	667	675	400	391	383	375	366	358	350	341	333	325	
521	684	692	700	709	717	725	734	742	750	759	316	307	300	292	283	275	266	258	250	241	
522	767	775	784	792	800	809	817	825	834	842	233	225	216	208	200	191	183	175	166	158	
523	850	858	867	875	883	892	900	908	916	925	150	142	133	125	117	108	100	092	083	075	
524	939	941	950	958	966	974	983	991	999	1	067	059	050	042	034	026	017	009	001	992	
525	72016	024	032	041	049	057	065	074	082	090	27984	976	968	959	951	943	935	926	918	910	
526	099	107	115	123	132	140	148	156	165	173	901	893	885	876	868	860	852	844	835	827	
527	181	189	197	206	214	222	230	239	247	255	819	811	803	794	786	778	770	761	753	745	
528	265	272	280	288	296	304	313	321	329	337	737	728	720	712	704	696	687	679	671	663	
529	348	356	364	370	378	387	395	403	411	419	654	646	638	630	622	613	605	597	589	581	
530	428	436	444	452	460	468	477	485	493	501	572	564	556	548	540	532	523	515	507	499	
531	509	518	526	534	542	550	558	567	575	583	491	480	474	466	458	450	442	433	425	417	
532	591	599	607	616	624	632	640	648	656	665	409	401	393	384	376	368	360	352	344	335	
533	673	681	689	697	705	713	722	730	738	746	321	313	305	297	288	280	271	263	254	246	
534	754	762	770	778	787	795	803	811	819	827	246	238	230	222	213	205	197	189	181	173	
535	835	843	852	860	868	876	884	892	900	908	165	157	148	140	132	124	116	108	100	092	
536	916	925	933	941	949	957	965	973	981	989	084	075	067	059	051	043	035	027	019	011	
537	997	1	5	14	22	30	38	46	54	62	005	995	987	978	970	962	954	946	938	930	
538	75078	086	094	102	110	119	127	135	143	151	26922	914	906	898	890	881	873	865	857	849	
539	159	167	175	183	191	199	207	215	223	231	871	863	855	847	839	831	823	815	807	799	
540	249	247	255	262	271	280	288	296	304	312	761	753	745	737	729	720	712	704	696	688	
541	320	328	336	344	352	360	368	376	384	392	680	672	664	656	648	640	632	624	616	608	
542	400	408	416	424	432	440	448	456	464	472	600	592	584	576	568	560	552	544	536	528	
543	490	488	496	504	512	520	528	536	544	552	520	512	504	496	488	480	472	464	456	448	
544	560	568	576	584	592	600	608	616	624	632	440	432	424	416	408	400	392	384	376	368	
545	640	648	656	663	671	679	687	695	703	711	360	352	344	337	329	321	313	305	297	289	
546	719	727	735	743	751	759	767	775	783	791	281	273	265	257	249	241	233	225	217	209	
547	799	807	815	823	830	838	846	854	862	870	201	193	185	178	170	162	154	146	138	130	
548	878	886	894	902	910	918	926	933	941	949	122	114	106	098	090	082	074	067	059	051	
549	957	965	973	981	989	997	1	9	17	25	043	035	027	019	011	003	995	987	980	972	

TABLE XVIII.

CONTAINING

THE LOGARITHMIC SINES AND COSINES

TO EVERY MINUTE OF THE QUADRANT,

WITH THEIR COMPLEMENTS,

AND DIFFERENCES ANSWERING TO EVERY 10";

ALSO

THE LOGARITHMIC TANGENTS AND COTANGENTS,

WITH THEIR

DIFFERENCES CORRESPONDING TO THE SAME ARC OF 10".

° Deg.

Tab. 18.

Sine	D. 10'	Comp. sin.	Cosine	D10'	Comp. cos.	Tangent	D. 10'	Cotangent
0 Inf. neg.	Infinit.	Infinit.	10.0000000	0.0	0.0000000	Inf. neg.	Infinit.	Infinit.
1 67437261	501717	9.5362739	10.0000000	0.1	0.0000000	6.4637261	501717	19.5362739
2 67647561	293485	9.2332439	9.9999999	0.2	0.0000000	6.7647562	293485	19.2332438
3 69408473	208231	9.0391527	9.9999999	0.3	0.0000000	6.9408473	208231	19.0391525
4 70657860	161517	9.9342140	9.9999999	0.4	0.0000000	7.0657865	161517	18.9342137
5 71626946	119668	9.8373040	9.9999999	0.5	0.0000000	7.1626946	119668	18.8373036
6 72418773	96653	9.7581229	9.9999999	0.6	0.0000000	7.2418778	96653	18.7581229
7 73088239	76262	9.6911761	9.9999999	0.7	0.0000000	7.3088239	76262	18.6911752
8 73668157	85254	9.6381843	9.9999998	0.8	0.0000001	7.3668159	85254	18.6381831
9 74179681	76262	9.5820319	9.9999998	0.9	0.0000001	7.4179680	76262	18.5820304
10 74637255	68988	9.5362745	9.9999998	1.0	0.0000018	7.4637255	68988	18.5362727
11 75031181	62981	9.488819	9.9999998	1.1	0.0000022	7.5031203	62981	18.4888197
12 75429065	57936	9.4579935	9.9999997	1.2	0.0000026	7.5429091	57936	18.4579909
13 75776684	53641	9.4223316	9.9999996	1.3	0.0000031	7.5777656	53641	18.4223285
14 76098530	49938	9.3981470	9.9999996	1.4	0.0000036	7.6098566	49938	18.3901434
15 76398160	46714	9.3601840	9.9999995	1.5	0.0000041	7.6398201	46714	18.3601799
16 76678445	43881	9.3321555	9.9999995	1.6	0.0000047	7.6678499	43881	18.3321508
17 76941733	41392	9.3058267	9.9999994	1.7	0.0000053	7.6941786	41392	18.3058214
18 77189960	39135	9.2810034	9.9999994	1.8	0.0000060	7.7189966	39135	18.2809974
19 77424773	37127	9.2575223	9.9999993	1.9	0.0000066	7.7424781	37127	18.2575159
20 77647537	35315	9.2352463	9.9999992	2.0	0.0000073	7.7647610	35315	18.2352390
21 77859427	33672	9.2140372	9.9999991	2.1	0.0000081	7.7854508	33672	18.2140492
22 78061458	32175	9.1938542	9.9999991	2.2	0.0000089	7.8061537	32175	18.1938453
23 78254507	30805	9.1745193	9.9999990	2.3	0.0000097	7.8254504	30805	18.1745396
24 78439338	29547	9.1560662	9.9999989	2.4	0.0000106	7.8439344	29547	18.1560536
25 78616623	28388	9.1383977	9.9999988	2.5	0.0000115	7.8616758	28388	18.1383962
26 78786953	27317	9.1218047	9.9999987	2.6	0.0000124	7.8786707	27317	18.1218293
27 78950854	26323	9.1049146	9.9999986	2.7	0.0000134	7.8950988	26323	18.1049123
28 79108793	25399	9.0891207	9.9999985	2.8	0.0000144	7.9108933	25399	18.0891062
29 79261196	24538	9.0738810	9.9999984	2.9	0.0000155	7.9261344	24538	18.0738656
30 79408419	23733	9.0591581	9.9999983	3.0	0.0000165	7.9408584	23733	18.0591416
31 79550819	22960	9.0449181	9.9999982	3.1	0.0000177	7.9550996	22960	18.0449004
32 79688698	22273	9.0311302	9.9999981	3.2	0.0000188	7.9688886	22273	18.0311348
33 79822354	21608	9.0177666	9.9999980	3.3	0.0000200	7.9822534	21608	18.0177466
34 79951980	20981	9.0048020	9.9999978	3.4	0.0000212	7.9952192	20981	18.0047808
35 80077867	20390	9.9922135	9.9999977	3.5	0.0000225	8.0077809	20390	17.9921908
36 80200207	19831	9.9797993	9.9999976	3.6	0.0000238	8.0200445	19831	17.9797554
37 80319195	19302	9.9680805	9.9999974	3.7	0.0000252	8.0319446	19302	17.9680354
38 80435009	18801	9.9564991	9.9999973	3.8	0.0000265	8.0435274	18801	17.9564726
39 80547814	18325	9.9452150	9.9999972	3.9	0.0000279	8.0548094	18325	17.9451906
40 80657763	17872	9.9342237	9.9999970	4.0	0.0000294	8.0658057	17872	17.9341945
41 80764997	17441	9.9235003	9.9999969	4.1	0.0000309	8.0765306	17441	17.9234694
42 80869646	17031	9.9130354	9.9999967	4.2	0.0000324	8.0869970	17031	17.9130003
43 80971832	16634	9.9028168	9.9999966	4.3	0.0000340	8.0972172	16634	17.9027828
44 81071669	16251	9.8933331	9.9999964	4.4	0.0000356	8.1072025	16251	17.8927975
45 81169266	15890	9.8830738	9.9999962	4.5	0.0000372	8.1169634	15890	17.8830366
46 81264710	15566	9.8735290	9.9999961	4.6	0.0000389	8.1265099	15566	17.8734901
47 81358104	15258	9.8641896	9.9999959	4.7	0.0000406	8.1358510	15258	17.8641490
48 81449573	14924	9.8550468	9.9999957	4.8	0.0000423	8.1449956	14924	17.8550044
49 81539075	14622	9.8460923	9.9999955	4.9	0.0000441	8.1539916	14622	17.8460484
50 81626808	14333	9.8373192	9.9999954	5.0	0.0000459	8.1627262	14333	17.8372733
51 81712404	14054	9.8287196	9.9999952	5.1	0.0000478	8.1713282	14054	17.8286718
52 81797124	13786	9.8202871	9.9999950	5.2	0.0000497	8.1797626	13786	17.8202374
53 81879948	13529	9.8120152	9.9999948	5.3	0.0000516	8.1880364	13529	17.8119636
54 81961020	13280	9.8038980	9.9999946	5.4	0.0000536	8.1961556	13280	17.8038444
55 82040701	13041	9.7959297	9.9999944	5.5	0.0000556	8.2041240	13041	17.7958741
56 82118919	12810	9.7881691	9.9999942	5.6	0.0000576	8.2119526	12810	17.7880474
57 82195811	12587	9.7804159	9.9999940	5.7	0.0000597	8.2196408	12587	17.7803592
58 82271325	12372	9.7728665	9.9999938	5.8	0.0000618	8.2271953	12372	17.7728047
59 82345568	12164	9.7654432	9.9999936	5.9	0.0000640	8.2346308	12164	17.7653792
60 82418553		9.7581447	9.9999933	6.0	0.0000662	8.2419215		17.7580785
Cosine	D. 10'	Comp. cos.	Sine	D10'	Comp. sin.	Cotang.	D. 10'	Tangent

Tab. 18.

Deg. 29.

1 Deg.

Tab. 18.

Sine	D. 10'	Comp. sin.	Cotang.	D. 10'	Comp. cos.	Tangent	D. 10'	Cotangent
18-2490322	11763	1-7509668	9-9999312	3-7	0-0000662	8-2417843	11967	11-7509783
18-2500943	11769	1-7490579	9-9999421	3-7	0-0000684	8-2491615	11772	11-7509823
18-2510424	11775	1-7479057	9-9999529	3-7	0-0000706	8-2561649	11778	11-7483311
18-2520424	11781	1-7467566	9-9999637	3-7	0-0000729	8-2631153	11784	11-7509857
18-2530424	11787	1-7456110	9-9999745	3-7	0-0000752	8-2699563	11790	11-7509897
18-2540424	11793	1-7444689	9-9999853	3-7	0-0000776	8-2766913	11796	11-7509937
18-2550424	11799	1-7433304	9-9999961	3-7	0-0000800	8-2833284	11802	11-7509977
18-2560424	11805	1-7421956	9-9999969	3-7	0-0000824	8-2899559	11808	11-7509985
18-2570424	11811	1-7410644	9-9999977	3-7	0-0000848	8-2965759	11814	11-7509993
18-2580424	11817	1-7399368	9-9999985	3-7	0-0000872	8-3031884	11820	11-7509997
18-2590424	11823	1-7388128	9-9999993	3-7	0-0000896	8-3097934	11826	11-7509999
18-2600424	11829	1-7376924	9-9999997	3-7	0-0000920	8-3163909	11832	11-7509999
18-2610424	11835	1-7365756	9-9999999	3-7	0-0000944	8-3229809	11838	11-7509999
18-2620424	11841	1-7354624	9-9999999	3-7	0-0000968	8-3295634	11844	11-7509999
18-2630424	11847	1-7343528	9-9999999	3-7	0-0000992	8-3361384	11850	11-7509999
18-2640424	11853	1-7332468	9-9999999	3-7	0-0001016	8-3427059	11856	11-7509999
18-2650424	11859	1-7321444	9-9999999	3-7	0-0001040	8-3492659	11862	11-7509999
18-2660424	11865	1-7310456	9-9999999	3-7	0-0001064	8-3558184	11868	11-7509999
18-2670424	11871	1-7300004	9-9999999	3-7	0-0001088	8-3623634	11874	11-7509999
18-2680424	11877	1-7289588	9-9999999	3-7	0-0001112	8-3689009	11880	11-7509999
18-2690424	11883	1-7279208	9-9999999	3-7	0-0001136	8-3754309	11886	11-7509999
18-2700424	11889	1-7268864	9-9999999	3-7	0-0001160	8-3819534	11892	11-7509999
18-2710424	11895	1-7258556	9-9999999	3-7	0-0001184	8-3884684	11898	11-7509999
18-2720424	11901	1-7248284	9-9999999	3-7	0-0001208	8-3949759	11904	11-7509999
18-2730424	11907	1-7238048	9-9999999	3-7	0-0001232	8-4014759	11910	11-7509999
18-2740424	11913	1-7227848	9-9999999	3-7	0-0001256	8-4079684	11916	11-7509999
18-2750424	11919	1-7217684	9-9999999	3-7	0-0001280	8-4144534	11922	11-7509999
18-2760424	11925	1-7207556	9-9999999	3-7	0-0001304	8-4209309	11928	11-7509999
18-2770424	11931	1-7197464	9-9999999	3-7	0-0001328	8-4274009	11934	11-7509999
18-2780424	11937	1-7187408	9-9999999	3-7	0-0001352	8-4338634	11940	11-7509999
18-2790424	11943	1-7177388	9-9999999	3-7	0-0001376	8-4403184	11946	11-7509999
18-2800424	11949	1-7167404	9-9999999	3-7	0-0001400	8-4467659	11952	11-7509999
18-2810424	11955	1-7157456	9-9999999	3-7	0-0001424	8-4532059	11958	11-7509999
18-2820424	11961	1-7147544	9-9999999	3-7	0-0001448	8-4596384	11964	11-7509999
18-2830424	11967	1-7137668	9-9999999	3-7	0-0001472	8-4660634	11970	11-7509999
18-2840424	11973	1-7127828	9-9999999	3-7	0-0001496	8-4724809	11976	11-7509999
18-2850424	11979	1-7118024	9-9999999	3-7	0-0001520	8-4788909	11982	11-7509999
18-2860424	11985	1-7108256	9-9999999	3-7	0-0001544	8-4852934	11988	11-7509999
18-2870424	11991	1-7098524	9-9999999	3-7	0-0001568	8-4916884	11994	11-7509999
18-2880424	11997	1-7088828	9-9999999	3-7	0-0001592	8-4980759	12000	11-7509999
18-2890424	12003	1-7079168	9-9999999	3-7	0-0001616	8-5044559	12009	11-7509999
18-2900424	12009	1-7069544	9-9999999	3-7	0-0001640	8-5108284	12015	11-7509999
18-2910424	12015	1-7059956	9-9999999	3-7	0-0001664	8-5171934	12021	11-7509999
18-2920424	12021	1-7050404	9-9999999	3-7	0-0001688	8-5235509	12027	11-7509999
18-2930424	12027	1-7040888	9-9999999	3-7	0-0001712	8-5299009	12033	11-7509999
18-2940424	12033	1-7031408	9-9999999	3-7	0-0001736	8-5362434	12039	11-7509999
18-2950424	12039	1-7021964	9-9999999	3-7	0-0001760	8-5425784	12045	11-7509999
18-2960424	12045	1-7012556	9-9999999	3-7	0-0001784	8-5489059	12051	11-7509999
18-2970424	12051	1-7003184	9-9999999	3-7	0-0001808	8-5552259	12057	11-7509999
18-2980424	12057	1-6993848	9-9999999	3-7	0-0001832	8-5615384	12063	11-7509999
18-2990424	12063	1-6984548	9-9999999	3-7	0-0001856	8-5678434	12069	11-7509999
18-3000424	12069	1-6975284	9-9999999	3-7	0-0001880	8-5741409	12075	11-7509999
18-3010424	12075	1-6966056	9-9999999	3-7	0-0001904	8-5804309	12081	11-7509999
18-3020424	12081	1-6956864	9-9999999	3-7	0-0001928	8-5867134	12087	11-7509999
18-3030424	12087	1-6947708	9-9999999	3-7	0-0001952	8-5929884	12093	11-7509999
18-3040424	12093	1-6938588	9-9999999	3-7	0-0001976	8-5992559	12099	11-7509999
18-3050424	12099	1-6929504	9-9999999	3-7	0-0002000	8-6055159	12105	11-7509999
18-3060424	12105	1-6920456	9-9999999	3-7	0-0002024	8-6117684	12111	11-7509999
18-3070424	12111	1-6911444	9-9999999	3-7	0-0002048	8-6180134	12117	11-7509999
18-3080424	12117	1-6902468	9-9999999	3-7	0-0002072	8-6242509	12123	11-7509999
18-3090424	12123	1-6893528	9-9999999	3-7	0-0002096	8-6304809	12129	11-7509999
18-3100424	12129	1-6884624	9-9999999	3-7	0-0002120	8-6367034	12135	11-7509999
18-3110424	12135	1-6875756	9-9999999	3-7	0-0002144	8-6429184	12141	11-7509999
18-3120424	12141	1-6866924	9-9999999	3-7	0-0002168	8-6491259	12147	11-7509999
18-3130424	12147	1-6858128	9-9999999	3-7	0-0002192	8-6553259	12153	11-7509999
18-3140424	12153	1-6849368	9-9999999	3-7	0-0002216	8-6615184	12159	11-7509999
18-3150424	12159	1-6840644	9-9999999	3-7	0-0002240	8-6677034	12165	11-7509999
18-3160424	12165	1-6831956	9-9999999	3-7	0-0002264	8-6738809	12171	11-7509999
18-3170424	12171	1-6823304	9-9999999	3-7	0-0002288	8-6800509	12177	11-7509999
18-3180424	12177	1-6814688	9-9999999	3-7	0-0002312	8-6862134	12183	11-7509999
18-3190424	12183	1-6806108	9-9999999	3-7	0-0002336	8-6923684	12189	11-7509999
18-3200424	12189	1-6797564	9-9999999	3-7	0-0002360	8-6985159	12195	11-7509999
18-3210424	12195	1-6789056	9-9999999	3-7	0-0002384	8-7046559	12201	11-7509999
18-3220424	12201	1-6780584	9-9999999	3-7	0-0002408	8-7107884	12207	11-7509999
18-3230424	12207	1-6772148	9-9999999	3-7	0-0002432	8-7169134	12213	11-7509999
18-3240424	12213	1-6763748	9-9999999	3-7	0-0002456	8-7230309	12219	11-7509999
18-3250424	12219	1-6755384	9-9999999	3-7	0-0002480	8-7291409	12225	11-7509999
18-3260424	12225	1-6747056	9-9999999	3-7	0-0002504	8-7352434	12231	11-7509999
18-3270424	12231	1-6738764	9-9999999	3-7	0-0002528	8-7413384	12237	11-7509999
18-3280424	12237	1-6730508	9-9999999	3-7	0-0002552	8-7474259	12243	11-7509999
18-3290424	12243	1-6722288	9-9999999	3-7	0-0002576	8-7535059	12249	11-7509999
18-3300424	12249	1-6714104	9-9999999	3-7	0-0002600	8-7595784	12255	11-7509999
18-3310424	12255	1-6705956	9-9999999	3-7	0-0002624	8-7656434	12261	11-7509999
18-3320424	12261	1-6697844	9-9999999	3-7	0-0002648	8-7716984	12267	11-7509999
18-3330424	12267	1-6689768	9-9999999	3-7	0-0002672	8-7777459	12273	11-7509999
18-3340424	12273	1-6681728	9-9999999	3-7	0-0002696	8-7837859	12279	11-7509999
18-3350424	12279	1-6673724	9-9999999	3-7	0-0002720	8-7898184	12285	11-7509999
18-3360424	12285	1-6665756	9-9999999	3-7	0-0002744	8-7958434	12291	11-7509999
18-3370424	12291	1-6657824	9-9999999	3-7	0-0002768	8-8018609	12297	11-7509999
18-3380424	12297	1-6649928	9-9999999	3-7	0-0002792	8-8078709	12303	11-7509999
18-3390424	12303	1-6642068	9-9999999	3-7	0-0002816	8-8138734	12309	11-7509999
18-3400424	12309	1-6634244	9-9999999	3-7	0-0002840	8-8198684	12315	11-7509999
18-3410424	12315	1-6626456	9-9999999	3-7	0-0002864	8-8258559	12321	11-7509999
18-3420424	12321	1-6618704	9-9999999	3-7	0-0002888	8-8318359	12327	11-7509999
18-3430424	12327	1-6610988	9-9999999	3-7	0-0002912	8-8378084	12333	11-7509999
18-3440424	12333	1-6603308	9-9999999	3-7	0-0002936	8-8437734	12339	11-7509999
18-3450424	12339	1-6595664	9-9999999	3-7	0-0002960	8-8497309	12345	11-7509999
18-3460424	12345	1-6588056	9-9999999	3-7	0-0002984	8-8556809	12351	11-7509999
18-3470424	12351	1-6580484	9-9999999	3-7	0-0003008	8-8616234	12357	11-7509999
18-3480424	12357	1-6572948	9-9999999	3-7	0-0003032	8-8675584	12363	11-7509999
18-3490424	12363	1-6565448	9-9999999	3-7	0-0003056	8-8734859	12369	11-7509999
18-3500424	12369	1-6557984	9-9999999	3-7	0-0003080	8-8794059	12375	11-7509999
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2 Deg.

Tab. 18.

Sine	D10	Comp. sin.	Cosine	D10	Comp. cos.	Tangent	Cotangent
08-5428192	5004	1-4337308	9-9997334	7-5	0-0002696	8-5430833	11-4569162
18-5464218	5954	1-4335782	9-9997339	7-6	0-0002688	8-5466903	11-4533091
28-5499948	5906	1-4300052	9-9997365	7-7	0-0002680	8-5502683	11-4497317
38-5535388	5858	1-4464614	9-9997220	7-8	0-0002672	8-5538156	11-4461834
48-5570356	5811	1-4429464	9-9997224	7-9	0-0002664	8-5573862	11-4426566
58-5605324	5763	1-4394596	9-9997228	8-0	0-0002656	8-5609576	11-4391733
68-5639956	5715	1-4360006	9-9997232	8-1	0-0002648	8-5645292	11-4357088
78-5674310	5667	1-4325690	9-9997236	8-2	0-0002640	8-5681008	11-4322533
88-5708357	5619	1-4291622	9-9997240	8-3	0-0002632	8-5716724	11-4288038
98-5742139	5571	1-4257861	9-9997244	8-4	0-0002624	8-5752440	11-4253543
108-5775660	5523	1-4224320	9-9997248	8-5	0-0002616	8-5788156	11-4219048
118-5808923	5475	1-4191077	9-9997252	8-6	0-0002608	8-5823872	11-4184553
128-5841933	5427	1-4158067	9-9997256	8-7	0-0002600	8-5859588	11-4150058
138-5874694	5379	1-4125306	9-9997260	8-8	0-0002592	8-5895304	11-4115563
148-5907209	5331	1-4092811	9-9997264	8-9	0-0002584	8-5931020	11-4081068
158-5939483	5283	1-4060517	9-9997268	9-0	0-0002576	8-5966736	11-4046573
168-5971517	5235	1-4028483	9-9997272	9-1	0-0002568	8-6002452	11-4012078
178-6003317	5187	1-3996683	9-9997276	9-2	0-0002560	8-6038168	11-3977583
188-6034896	5139	1-3965114	9-9997280	9-3	0-0002552	8-6073884	11-3943088
198-6066286	5091	1-3933774	9-9997284	9-4	0-0002544	8-6109600	11-3908593
208-6097341	5043	1-3902659	9-9997288	9-5	0-0002536	8-6145316	11-3874098
218-6128233	5000	1-3871765	9-9997292	9-6	0-0002528	8-6181032	11-3839603
228-6158940	5006	1-3841090	9-9997296	9-7	0-0002520	8-6216748	11-3805108
238-6189366	5012	1-3810631	9-9997300	9-8	0-0002512	8-6252464	11-3770613
248-6219616	5018	1-3780384	9-9997304	9-9	0-0002504	8-6288180	11-3736118
258-6249633	5024	1-3750349	9-9997308	10-0	0-0002496	8-6323896	11-3701623
268-6279448	5030	1-3720516	9-9997312	10-1	0-0002488	8-6359612	11-3667128
278-6309111	5036	1-3690889	9-9997316	10-2	0-0002480	8-6395328	11-3632633
288-6338537	5042	1-3661463	9-9997320	10-3	0-0002472	8-6431044	11-3598138
298-6367764	5048	1-3632236	9-9997324	10-4	0-0002464	8-6466760	11-3563643
308-6396796	5054	1-3603204	9-9997328	10-5	0-0002456	8-6502476	11-3529148
318-6425634	5060	1-3574366	9-9997332	10-6	0-0002448	8-6538192	11-3494653
328-6454282	5066	1-3545718	9-9997336	10-7	0-0002440	8-6573908	11-3460158
338-6482742	5072	1-3517259	9-9997340	10-8	0-0002432	8-6609624	11-3425663
348-6511016	5078	1-3488983	9-9997344	10-9	0-0002424	8-6645340	11-3391168
358-6539107	5084	1-3460899	9-9997348	11-0	0-0002416	8-6681056	11-3356673
368-6566917	5090	1-3433003	9-9997352	11-1	0-0002408	8-6716772	11-3322178
378-6594478	5096	1-3405299	9-9997356	11-2	0-0002400	8-6752488	11-3287683
388-6621830	5102	1-3377787	9-9997360	11-3	0-0002392	8-6788204	11-3253188
398-6649684	5108	1-3350316	9-9997364	11-4	0-0002384	8-6823920	11-3218693
408-6676893	5114	1-3322910	9-9997368	11-5	0-0002376	8-6859636	11-3184198
418-6703932	5120	1-3295608	9-9997372	11-6	0-0002368	8-6895352	11-3149703
428-6730804	5126	1-3268416	9-9997376	11-7	0-0002360	8-6931068	11-3115208
438-6757510	5132	1-3241324	9-9997380	11-8	0-0002352	8-6966784	11-3080713
448-6784053	5138	1-3214332	9-9997384	11-9	0-0002344	8-7002500	11-3046218
458-6810433	5144	1-3187440	9-9997388	12-0	0-0002336	8-7038216	11-3011723
468-6836654	5150	1-3160648	9-9997392	12-1	0-0002328	8-7073932	11-2977228
478-6862718	5156	1-3133956	9-9997396	12-2	0-0002320	8-7109648	11-2942733
488-6888625	5162	1-3107364	9-9997400	12-3	0-0002312	8-7145364	11-2908238
498-6914379	5168	1-3080872	9-9997404	12-4	0-0002304	8-7181080	11-2873743
508-6939980	5174	1-3060020	9-9997408	12-5	0-0002296	8-7216796	11-2839248
518-6965431	5180	1-3039268	9-9997412	12-6	0-0002288	8-7252512	11-2804753
528-6990734	5186	1-3018616	9-9997416	12-7	0-0002280	8-7288228	11-2770258
538-7015889	5192	1-2998064	9-9997420	12-8	0-0002272	8-7323944	11-2735763
548-7040899	5198	1-2977612	9-9997424	12-9	0-0002264	8-7359660	11-2701268
558-7065766	5204	1-2957260	9-9997428	13-0	0-0002256	8-7395376	11-2666773
568-7090491	5210	1-2937008	9-9997432	13-1	0-0002248	8-7431092	11-2632278
578-7115075	5216	1-2916856	9-9997436	13-2	0-0002240	8-7466808	11-2597783
588-7139529	5222	1-2896804	9-9997440	13-3	0-0002232	8-7502524	11-2563288
598-7163883	5228	1-2876852	9-9997444	13-4	0-0002224	8-7538240	11-2528793
608-7188002	5234	1-2857000	9-9997448	13-5	0-0002216	8-7573956	11-2494298

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3 Deg.

Sine	D10	Comp. Sin.	Cosine	D10	Comp. Cos.	Tangent	D10	Cotangent
018 7188002	4006	12611998	99993044	11-0	0-0005256	8-7123958	4017	11-2806042
18 7212040	3984	12787960	99993978	11-2	0-0006022	8-7218063	3995	11-2781937
28 7235046	3963	12976405	99993951	11-2	0-0006089	8-7242035	3974	11-2757965
38 7259721	3941	12746279	99993844	11-3	0-0006156	8-7265877	3952	11-2734122
48 7284360	3919	12716634	99993770	11-3	0-0006224	8-7289589	3931	11-2710411
58 7308882	3898	12693118	99993708	11-3	0-0006292	8-7313174	3909	11-2686826
68 7330272	3877	12669728	99993610	11-3	0-0006360	8-7336631	3888	11-2663369
78 7351535	3857	12646463	99993577	11-5	0-0006428	8-7359964	3866	11-2640030
88 7376675	3836	12623328	99993505	11-5	0-0006497	8-7383172	3845	11-2616828
98 7399691	3816	12600309	99993433	11-5	0-0006567	8-7406358	3824	11-2593742
108 7422586	3796	12577411	99993364	11-8	0-0006636	8-7429222	3803	11-2570778
118 7445360	3776	12554640	99993293	11-7	0-0006707	8-7452067	3782	11-2547933
128 7468015	3756	12531985	99993223	11-8	0-0006777	8-7474792	3761	11-2525200
138 7490553	3737	12509447	99993152	11-8	0-0006848	8-7497400	3740	11-2502600
148 7512973	3717	12487027	99993081	12-0	0-0006919	8-7519892	3719	11-2480108
158 7535278	3698	12464722	99993009	12-0	0-0006991	8-7542280	3698	11-2457731
168 7557469	3679	12442531	99992938	12-0	0-0007063	8-7564551	3677	11-2435369
178 7579546	3660	12420454	99992865	12-0	0-0007135	8-7586681	3656	11-2413019
188 7601512	3642	12398488	99992793	12-2	0-0007207	8-7608719	3635	11-2390681
198 7623366	3624	12376634	99992720	12-2	0-0007280	8-7630657	3614	11-2368383
208 7645111	3606	12354880	99992646	12-3	0-0007353	8-7652485	3593	11-2346135
218 7666747	3588	12333253	99992572	12-3	0-0007428	8-7674175	3572	11-2323925
228 7688275	3570	12311725	99992498	12-3	0-0007502	8-7695777	3551	11-2301763
238 7709697	3553	12290303	99992424	12-5	0-0007577	8-7717274	3530	11-2279647
248 7731014	3535	12268986	99992349	12-5	0-0007651	8-7738663	3509	11-2257576
258 7752226	3518	12247774	99992274	12-7	0-0007726	8-7759953	3488	11-2235549
268 7773334	3501	12226666	99992198	12-7	0-0007802	8-7781156	3467	11-2213564
278 7794340	3484	12205660	99992122	12-7	0-0007878	8-7802218	3446	11-2191620
288 7815244	3467	12184756	99992046	12-8	0-0007954	8-7823199	3425	11-2169712
298 7836048	3451	12163952	99991969	12-8	0-0008031	8-7844079	3404	11-2147841
308 7856753	3434	12143247	99991899	12-8	0-0008108	8-7864861	3383	11-2126009
318 7877359	3418	12122641	99991815	13-0	0-0008185	8-7885544	3362	11-2104215
328 7897867	3402	12102133	99991737	13-0	0-0008265	8-7906120	3341	11-2082458
338 7918278	3386	12081722	99991659	13-2	0-0008344	8-7926620	3320	11-2060730
348 7938591	3370	12061406	99991580	13-2	0-0008424	8-7947011	3299	11-2039036
358 7958814	3354	12041186	99991501	13-2	0-0008504	8-7967319	3278	11-2017376
368 7978941	3339	12021059	99991422	13-3	0-0008585	8-7987519	3257	11-2005748
378 7998974	3323	12001026	99991342	13-3	0-0008668	8-8007633	3236	11-1984152
388 8018915	3308	11981085	99991262	13-3	0-0008752	8-8027653	3215	11-1962587
398 8038764	3293	11961236	99991182	13-5	0-0008838	8-8047583	3194	11-1941052
408 8058522	3278	11941477	99991101	13-5	0-0008925	8-8067422	3173	11-1919547
418 8078192	3263	11921808	99991021	13-7	0-0009012	8-8087171	3152	11-1898071
428 8097772	3249	11902228	99990938	13-7	0-0009100	8-8106834	3131	11-1876624
438 8117264	3234	11882736	99990856	13-7	0-0009189	8-8126407	3110	11-1855205
448 8136668	3219	11863332	99990774	13-8	0-0009278	8-8145891	3089	11-1833816
458 8155985	3205	11844015	99990691	13-8	0-0009369	8-8165294	3068	11-1812456
468 8175217	3191	11824783	99990608	13-8	0-0009458	8-8184608	3047	11-1791124
478 8194363	3177	11805637	99990525	14-0	0-0009547	8-8203838	3026	11-1769821
488 8213425	3163	11786575	99990441	14-0	0-0009638	8-8222988	3005	11-1748546
498 8232404	3149	11767596	99990357	14-0	0-0009727	8-8242046	3084	11-1727300
508 8251299	3135	11748701	99990273	14-2	0-0009818	8-8261026	3063	11-1706081
518 8270112	3122	11729888	99990188	14-2	0-0009912	8-8279923	3042	11-1684890
528 8288844	3108	11711156	99990103	14-3	0-0009997	8-8298741	3021	11-1663727
538 8307495	3095	11692505	99990017	14-3	0-0010093	8-8317478	3000	11-1642592
548 8326066	3082	11673934	99989931	14-3	0-0010189	8-8336134	2979	11-1621486
558 8344557	3069	11655443	99989845	14-5	0-0010289	8-8354712	2958	11-1600409
568 8362969	3056	11637031	99989758	14-5	0-0010389	8-8373211	2937	11-1579360
578 8381304	3043	11618696	99989671	14-5	0-0010489	8-8391633	2916	11-1558339
588 8399561	3030	11600429	99989584	14-7	0-0010590	8-8409977	2895	11-1537345
598 8417741	3017	11582229	99989496	14-7	0-0010692	8-8428245	2874	11-1516375
608 8435845		11564155	99989408	14-7	0-0010795	8-8446437	2853	11-1495429
Cosine	D10	Comp. cos.	Sine	D10	Comp. sin.	Cotang.	D10	Tangent

4 Deg.

Tab. 18.

	Sine.	D10'	Comp. sin.	Cosine.	D10'	Comp. cos.	Tangent.	D10'	Cotangent.
0	8433815	3005	1564155	99989408	14-8	0-0010592	8-8446437	3019	11-1533563
1	8433824	2992	1546126	99989319	14-8	0-0010601	8-8446454	3005	11-1535446
2	8433837	2980	1528173	99989230	14-8	0-0010610	8-8446471	2992	11-1537405
3	8433850	2967	1510293	99989142	14-8	0-0010619	8-8446488	2979	11-1499436
4	8433863	2955	1492488	99989053	14-8	0-0010628	8-8446505	2967	11-1481359
5	8433876	2943	1474755	99988965	15-0	0-0011038	8-8446522	2955	11-1463717
6	8433889	2931	1457093	99988877	15-2	0-0011129	8-8446539	2943	11-1445968
7	8433902	2919	1439508	99988789	15-2	0-0011220	8-8446556	2931	11-1428287
8	8433915	2908	1421990	99988701	15-2	0-0011311	8-8446573	2919	11-1410679
9	8433928	2896	1404543	99988613	15-3	0-0011403	8-8446590	2908	11-1393141
10	8433941	2884	1387167	99988525	15-3	0-0011494	8-8446607	2896	11-1375678
11	8433954	2873	1369861	99988437	15-3	0-0011586	8-8446624	2884	11-1358254
12	8433967	2861	1352624	99988349	15-3	0-0011679	8-8446641	2873	11-1340945
13	8433980	2850	1335455	99988261	15-5	0-0011772	8-8446658	2861	11-1323687
14	8433993	2839	1318354	99988173	15-5	0-0011865	8-8446675	2850	11-1306489
15	8434006	2828	1301320	99988085	15-7	0-0011959	8-8446692	2839	11-1289352
16	8434019	2817	1284354	99987997	15-7	0-0012053	8-8446709	2828	11-1272301
17	8434032	2806	1267454	99987909	15-8	0-0012147	8-8446726	2817	11-1255306
18	8434045	2795	1250619	99987821	15-8	0-0012242	8-8446743	2806	11-1238377
19	8434058	2784	1233850	99987733	16-0	0-0012337	8-8446760	2795	11-1221519
20	8434071	2773	1217146	99987645	16-0	0-0012433	8-8446777	2784	11-1204714
21	8434084	2763	1200507	99987557	16-0	0-0012529	8-8446794	2773	11-1187939
22	8434097	2752	1183931	99987469	16-2	0-0012625	8-8446811	2763	11-1171308
23	8434110	2742	1167419	99987381	16-2	0-0012722	8-8446828	2752	11-1154697
24	8434123	2731	1150969	99987293	16-2	0-0012819	8-8446845	2742	11-1138150
25	8434136	2721	1134589	99987205	16-3	0-0012916	8-8446862	2731	11-1121668
26	8434149	2711	1118257	99987117	16-3	0-0013013	8-8446879	2721	11-1105243
27	8434162	2700	1101993	99987029	16-3	0-0013110	8-8446896	2711	11-1088881
28	8434175	2690	1085791	99986941	16-5	0-0013210	8-8446913	2700	11-1072580
29	8434188	2680	1069564	99986853	16-5	0-0013308	8-8446930	2690	11-1056340
30	8434201	2670	1053367	99986765	16-5	0-0013409	8-8446947	2680	11-1040158
31	8434214	2660	1037154	99986677	16-7	0-0013508	8-8446964	2670	11-1024037
32	8434227	2651	1020982	99986589	16-7	0-0013608	8-8446981	2660	11-1007974
33	8434240	2641	1004789	99986501	16-8	0-0013708	8-8446998	2651	11-0991970
34	8434253	2631	988583	99986413	16-8	0-0013809	8-8447015	2641	11-0976023
35	8434266	2622	972403	99986325	17-0	0-0013910	8-8447032	2631	11-0960134
36	8434279	2612	956253	99986237	17-0	0-0014012	8-8447049	2622	11-0944200
37	8434292	2603	940126	99986149	17-0	0-0014114	8-8447066	2612	11-0928328
38	8434305	2593	924022	99986061	17-0	0-0014216	8-8447083	2603	11-0912410
39	8434318	2584	907943	99985973	17-2	0-0014318	8-8447100	2593	11-0896477
40	8434331	2575	891889	99985885	17-2	0-0014421	8-8447117	2584	11-0880540
41	8434344	2566	875861	99985797	17-3	0-0014523	8-8447134	2575	11-0864598
42	8434357	2556	860057	99985709	17-3	0-0014626	8-8447151	2566	11-0848651
43	8434370	2547	844278	99985621	17-5	0-0014729	8-8447168	2556	11-0832704
44	8434383	2538	828524	99985533	17-5	0-0014832	8-8447185	2547	11-0816757
45	8434396	2529	812796	99985445	17-5	0-0014935	8-8447202	2538	11-0800809
46	8434409	2520	797093	99985357	17-7	0-0015038	8-8447219	2529	11-0784862
47	8434422	2512	781415	99985269	17-7	0-0015141	8-8447236	2520	11-0768914
48	8434435	2503	765761	99985181	17-8	0-0015244	8-8447253	2512	11-0752967
49	8434448	2494	750132	99985093	17-8	0-0015347	8-8447270	2503	11-0737019
50	8434461	2486	734528	99985005	17-8	0-0015450	8-8447287	2494	11-0721072
51	8434474	2477	718949	99984917	18-0	0-0015553	8-8447304	2486	11-0705124
52	8434487	2469	703395	99984829	18-0	0-0015656	8-8447321	2477	11-0689177
53	8434500	2460	687866	99984741	18-2	0-0015759	8-8447338	2469	11-0673229
54	8434513	2452	672361	99984653	18-2	0-0015862	8-8447355	2460	11-0657282
55	8434526	2443	656880	99984565	18-2	0-0015965	8-8447372	2452	11-0641334
56	8434539	2435	641423	99984477	18-2	0-0016068	8-8447389	2443	11-0625387
57	8434552	2427	625990	99984389	18-3	0-0016171	8-8447406	2435	11-0609439
58	8434565	2419	610581	99984301	18-3	0-0016274	8-8447423	2427	11-0593492
59	8434578	2411	595196	99984213	18-3	0-0016377	8-8447440	2419	11-0577544
60	8434591	2403	579835	99984125		0-0016480	8-8447457	2411	11-0561597
	Cosine.	D10'	Comp. cos.	Sine.	D10'	Comp. sin.	Cotang.	D10'	Tangent.

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Sine	D10"	Comp. sin.	Cosine	D10"	Comp. cos.	Tangent	D10"	Cotangent
08 9402960	2403	1 0597034	9 9983442	18-7	0 0016558	8 9419518	2421	11 0580182
118 9417376	2395	1 0589624	9 9983332	18-7	0 0016668	8 9424044	2414	11 0565956
2 8 941517 3	2387	1 0568257	9 9983220	18 5	0 0016780	8 9448523	2405	11 0551477
3 8 9446063	2379	1 0553937	9 9983109	18-7	0 0016891	8 9462954	2397	11 0537046
4 8 9460573	2371	1 0539663	9 9982997	18-7	0 0017003	8 9477 38	2390	11 0522662
5 8 9474561	2363	1 0525439	9 9982885	18-8	0 0017115	8 9491676	2382	11 0508324
6 8 9488739	2355	1 0511261	9 9982772	18-8	0 0017228	8 9505967	2374	11 0494033
7 8 9502871	2348	1 0497125	9 9982660	18-7	0 0017340	8 9520211	2366	11 0479789
8 8 9516957	2340	1 0483043	9 9982546	19-0	0 0017453	8 9534410	2359	11 0465590
9 8 9530996	2332	1 0468904	9 9982433	18-8	0 0017565	8 9548564	2351	11 0451436
10 8 9544991	2325	1 0455009	9 9982318	19-2	0 0017682	8 9562672	2344	11 0437328
11 8 9558940	2317	1 0441106	9 9982204	19-0	0 0017796	8 9576735	2336	11 0423265
12 8 9572843	2310	1 0427155	9 9982089	19-2	0 0017911	8 9590764	2329	11 0409246
13 8 9586703	2302	1 0413207	9 9981974	19-0	0 0018026	8 9604728	2322	11 0395272
14 8 9600517	2295	1 0399248	9 9981859	19-2	0 0018141	8 9618659	2314	11 0381341
15 8 9614288	2288	1 0385271	9 9981742	19-3	0 0018257	8 9632545	2307	11 0367455
16 8 9628014	2280	1 0371286	9 9981626	19-3	0 0018374	8 9646388	2300	11 0353612
17 8 9641697	2273	1 0357303	9 9981510	19-5	0 0018490	8 9660188	2293	11 0339811
18 8 9655337	2266	1 0343366	9 9981393	19-7	0 0018607	8 9673944	2286	11 0326056
19 8 9668934	2259	1 0329406	9 9981275	19-3	0 0018725	8 9687655	2279	11 0312344
20 8 9682487	2252	1 0315451	9 9981158	19-7	0 0018842	8 9701330	2272	11 0298670
21 8 9695999	2245	1 0301501	9 9981040	19-8	0 0018960	8 9714954	2265	11 0285041
22 8 9709463	2238	1 0287533	9 9980921	19-8	0 0019077	8 9728517	2258	11 0271453
23 8 9722895	2231	1 0273565	9 9980802	19-8	0 0019195	8 9742093	2251	11 0257908
24 8 9736280	2224	1 0259572	9 9980683	19-8	0 0019312	8 9755597	2244	11 0244405
25 8 9749624	2217	1 0245563	9 9980563	20-0	0 0019437	8 9769000	2237	11 0230949
26 8 9762926	2210	1 0231574	9 9980442	20-0	0 0019557	8 9782433	2230	11 0217517
27 8 9776168	2203	1 0217581	9 9980323	20-0	0 0019677	8 9795865	2223	11 0204133
28 8 9789463	2197	1 0203592	9 9980202	20-2	0 0019798	8 9809266	2216	11 0190774
29 8 9802789	2190	1 0189611	9 9980081	20-2	0 0019919	8 9822507	2210	11 0177492
30 8 9816729	2183	1 0175621	9 9979963	20-2	0 0020040	8 9835769	2204	11 0164213
31 8 9830829	2177	1 0161631	9 9979843	20-3	0 0020162	8 9848991	2197	11 0150999
32 8 9844850	2170	1 0147641	9 9979721	20 3	0 0020285	8 9862173	2191	11 0137528
33 8 9858910	2163	1 0133651	9 9979599	20 3	0 0020407	8 9875317	2184	11 0124085
34 8 9872981	2157	1 0119661	9 9979477	20-5	0 0020530	8 9888421	2178	11 0110579
35 8 9887085	2150	1 0105671	9 9979355	20 5	0 0020653	8 9901487	2171	11 0098135
36 8 9901137	2144	1 0091681	9 9979232	20 7	0 0020777	8 9914514	2165	11 0085486
37 8 9915200	2138	1 0077691	9 9979109	20 7	0 0020901	8 9927503	2159	11 0072497
38 8 9929242	2131	1 0063701	9 9978987	20 7	0 0021025	8 9940454	2152	11 0059516
39 8 9943217	2125	1 0049711	9 9978865	20 8	0 0021149	8 9953367	2146	11 0046623
40 8 9957198	2119	1 0035721	9 9978743	21 0	0 0021273	8 9966243	2140	11 0033757
41 8 9971173	2112	1 0021731	9 9978621	21 0	0 0021397	8 9979081	2134	11 0020919
42 8 9985155	2106	1 0007741	9 9978499	21-0	0 0021521	8 9991887	2127	11 0008117
43 8 9999136	2100	1 0003751	9 9978377	21-0	0 0021645	9 0004677	2121	11 0005353
44 8 9999595	2094	1 0003751	9 9978255	21-2	0 0021769	9 0017375	2115	11 0002657
45 8 9999816	2088	1 0003751	9 9978133	21-2	0 0021893	9 0030066	2108	11 0000004
46 8 9999968	2082	1 0003751	9 9978011	21-2	0 0022017	9 0042721	2102	11 0005729
47 8 9999968	2076	1 0003751	9 9977889	21-3	0 0022141	9 0055340	2097	11 0005466
48 8 9999968	2070	1 0003751	9 9977767	21-3	0 0022265	9 0067924	2091	11 0005203
49 8 9999968	2064	1 0003751	9 9977645	21 5	0 0022389	9 0080471	2085	11 0005040
50 8 9999968	2058	1 0003751	9 9977523	21 5	0 0022513	9 0093068	2079	11 0004877
51 8 9999968	2052	1 0003751	9 9977401	21 7	0 0022637	9 0105664	2074	11 0004714
52 8 9999968	2046	1 0003751	9 9977279	21 7	0 0022761	9 0118260	2068	11 0004551
53 8 9999968	2040	1 0003751	9 9977157	21 8	0 0022885	9 0130856	2062	11 0004388
54 8 9999968	2034	1 0003751	9 9977035	21 8	0 0023009	9 0143452	2056	11 0004225
55 8 9999968	2028	1 0003751	9 9976913	21 8	0 0023133	9 0156048	2051	11 0004062
56 8 9999968	2022	1 0003751	9 9976791	22-0	0 0023257	9 0168644	2045	11 0003899
57 8 9999968	2017	1 0003751	9 9976669	22-0	0 0023381	9 0181240	2040	11 0003736
58 8 9999968	2011	1 0003751	9 9976547	22-0	0 0023505	9 0193836	2034	11 0003573
59 8 9999968	2006	1 0003751	9 9976425	22-2	0 0023629	9 0206432	2028	11 0003410
60 8 9999968	2000	1 0003751	9 9976303		0 0023753	9 0219028		11 0003247
Cosine	D10"	Comp. cos.	Sine	D10"	Comp. sin.	Cotang.	D10"	Tangent

6 Deg.

Tab. 18

	Sine	D10"	Comp sin.	Cosine	D10"	Comps cos.	Tangent	D10"	Cotangent
0	9-0192946	2000	0-9807654	9-9976143	22-0	0-0023857	9-0216282	2023	10-9783718
1	9-0204348	1995	0-9795552	9-9976011	22-3	0-0023988	9-0228336	2017	10-9771662
2	9-0216318	1989	0-9783682	9-9975877	22-5	0-0024122	9-0240441	2011	10-9759559
3	9-0228254	1984	0-9771746	9-9975743	22-8	0-0024257	9-0252510	2006	10-9747490
4	9-0240157	1978	0-9759833	9-9975609	22-9	0-0024391	9-0264548	2000	10-9735452
5	9-0252027	1973	0-9747973	9-9975475	22-9	0-0024527	9-0276553	1995	10-9723448
6	9-0263865	1967	0-9736135	9-9975340	22-7	0-0024666	9-0288524	1990	10-9711476
7	9-0275669	1962	0-9724321	9-9975205	22-7	0-0024799	9-0300464	1985	10-9699536
8	9-0287442	1957	0-9712558	9-9975069	22-7	0-0024931	9-0312377	1979	10-9687627
9	9-0299182	1951	0-9700818	9-9974933	22-7	0-0025067	9-0324249	1974	10-9675751
10	9-0310890	1946	0-9689110	9-9974797	22-8	0-0025203	9-0336093	1969	10-9663907
11	9-0322567	1941	0-9677433	9-9974660	22-8	0-0025340	9-0347906	1964	10-9652094
12	9-0334213	1935	0-9665788	9-9974523	22-8	0-0025477	9-0359688	1958	10-9640319
13	9-0345828	1930	0-9654175	9-9974386	22-8	0-0025614	9-0371439	1953	10-9628561
14	9-0357407	1925	0-9642595	9-9974248	22-8	0-0025752	9-0383159	1948	10-9616841
15	9-0368958	1920	0-9631042	9-9974110	22-8	0-0025890	9-0394848	1943	10-9605152
16	9-0380477	1915	0-9619523	9-9973971	22-8	0-0026029	9-0406506	1938	10-9593494
17	9-0391966	1910	0-9608034	9-9973833	22-8	0-0026167	9-0418134	1933	10-9581866
18	9-0403424	1905	0-9596576	9-9973695	22-8	0-0026307	9-0429731	1928	10-9570269
19	9-0414852	1900	0-9585148	9-9973557	22-8	0-0026446	9-0441299	1923	10-9558701
20	9-0426249	1895	0-9573751	9-9973419	22-8	0-0026585	9-0452836	1918	10-9547163
21	9-0437617	1889	0-9562383	9-9973273	22-8	0-0026727	9-0464343	1913	10-9535657
22	9-0448954	1884	0-9551044	9-9973132	22-8	0-0026868	9-0475821	1908	10-9524179
23	9-0460261	1879	0-9539739	9-9972991	22-8	0-0027009	9-0487270	1903	10-9512730
24	9-0471558	1873	0-9528462	9-9972851	22-8	0-0027151	9-0498689	1898	10-9501311
25	9-0482836	1868	0-9517214	9-9972703	22-8	0-0027295	9-0510078	1893	10-9489922
26	9-0494085	1860	0-9505995	9-9972566	22-8	0-0027439	9-0521439	1889	10-9478561
27	9-0505319	1855	0-9494806	9-9972423	22-8	0-0027577	9-0532771	1884	10-9467229
28	9-0516535	1850	0-9483646	9-9972280	22-8	0-0027720	9-0544074	1879	10-9455926
29	9-0527745	1845	0-9472515	9-9972137	22-8	0-0027865	9-0555359	1875	10-9444651
30	9-0538988	1840	0-9461412	9-9971993	22-8	0-0028007	9-0566593	1870	10-9433405
31	9-0550161	1834	0-9450339	9-9971849	22-8	0-0028151	9-0577813	1865	10-9422187
32	9-0561270	1828	0-9439294	9-9971701	22-8	0-0028296	9-0589002	1860	10-9410998
33	9-0572312	1823	0-9428277	9-9971559	22-8	0-0028441	9-0600164	1855	10-9399830
34	9-0583281	1817	0-9417289	9-9971414	22-8	0-0028586	9-0611295	1851	10-9388703
35	9-0594176	1812	0-9406328	9-9971268	22-8	0-0028732	9-0622405	1846	10-9377595
36	9-0605000	1807	0-9395396	9-9971122	22-8	0-0028878	9-0633483	1842	10-9366518
37	9-0615753	1803	0-9384491	9-9970976	22-8	0-0029024	9-0644535	1837	10-9355467
38	9-0626536	1800	0-9373614	9-9970829	22-8	0-0029171	9-0655556	1833	10-9344444
39	9-0637231	1804	0-9362765	9-9970682	22-8	0-0029318	9-0666553	1829	10-9333447
40	9-0647947	1799	0-9351943	9-9970535	22-8	0-0029465	9-0677522	1824	10-9322478
41	9-0658685	1794	0-9341144	9-9970387	22-8	0-0029613	9-0688463	1819	10-9311551
42	9-0669441	1790	0-9330381	9-9970239	22-8	0-0029761	9-0699384	1815	10-9300679
43	9-0680210	1786	0-9319646	9-9970090	22-8	0-0029910	9-0710270	1810	10-9289850
44	9-0690994	1781	0-9308939	9-9969941	22-8	0-0030058	9-0721133	1806	10-9278967
45	9-0701761	1777	0-9298259	9-9969792	22-8	0-0030206	9-0731969	1802	10-9268031
46	9-0712512	1772	0-9287599	9-9969642	22-8	0-0030354	9-0742779	1797	10-9257044
47	9-0723245	1768	0-9276945	9-9969492	22-8	0-0030502	9-0753565	1793	10-9246037
48	9-0733963	1763	0-9266307	9-9969343	22-8	0-0030650	9-0764331	1789	10-9235079
49	9-0744654	1759	0-9255685	9-9969191	22-8	0-0030800	9-0775053	1784	10-9224147
50	9-0755329	1755	0-9245081	9-9969040	22-8	0-0030949	9-0785760	1780	10-9213240
51	9-0765989	1750	0-9234497	9-9968888	22-8	0-0031112	9-0796441	1776	10-9202359
52	9-0776632	1746	0-9223934	9-9968736	22-8	0-0031263	9-0807096	1772	10-9191490
53	9-0787261	1742	0-9213391	9-9968584	22-8	0-0031416	9-0817727	1767	10-9180644
54	9-0797876	1738	0-9202868	9-9968431	22-8	0-0031569	9-0828331	1763	10-9169819
55	9-0808479	1733	0-9192361	9-9968278	22-8	0-0031722	9-0838911	1759	10-9158995
56	9-0819070	1729	0-9181874	9-9968125	22-8	0-0031875	9-0849460	1755	10-9148181
57	9-0829649	1725	0-9171404	9-9967971	22-8	0-0032029	9-0859986	1751	10-9137376
58	9-0840217	1721	0-9160951	9-9967817	22-8	0-0032183	9-0870501	1747	10-9126589
59	9-0850774	1717	0-9150513	9-9967662	22-8	0-0032338	9-0880991	1743	10-9115819
60	9-0861320	1713	0-9140089	9-9967507	22-8	0-0032493	9-0891466	1739	10-9105065
	Cosine	D10"	Comp. cos.	Sine	D10"	Comp. sin.	Tangent	D10"	Cotangent

Tab. 18.

De 6

7 Deg.

Tab. 18.

	Sine	D10'	Comp. sin	Cosine	D10'	Comp. cos	Tangent	D10'	Cotangent
1	9 0858935	1713	0 9141055	9 9967507	25 8	0 0032193	9 0591438	1739	10 9108562
2	9 0869221	1709	0 9130779	9 9967352	26 0	0 0032648	9 0901869	1735	10 9098131
3	9 0879473	1704	0 9120527	9 9967196	26 0	0 0033204	9 0912222	1731	10 9087793
4	9 0889700	1700	0 9110300	9 9967040	26 0	0 0033760	9 0922600	1727	10 9077540
5	9 0899903	1696	0 9100097	9 9966884	26 0	0 0034316	9 0933020	1723	10 9066980
6	9 0910082	1692	0 9089918	9 9966727	26 0	0 0034873	9 0943355	1719	10 9056645
7	9 0920237	1688	0 9079763	9 9966570	26 0	0 0035430	9 0953667	1715	10 9046333
8	9 0930367	1684	0 9069633	9 9966412	26 0	0 0035988	9 0963955	1711	10 9036045
9	9 0940474	1680	0 9059526	9 9966254	26 0	0 0036546	9 0974219	1707	10 9025781
10	9 0950556	1676	0 9049444	9 9966096	26 0	0 0037104	9 0984460	1703	10 9015540
11	9 0960615	1673	0 9039385	9 9965937	26 0	0 0037662	9 0994678	1699	10 9005329
12	9 0970651	1669	0 9029342	9 9965778	26 0	0 0038220	9 1004872	1695	10 8995128
13	9 0980662	1665	0 9019328	9 9965619	26 0	0 0038778	9 1015044	1691	10 8984956
14	9 0990651	1661	0 9009349	9 9965459	26 0	0 0039336	9 1025192	1687	10 8974780
15	9 1000616	1657	0 8999384	9 9965299	26 0	0 0039894	9 1035317	1683	10 8964633
16	9 1010558	1653	0 8989442	9 9965138	26 0	0 0040452	9 1045420	1680	10 8954580
17	9 1020477	1649	0 8979523	9 9964977	26 0	0 0041010	9 1055500	1676	10 8944500
18	9 1030373	1645	0 8969627	9 9964816	26 0	0 0041568	9 1065557	1672	10 8934443
19	9 1040246	1642	0 8959754	9 9964655	26 0	0 0042126	9 1075591	1669	10 8924409
20	9 1050096	1638	0 8949904	9 9964493	27 2	0 0042684	9 1085604	1665	10 8914397
21	9 1059924	1634	0 8940076	9 9964330	27 2	0 0043242	9 1095594	1661	10 8904390
22	9 1069729	1630	0 8930271	9 9964167	27 2	0 0043800	9 1105562	1658	10 8894399
23	9 1079512	1627	0 8920488	9 9964004	27 2	0 0044358	9 1115508	1654	10 8884429
24	9 1089272	1623	0 8910728	9 9963841	27 2	0 0044916	9 1125431	1650	10 8874469
25	9 1099010	1619	0 8900990	9 9963677	27 2	0 0045474	9 1135333	1647	10 8864527
26	9 1108726	1616	0 8891274	9 9963513	27 2	0 0046032	9 1145213	1643	10 8854593
27	9 1118420	1612	0 8881580	9 9963348	27 2	0 0046590	9 1155072	1639	10 8844668
28	9 1128092	1608	0 8871908	9 9963183	27 2	0 0047148	9 1164909	1636	10 8834751
29	9 1137742	1605	0 8862258	9 9963018	27 2	0 0047706	9 1174724	1632	10 8824842
30	9 1147370	1601	0 8852630	9 9962852	27 2	0 0048264	9 1184518	1629	10 8814942
31	9 1156977	1597	0 8843023	9 9962686	27 8	0 0048822	9 1194291	1625	10 8805059
32	9 1166562	1594	0 8833438	9 9962519	27 8	0 0049380	9 1204043	1622	10 8795197
33	9 1176125	1590	0 8823875	9 9962352	27 8	0 0049938	9 1213773	1618	10 8785357
34	9 1185668	1587	0 8814333	9 9962185	28 0	0 0050496	9 1223482	1615	10 8775538
35	9 1195188	1583	0 8804812	9 9962017	28 0	0 0051054	9 1233171	1611	10 8765739
36	9 1204688	1580	0 8795312	9 9961849	28 0	0 0051612	9 1242839	1608	10 8755959
37	9 1214167	1576	0 8785833	9 9961681	28 0	0 0052170	9 1252486	1604	10 8746198
38	9 1223624	1573	0 8776376	9 9961512	28 0	0 0052728	9 1262112	1601	10 8736456
39	9 1233061	1569	0 8766939	9 9961343	28 0	0 0053286	9 1271718	1597	10 8726732
40	9 1242477	1566	0 8757523	9 9961174	28 0	0 0053844	9 1281303	1594	10 8717027
41	9 1251872	1562	0 8748126	9 9961004	28 0	0 0054402	9 1290868	1591	10 8707340
42	9 1261246	1559	0 8738750	9 9960834	28 0	0 0054960	9 1300413	1587	10 8697671
43	9 1270600	1556	0 8729393	9 9960663	28 0	0 0055518	9 1309937	1584	10 8688020
44	9 1279934	1552	0 8720066	9 9960492	28 0	0 0056076	9 1319442	1581	10 8678387
45	9 1289247	1549	0 8710753	9 9960321	28 0	0 0056634	9 1328926	1577	10 8668771
46	9 1298539	1545	0 8701461	9 9960149	28 0	0 0057192	9 1338391	1574	10 8659171
47	9 1307812	1542	0 8692188	9 9959977	28 0	0 0057750	9 1347833	1571	10 8649587
48	9 1317064	1539	0 8682936	9 9959804	28 0	0 0058308	9 1357260	1567	10 8640019
49	9 1326299	1535	0 8673703	9 9959631	28 0	0 0058866	9 1366665	1564	10 8630467
50	9 1335533	1532	0 8664491	9 9959458	29 0	0 0059424	9 1376051	1561	10 8620931
51	9 1344769	1529	0 8655298	9 9959284	29 0	0 0060000	9 1385417	1558	10 8611413
52	9 1353987	1525	0 8646125	9 9959111	29 0	0 0060576	9 1394764	1555	10 8601913
53	9 1363028	1522	0 8636972	9 9958936	29 0	0 0061152	9 1404092	1551	10 8592430
54	9 1372161	1519	0 8627839	9 9958761	29 0	0 0061728	9 1413400	1548	10 8582960
55	9 1381275	1516	0 8618725	9 9958586	29 0	0 0062304	9 1422688	1545	10 8573511
56	9 1390376	1513	0 8609630	9 9958411	29 0	0 0062880	9 1431959	1542	10 8564084
57	9 1399445	1509	0 8600555	9 9958235	29 0	0 0063456	9 1441210	1539	10 8554679
58	9 1408501	1506	0 8591499	9 9958059	29 0	0 0064032	9 1450442	1536	10 8545293
59	9 1417537	1503	0 8582463	9 9957882	29 0	0 0064608	9 1459655	1532	10 8535925
60	9 1426553	1500	0 8573447	9 9957705	29 0	0 0065184	9 1468849	1529	10 8526575
61	9 1435553		0 8564447	9 9957528		0 0065760	9 1478023		10 8517245
	Cosine	D10'	Comp. cos.	Sine	D10'	Comp. sin	Cotang.	D10'	Tangent

Tab. 18.

Deg. 82.

8 Deg.

Tab. 18.

Sine.	D10'	Comp. sin.	Cosine	D10'	Comp. cos.	Tangent	D10'	Cotangent
0	9-1435553	1496	0-8564447	9-9957328	29-7	0-0042472	9-1478025	1526
1	9-1444532	1493	0-8555468	9-9957350	29-7	0-0042650	9-1487182	1528
2	9-1453493	1490	0-8546507	9-9957172	29-8	0-0042828	9-1496321	1530
3	9-1462435	1487	0-8537565	9-9956993	29-8	0-0043007	9-1505441	1517
4	9-1471358	1484	0-8528642	9-9956815	30-0	0-0043185	9-1514545	1514
5	9-1480262	1481	0-8519738	9-9956635	30-0	0-0043365	9-1523627	1511
6	9-1489148	1478	0-8510852	9-9956456	30-0	0-0043544	9-1532692	1508
7	9-1498013	1475	0-8501985	9-9956276	30-2	0-0043724	9-1541739	1505
8	9-1506864	1472	0-8493128	9-9956095	30-2	0-0043905	9-1550769	1502
9	9-1515694	1469	0-8484280	9-9955915	30-2	0-0044085	9-1559780	1499
10	9-1524507	1466	0-8475438	9-9955734	30-3	0-0044266	9-1568773	1496
11	9-1533301	1463	0-8466699	9-9955552	30-3	0-0044448	9-1577748	1493
12	9-1542076	1460	0-8457924	9-9955370	30-3	0-0044630	9-1586706	1490
13	9-1550834	1457	0-8449161	9-9955188	30-5	0-0044812	9-1595646	1487
14	9-1559574	1454	0-8440426	9-9955005	30-5	0-0044995	9-1604569	1484
15	9-1568296	1451	0-8431704	9-9954822	30-5	0-0045178	9-1613471	1481
16	9-1577000	1448	0-8423000	9-9954639	30-7	0-0045361	9-1622351	1478
17	9-1585686	1445	0-8414314	9-9954455	30-7	0-0045545	9-1631231	1475
18	9-1594354	1442	0-8405646	9-9954271	30-7	0-0045729	9-1640083	1472
19	9-1603005	1439	0-8396995	9-9954087	30-8	0-0045913	9-1648919	1470
20	9-1611639	1436	0-8388361	9-9953903	30-8	0-0046098	9-1657737	1467
21	9-1620254	1433	0-8379746	9-9953717	31-0	0-0046283	9-1666538	1464
22	9-1628853	1430	0-8371147	9-9953531	31-0	0-0046469	9-1675322	1461
23	9-1637434	1427	0-8362569	9-9953345	31-0	0-0046655	9-1684089	1458
24	9-1645998	1424	0-8354002	9-9953159	31-0	0-0046841	9-1692839	1455
25	9-1654544	1422	0-8345456	9-9952972	31-2	0-0047028	9-1701572	1453
26	9-1663074	1419	0-8336926	9-9952785	31-2	0-0047215	9-1710289	1450
27	9-1671586	1416	0-8328414	9-9952597	31-3	0-0047403	9-1718989	1447
28	9-1680081	1413	0-8319919	9-9952409	31-3	0-0047591	9-1727672	1444
29	9-1688559	1410	0-8311441	9-9952221	31-3	0-0047779	9-1736338	1442
30	9-1697021	1407	0-8302979	9-9952033	31-5	0-0047967	9-1744988	1439
31	9-1705465	1405	0-8294535	9-9951844	31-7	0-0048156	9-1753622	1436
32	9-1713893	1402	0-8286107	9-9951654	31-7	0-0048346	9-1762239	1433
33	9-1722305	1399	0-8277695	9-9951464	31-7	0-0048536	9-1770840	1431
34	9-1730699	1396	0-8269303	9-9951274	31-7	0-0048726	9-1779425	1428
35	9-1739077	1394	0-8260932	9-9951084	31-8	0-0048916	9-1787998	1425
36	9-1747439	1391	0-8252561	9-9950893	31-8	0-0049107	9-1796546	1423
37	9-1755784	1388	0-8244216	9-9950702	32-0	0-0049298	9-1805082	1420
38	9-1764112	1385	0-8235888	9-9950510	32-0	0-0049490	9-1813602	1417
39	9-1772425	1383	0-8227575	9-9950318	32-0	0-0049682	9-1822106	1415
40	9-1780721	1380	0-8219279	9-9950126	32-2	0-0049874	9-1830595	1412
41	9-1789001	1377	0-8210999	9-9949933	32-2	0-0050066	9-1839068	1409
42	9-1797265	1374	0-8202735	9-9949740	32-3	0-0050258	9-1847525	1407
43	9-1805512	1372	0-8194488	9-9949546	32-3	0-0050450	9-1855966	1404
44	9-1813744	1369	0-8186256	9-9949352	32-3	0-0050642	9-1864392	1402
45	9-1821960	1366	0-8178040	9-9949158	32-3	0-0050834	9-1872802	1399
46	9-1830160	1364	0-8169840	9-9948964	32-5	0-0051036	9-1881196	1396
47	9-1838344	1361	0-8161656	9-9948769	32-7	0-0051231	9-1889575	1394
48	9-1846512	1359	0-8153488	9-9948575	32-7	0-0051427	9-1897939	1391
49	9-1854665	1356	0-8145335	9-9948377	32-7	0-0051623	9-1906287	1389
50	9-1862802	1353	0-8137196	9-9948181	32-7	0-0051819	9-1914621	1386
51	9-1870923	1351	0-8129077	9-9947985	32-8	0-0052015	9-1922939	1384
52	9-1879029	1348	0-8120971	9-9947788	32-8	0-0052212	9-1931244	1381
53	9-1887120	1346	0-8112880	9-9947591	33-0	0-0052409	9-1939529	1379
54	9-1895195	1343	0-8104806	9-9947393	33-0	0-0052607	9-1947802	1376
55	9-1903254	1341	0-8096746	9-9947195	33-0	0-0052805	9-1956059	1374
56	9-1911299	1338	0-8088701	9-9946997	33-0	0-0053003	9-1964302	1371
57	9-1919328	1336	0-8080672	9-9946798	33-2	0-0053202	9-1972530	1369
58	9-1927342	1333	0-8072658	9-9946599	33-3	0-0053401	9-1980743	1366
59	9-1935341	1330	0-8064659	9-9946399	33-3	0-0053601	9-1988941	1364
60	9-1943324		0-8056676	9-9946199		0-0053801	9-1997125	
Cosine.	D10'	Comp. cos.	Sine	D10'	Comp. sin.	Cotang.	D10'	Tangent

Tab. 18.

Dec. 81.

9 Deg.

Tab. 18.

Sine	D10"	Comp. sin.	Cosine	D10"	Comp. cos.	Tangent	D10"	Cotangent
09-1843324	1328	0-8056676	9-9946199	33-3	0-0053801	9-1997125	1361	10-8002875
19-1951293	1326	0-8048707	9-9945999	33-5	0-0054001	9-2005294	1359	10-7994706
29-1959247	1323	0-8040753	9-9945798	33-5	0-0054202	9-2013449	1356	10-7986551
39-1967186	1321	0-8032814	9-9945596	33-5	0-0054403	9-2021688	1354	10-7978412
49-1975110	1318	0-8024890	9-9945396	33-7	0-0054604	9-2029914	1352	10-7970286
59-1983019	1316	0-8016981	9-9945194	33-7	0-0054806	9-2037827	1349	10-7962175
69-1990913	1313	0-8009087	9-9944992	33-8	0-0055008	9-2045922	1347	10-7954078
79-1998793	1310	0-8001207	9-9944789	33-8	0-0055211	9-2054004	1342	10-7945996
89-2006658	1311	0-7993342	9-9944587	33-8	0-0055413	9-2062072	1343	10-7937928
99-2014509	1308	0-7985491	9-9944383	34-0	0-0055617	9-2070126	1342	10-7929874
10-2022385	1306	0-7977655	9-9944180	34-0	0-0055820	9-2078165	1340	10-7921835
11-2030167	1304	0-7969833	9-9943975	34-0	0-0056025	9-2086191	1338	10-7913809
12-2037974	1301	0-7962026	9-9943771	34-2	0-0056229	9-2094203	1335	10-7905797
13-2045766	1296	0-7954234	9-9943566	34-2	0-0056431	9-2102200	1332	10-7897806
14-2053545	1294	0-7946455	9-9943361	34-2	0-0056634	9-2110184	1331	10-7889816
15-2061309	1294	0-7938691	9-9943156	34-2	0-0056844	9-2118153	1328	10-7881847
16-2069059	1292	0-7930941	9-9942950	34-3	0-0057050	9-2126109	1326	10-7873891
17-2076795	1289	0-7923205	9-9942745	34-3	0-0057257	9-2134051	1324	10-7865949
18-2084516	1287	0-7915484	9-9942537	34-5	0-0057463	9-2141980	1321	10-7858020
19-2092224	1283	0-7907776	9-9942330	34-5	0-0057670	9-2149894	1319	10-7850096
20-2099917	1282	0-7900083	9-9942122	34-7	0-0057878	9-2157795	1317	10-7842205
21-2107597	1280	0-7892403	9-9941914	34-7	0-0058086	9-2165683	1315	10-7834337
22-2115262	1278	0-7884737	9-9941706	34-7	0-0058294	9-2173566	1312	10-7826444
23-2122914	1275	0-7877086	9-9941498	34-8	0-0058502	9-2181447	1310	10-7818538
24-2130552	1273	0-7869449	9-9941289	34-8	0-0058711	9-2189321	1308	10-7810630
25-2138176	1271	0-7861824	9-9941079	35-0	0-0058921	9-2197197	1305	10-7802733
26-2145787	1268	0-7854213	9-9940870	35-0	0-0059130	9-2205097	1303	10-7794838
27-2153384	1266	0-7846616	9-9940679	35-0	0-0059341	9-2212974	1301	10-7786943
28-2160907	1264	0-7839033	9-9940449	35-2	0-0059551	9-2220838	1299	10-7779048
29-2168456	1261	0-7831464	9-9940238	35-2	0-0059762	9-2228698	1297	10-7771152
30-2176092	1259	0-7823908	9-9940027	35-2	0-0059973	9-2236565	1294	10-7763256
31-2183635	1257	0-7816363	9-9939817	35-3	0-0060185	9-2244389	1292	10-7755351
32-2191164	1255	0-7808836	9-9939603	35-3	0-0060397	9-2252161	1290	10-7747439
33-2198680	1253	0-7801320	9-9939391	35-3	0-0060609	9-2259959	1288	10-7739511
34-2206182	1250	0-7793818	9-9939178	35-5	0-0060822	9-2267700	1285	10-7731580
35-2213671	1248	0-7786329	9-9938963	35-5	0-0061035	9-2275476	1284	10-7723648
36-2221147	1246	0-7778853	9-9938752	35-5	0-0061248	9-2283255	1281	10-7715715
37-2228606	1244	0-7771391	9-9938538	35-7	0-0061462	9-2291051	1279	10-7707782
38-2236059	1242	0-7763941	9-9938324	35-7	0-0061676	9-2298775	1277	10-7700000
39-2243495	1239	0-7756505	9-9938109	35-7	0-0061891	9-2306538	1275	10-7692167
40-2250918	1237	0-7749082	9-9937894	35-8	0-0062106	9-2314302	1273	10-7684332
41-2258329	1235	0-7741672	9-9937679	36-0	0-0062321	9-2322066	1271	10-7676495
42-2265725	1233	0-7734275	9-9937463	36-0	0-0062537	9-2329829	1269	10-7668658
43-2273110	1231	0-7726890	9-9937247	36-0	0-0062753	9-2337583	1267	10-7660811
44-2280481	1228	0-7719519	9-9937030	36-2	0-0062970	9-2345341	1265	10-7652964
45-2287839	1226	0-7712161	9-9936813	36-2	0-0063187	9-2353106	1262	10-7645117
46-2295185	1224	0-7704815	9-9936596	36-2	0-0063404	9-2360859	1260	10-7637270
47-2302518	1222	0-7697482	9-9936375	36-3	0-0063622	9-2368619	1258	10-7629423
48-2309834	1220	0-7690162	9-9936160	36-3	0-0063840	9-2376378	1254	10-7621576
49-2317145	1218	0-7682853	9-9935942	36-5	0-0064058	9-2384123	1252	10-7613729
50-2324440	1216	0-7675560	9-9935723	36-5	0-0064277	9-2391877	1250	10-7605882
51-2331722	1214	0-7668278	9-9935504	36-5	0-0064496	9-2399621	1248	10-7598035
52-2338992	1212	0-7660908	9-9935285	36-7	0-0064715	9-2407378	1246	10-7590188
53-2346244	1209	0-7653571	9-9935065	36-7	0-0064935	9-2415135	1244	10-7582341
54-2353495	1207	0-7646260	9-9934844	36-7	0-0065156	9-2422890	1242	10-7574494
55-2360726	1205	0-7638974	9-9934624	36-8	0-0065376	9-2430643	1240	10-7566647
56-2367946	1203	0-7631704	9-9934403	36-8	0-0065597	9-2438393	1238	10-7558800
57-2375155	1201	0-7624447	9-9934181	37-0	0-0065819	9-2446147	1236	10-7550953
58-2382349	1199	0-7617165	9-9933959	37-0	0-0066041	9-2453898	1234	10-7543106
59-2389532	1197	0-7610468	9-9933737	37-0	0-0066263	9-2461644	1232	10-7535259
60-2396702	1195	0-7603298	9-9933515	37-0	0-0066485	9-2469388	1230	10-7527412
Cosine	D10"	Comp. cos.	Sine	D10"	Comp. sin.	Cotang.	D10"	Tangent

Tab. 18.

Deg. 80.

10 Deg.

Tab. 18.

Sine	D10"	Comp. sin.	Cosine	D10"	Comp. cos.	Tangent	D10"	Cotangent
0.9-2396702	1193	0.7603298	9.9933515	37.2	0.0066482	9.24463188	1230	10.7586812
1.9-2403861	1191	0.7596139	9.9933299	37.2	0.0066708	9.24470369	1228	10.7529431
2.9-2411007	1189	0.7588993	9.9933068	37.3	0.0066932	9.24477939	1226	10.7522061
3.9-2418141	1187	0.7581859	9.9932845	37.3	0.0067156	9.24485297	1224	10.7514703
4.9-2425264	1185	0.7574736	9.9932621	37.5	0.0067379	9.24492643	1222	10.7507357
5.9-2432374	1183	0.7567626	9.9932396	37.5	0.0067604	9.24499978	1220	10.7500022
6.9-2439472	1181	0.7560528	9.9932171	37.5	0.0067829	9.2507301	1218	10.7492699
7.9-2446558	1179	0.7553440	9.9931946	37.7	0.0068054	9.2514612	1216	10.7485388
8.9-2453632	1177	0.7546368	9.9931720	37.7	0.0068280	9.2521912	1215	10.7478068
9.9-2460695	1175	0.7539305	9.9931494	37.7	0.0068506	9.2529200	1213	10.7470760
10.9-2467746	1173	0.7532254	9.9931268	37.8	0.0068732	9.2536477	1211	10.7463453
11.9-2474784	1171	0.7525216	9.9931041	37.8	0.0068959	9.2543743	1209	10.7456157
12.9-2481811	1169	0.7518189	9.9930814	37.8	0.0069186	9.2551009	1207	10.7448860
13.9-2488827	1167	0.7511173	9.9930587	38.0	0.0069413	9.2558272	1205	10.7441564
14.9-2495850	1165	0.7504170	9.9930359	38.0	0.0069641	9.2565534	1203	10.7434268
15.9-2502822	1163	0.7497175	9.9930131	38.2	0.0069869	9.2572792	1201	10.7426973
16.9-2509803	1161	0.7490197	9.9929902	38.2	0.0070098	9.2579901	1199	10.7419679
17.9-2516772	1159	0.7483228	9.9929675	38.2	0.0070327	9.2587099	1197	10.7412384
18.9-2523729	1158	0.7476271	9.9929444	38.3	0.0070556	9.2594285	1196	10.7405091
19.9-2530675	1156	0.7469323	9.9929214	38.3	0.0070786	9.2601461	1194	10.7397798
20.9-2537600	1154	0.7462391	9.9928984	38.3	0.0071016	9.2608625	1192	10.7390505
21.9-2544533	1152	0.7455468	9.9928753	38.5	0.0071247	9.2615779	1190	10.7383212
22.9-2551444	1150	0.7448556	9.9928522	38.5	0.0071478	9.2622931	1188	10.7375919
23.9-2558344	1148	0.7441656	9.9928291	38.5	0.0071709	9.2630083	1186	10.7368626
24.9-2565233	1146	0.7434767	9.9928059	38.7	0.0071941	9.2637173	1185	10.7361333
25.9-2572110	1144	0.7427890	9.9927827	38.7	0.0072172	9.2644283	1183	10.7354040
26.9-2578977	1142	0.7421023	9.9927595	38.7	0.0072403	9.2651382	1181	10.7346747
27.9-2585832	1140	0.7414168	9.9927362	38.8	0.0072635	9.2658470	1179	10.7339453
28.9-2592676	1138	0.7407324	9.9927129	38.8	0.0072867	9.2665547	1177	10.7332160
29.9-2599509	1137	0.7400491	9.9926895	39.0	0.0073100	9.2672613	1176	10.7324867
30.9-2606350	1135	0.7393670	9.9926661	39.0	0.0073333	9.2679669	1174	10.7317574
31.9-2613141	1133	0.7386859	9.9926427	39.2	0.0073567	9.2686714	1172	10.7310281
32.9-2619941	1131	0.7380059	9.9926192	39.2	0.0073800	9.2693749	1170	10.7302988
33.9-2626729	1129	0.7373277	9.9925955	39.2	0.0074034	9.2700772	1169	10.7295695
34.9-2633507	1128	0.7366495	9.9925720	39.2	0.0074267	9.2707786	1167	10.7288402
35.9-2640274	1126	0.7359726	9.9925480	39.4	0.0074501	9.2714788	1165	10.7281109
36.9-2647020	1124	0.7352970	9.9925245	39.5	0.0074735	9.2721780	1164	10.7273816
37.9-2653775	1122	0.7346225	9.9925011	39.5	0.0074969	9.2728762	1162	10.7266523
38.9-2660509	1120	0.7339491	9.9924776	39.5	0.0075203	9.2735733	1160	10.7259230
39.9-2667232	1119	0.7332768	9.9924539	39.7	0.0075436	9.2742694	1158	10.7251937
40.9-2673945	1117	0.7326055	9.9924304	39.7	0.0075669	9.2749644	1156	10.7244644
41.9-2680637	1115	0.7319353	9.9924068	39.8	0.0075902	9.2756584	1155	10.7237351
42.9-2687338	1113	0.7312662	9.9923832	39.8	0.0076136	9.2763514	1153	10.7230058
43.9-2694019	1112	0.7305981	9.9923595	39.8	0.0076369	9.2770434	1151	10.7222765
44.9-2700689	1110	0.7299311	9.9923358	40.0	0.0076603	9.2777343	1150	10.7215472
45.9-2707348	1108	0.7292652	9.9923120	40.0	0.0076836	9.2784242	1148	10.7208179
46.9-2713997	1106	0.7286003	9.9922882	40.0	0.0077070	9.2791131	1146	10.7200886
47.9-2720635	1105	0.7279355	9.9922645	40.2	0.0077304	9.2798009	1145	10.7193593
48.9-2727263	1103	0.7272723	9.9922408	40.2	0.0077538	9.2804878	1143	10.7186300
49.9-2733880	1101	0.7266120	9.9922171	40.3	0.0077772	9.2811736	1141	10.7179007
50.9-2740487	1100	0.7259553	9.9921932	40.3	0.0078006	9.2818585	1140	10.7171714
51.9-2747083	1098	0.7252917	9.9921693	40.3	0.0078240	9.2825423	1138	10.7164421
52.9-2753669	1096	0.7246311	9.9921454	40.5	0.0078474	9.2832251	1136	10.7157128
53.9-2760245	1094	0.7239735	9.9921215	40.5	0.0078708	9.2839079	1135	10.7149835
54.9-2766811	1092	0.7233189	9.9920976	40.5	0.0078942	9.2845907	1133	10.7142542
55.9-2773366	1090	0.7226663	9.9920737	40.7	0.0079176	9.2852735	1131	10.7135249
56.9-2779911	1089	0.7220169	9.9920498	40.7	0.0079410	9.2859563	1130	10.7127956
57.9-2786444	1087	0.7213695	9.9920259	40.8	0.0079644	9.2866391	1128	10.7120663
58.9-2792970	1086	0.7207230	9.9919999	40.8	0.0079878	9.2873219	1126	10.7113370
59.9-2799484	1084	0.7200784	9.9919740	40.8	0.0080112	9.2880047	1125	10.7106077
60.9-2805988		0.7194357	9.9919481	40.8	0.0080346	9.2886875		10.7098784
Cosine	D10"	Comp. cos.	Sine	D10"	Comp. sin.	Cotang.	D10"	Tangent.

Tab. 18.

Deg. 79.

Sine	D10'	Comp. sin	Cosine	D10'	Comp. cos.	Tangent	D10'	Cotangent
0.9235588	1092	0.7194017	0.9919466	41	0.0080531	0.2886523	1123	10.7115477 60
1.9235588	1081	0.7185151	0.9919230	41	0.0080780	0.2893263	1122	10.7106737 59
2.9235588	1071	0.7176359	0.9918974	41	0.0081026	0.2899993	1121	10.7100007 58
3.9235588	1061	0.7167559	0.9918727	41	0.0081273	0.2906713	1120	10.7093287 57
4.9235588	1057	0.7168929	0.9918480	41	0.0081520	0.2913424	1119	10.7086567 56
5.9235588	1056	0.7161641	0.9918233	41	0.0081768	0.2920126	1117	10.7079847 55
6.9235588	1076	0.7155197	0.9917986	41	0.0082014	0.2926817	1116	10.7073127 54
7.9235588	1072	0.7148763	0.9917737	41	0.0082263	0.2933500	1114	10.7066400 53
8.9235588	1071	0.7142339	0.9917489	41	0.0082511	0.2940172	1113	10.7059678 52
9.9235588	1069	0.7135924	0.9917240	41	0.0082760	0.2946833	1111	10.7052951 51
10.9235588	1067	0.7129520	0.9916991	41	0.0083009	0.2953489	1109	10.7046221 50
11.9235588	1066	0.7123125	0.9916741	42	0.0083259	0.2960134	1107	10.7039496 49
12.9235588	1064	0.7116740	0.9916492	42	0.0083509	0.2966769	1106	10.7032771 48
13.9235588	1063	0.7110364	0.9916241	42	0.0083759	0.2973395	1104	10.7026046 47
14.9235588	1061	0.7103999	0.9915990	42	0.0084010	0.2980011	1103	10.7019321 46
15.9235588	1059	0.7097643	0.9915739	42	0.0084261	0.2986618	1101	10.7012596 45
16.9235588	1057	0.7091296	0.9915488	42	0.0084512	0.2993216	1100	10.7005871 44
17.9235588	1056	0.7084960	0.9915236	42	0.0084764	0.2999804	1098	10.7000146 43
18.9235588	1054	0.7078633	0.9914984	42	0.0085016	0.3006383	1097	10.6993421 42
19.9235588	1053	0.7072315	0.9914731	42	0.0085269	0.3012954	1095	10.6986696 41
20.9235588	1051	0.7066017	0.9914476	42	0.0085522	0.3019514	1094	10.6979971 40
21.9235588	1050	0.7059709	0.9914223	42	0.0085775	0.3026066	1092	10.6973246 39
22.9235588	1048	0.7053420	0.9913971	42	0.0086029	0.3032609	1090	10.6966521 38
23.9235588	1046	0.7047141	0.9913717	42	0.0086283	0.3039143	1089	10.6959796 37
24.9235588	1045	0.7040871	0.9913462	42	0.0086538	0.3045667	1087	10.6953071 36
25.9235588	1043	0.7034610	0.9913207	42	0.0086793	0.3052183	1086	10.6946346 35
26.9235588	1042	0.7028359	0.9912952	42	0.0087048	0.3058688	1084	10.6939621 34
27.9235588	1040	0.7022117	0.9912696	43	0.0087304	0.3065187	1083	10.6932896 33
28.9235588	1039	0.7015884	0.9912440	43	0.0087560	0.3071673	1081	10.6926171 32
29.9235588	1037	0.7009661	0.9912184	43	0.0087816	0.3078153	1080	10.6919446 31
30.9235588	1036	0.7003447	0.9911927	43	0.0088073	0.3084626	1078	10.6912721 30
31.9235588	1034	0.6997242	0.9911670	43	0.0088330	0.3091098	1077	10.6905996 29
32.9235588	1032	0.6991047	0.9911412	43	0.0088588	0.3097564	1075	10.6899271 28
33.9235588	1031	0.6984860	0.9911154	43	0.0088846	0.3104015	1074	10.6892546 27
34.9235588	1030	0.6978688	0.9910896	43	0.0089104	0.3110461	1072	10.6885821 26
35.9235588	1028	0.6972515	0.9910637	43	0.0089363	0.3116888	1071	10.6879096 25
36.9235588	1026	0.6966353	0.9910378	43	0.0089622	0.3123306	1070	10.6872371 24
37.9235588	1025	0.6960206	0.9910119	43	0.0089881	0.3129707	1068	10.6865646 23
38.9235588	1023	0.6954066	0.9909859	43	0.0090141	0.3136076	1067	10.6858921 22
39.9235588	1022	0.6947934	0.9909596	43	0.0090402	0.3142428	1065	10.6852196 21
40.9235588	1020	0.6941811	0.9909338	43	0.0090662	0.3148831	1064	10.6845471 20
41.9235588	1019	0.6935697	0.9909077	44	0.0090923	0.3155226	1062	10.6838746 19
42.9235588	1017	0.6929593	0.9908811	44	0.0091185	0.3161592	1061	10.6832021 18
43.9235588	1016	0.6923497	0.9908553	44	0.0091447	0.3167950	1059	10.6825296 17
44.9235588	1014	0.6917410	0.9908291	44	0.0091709	0.3174299	1058	10.6818571 16
45.9235588	1013	0.6911332	0.9908030	44	0.0091971	0.3180640	1057	10.6811846 15
46.9235588	1011	0.6905263	0.9907766	44	0.0092234	0.3186972	1055	10.6805121 14
47.9235588	1010	0.6899202	0.9907502	44	0.0092498	0.3193295	1054	10.6798396 13
48.9235588	1008	0.6893151	0.9907239	44	0.0092761	0.3199611	1053	10.6791671 12
49.9235588	1007	0.6887108	0.9906974	44	0.0093026	0.3205918	1051	10.6784946 11
50.9235588	1006	0.6881074	0.9906710	44	0.0093290	0.3212216	1050	10.6778221 10
51.9235588	1004	0.6875049	0.9906443	44	0.0093555	0.3218506	1048	10.6771496 9
52.9235588	1003	0.6869032	0.9906180	44	0.0093820	0.3224788	1047	10.6764771 8
53.9235588	1001	0.6863024	0.9905914	44	0.0094086	0.3231061	1045	10.6758046 7
54.9235588	1000	0.6857025	0.9905648	44	0.0094352	0.3237325	1044	10.6751321 6
55.9235588	998	0.6851035	0.9905382	44	0.0094618	0.3243584	1043	10.6744596 5
56.9235588	997	0.6845053	0.9905115	44	0.0094885	0.3249832	1041	10.6737871 4
57.9235588	996	0.6839079	0.9904848	44	0.0095152	0.3256073	1040	10.6731146 3
58.9235588	994	0.6833115	0.9904580	45	0.0095420	0.3262305	1039	10.6724421 2
59.9235588	992	0.6827159	0.9904312	45	0.0095688	0.3268529	1037	10.6717696 1
60.9235588	991	0.6821211	0.9904044	45	0.0095956	0.3274745	1036	10.6710971 0

Cosine	D10'	Comp. cos.	Sine	D10'	Comp. sin	Cotangent	D10'	Tangent
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12 Deg.

Tab. 18.

Sine	D10"	Comp. sin.	Cosine	D10"	Comp. cos.	Tangent	D10"	Cotangent
0° 3178789	990	0.6821211	9.9904044	45	0.0095956	9.3274745	1035	10.6733255
1° 3184728	988	0.6815272	9.9903775	45	0.0096225	9.3280553	1038	10.6719047
2° 3190659	987	0.6809341	9.9903506	45	0.0096494	9.3287153	1032	10.6712247
3° 3196581	986	0.6803419	9.9903237	45	0.0096763	9.3293545	1030	10.6706653
4° 3202495	984	0.6797505	9.9902967	45	0.0097033	9.3299928	1029	10.6700475
5° 3208400	983	0.6791600	9.9902697	45	0.0097303	9.3305704	1028	10.6694296
6° 3214297	981	0.6785703	9.9902426	45	0.0097574	9.3311872	1027	10.6688138
7° 3220186	980	0.6779814	9.9902155	45	0.0097845	9.3318031	1025	10.6681969
8° 3226066	979	0.6773924	9.9901883	45	0.0098117	9.3324189	1024	10.6675817
9° 3231938	977	0.6768035	9.9901612	45	0.0098388	9.3330327	1022	10.6669673
10° 3237802	976	0.6762198	9.9901339	45	0.0098661	9.3336463	1021	10.6663537
11° 3243657	975	0.6756343	9.9901067	45	0.0098933	9.3342599	1020	10.6657409
12° 3249505	973	0.6750485	9.9900794	45	0.0099206	9.3348731	1019	10.6651289
13° 3255344	972	0.6744656	9.9900521	45	0.0099479	9.3354862	1017	10.6645177
14° 3261174	970	0.6738826	9.9900247	46	0.0099753	9.3360997	1016	10.6639073
15° 3266997	969	0.6733003	9.9899973	46	0.0100027	9.3367132	1015	10.6632976
16° 3272811	968	0.6727189	9.9899698	46	0.0100302	9.3373273	1014	10.6626887
17° 3278617	966	0.6721383	9.9899423	46	0.0100577	9.3379414	1013	10.6620806
18° 3284416	965	0.6715584	9.9899148	46	0.0100852	9.3385567	1012	10.6614733
19° 3290206	963	0.6709784	9.9898873	46	0.0101127	9.3391733	1010	10.6608667
20° 3295988	962	0.6704012	9.9898597	46	0.0101403	9.3397899	1008	10.6602609
21° 3301761	961	0.6698239	9.9898320	46	0.0101680	9.3404044	1007	10.6596559
22° 3307527	960	0.6692473	9.9898043	46	0.0101957	9.3410184	1006	10.6590516
23° 3313285	958	0.6686715	9.9897766	46	0.0102234	9.3415519	1004	10.6584481
24° 3319035	957	0.6680965	9.9897489	46	0.0102511	9.3421546	1003	10.6578454
25° 3324777	956	0.6675223	9.9897211	46	0.0102789	9.3427566	1002	10.6572433
26° 3330511	954	0.6669489	9.9896932	46	0.0103068	9.3433578	1001	10.6566414
27° 3336237	953	0.6663763	9.9896654	46	0.0103346	9.3439583	999	10.6560414
28° 3341955	952	0.6658045	9.9896375	47	0.0103626	9.3445580	998	10.6554420
29° 3347665	950	0.6652335	9.9896095	47	0.0103905	9.3451570	997	10.6548430
30° 3353368	949	0.6646632	9.9895815	47	0.0104183	9.3457552	996	10.6542448
31° 3359062	948	0.6640936	9.9895535	47	0.0104465	9.3463527	994	10.6536473
32° 3364749	946	0.6635251	9.9895254	47	0.0104746	9.3469494	992	10.6530506
33° 3370428	945	0.6629572	9.9894973	47	0.0105027	9.3475454	990	10.6524546
34° 3376099	944	0.6623901	9.9894692	47	0.0105308	9.3481407	989	10.6518593
35° 3381762	943	0.6618238	9.9894410	47	0.0105590	9.3487352	987	10.6512648
36° 3387415	941	0.6612582	9.9894128	47	0.0105872	9.3493290	986	10.6506710
37° 3393065	940	0.6606935	9.9893845	47	0.0106155	9.3499220	985	10.6500780
38° 3398706	939	0.6601294	9.9893562	47	0.0106438	9.3505143	984	10.6494857
39° 3404338	938	0.6595662	9.9893279	47	0.0106721	9.3511059	983	10.6488941
40° 3409963	936	0.6590037	9.9892995	47	0.0107005	9.3516968	982	10.6483032
41° 3415580	935	0.6584420	9.9892711	47	0.0107289	9.3522869	981	10.6477131
42° 3421190	934	0.6578816	9.9892427	47	0.0107573	9.3528763	980	10.6471237
43° 3426792	932	0.6573208	9.9892142	48	0.0107858	9.3534650	979	10.6465350
44° 3432386	931	0.6567614	9.9891856	48	0.0108144	9.3540530	978	10.6459470
45° 3437973	930	0.6562027	9.9891571	48	0.0108429	9.3546402	977	10.6453598
46° 3443552	929	0.6556448	9.9891285	48	0.0108715	9.3552267	976	10.6447733
47° 3449124	927	0.6550876	9.9890999	48	0.0109002	9.3558126	975	10.6441874
48° 3454688	926	0.6545312	9.9890711	48	0.0109289	9.3563977	974	10.6436023
49° 3460245	925	0.6539755	9.9890424	48	0.0109576	9.3569821	973	10.6430179
50° 3465794	924	0.6534206	9.9890137	48	0.0109863	9.3575658	972	10.6424342
51° 3471336	922	0.6528664	9.9889849	48	0.0110151	9.3581487	971	10.6418518
52° 3476870	921	0.6523130	9.9889560	48	0.0110440	9.3587310	970	10.6412690
53° 3482397	920	0.6517603	9.9889271	48	0.0110729	9.3593126	969	10.6406874
54° 3487917	919	0.6512083	9.9888982	48	0.0111018	9.3598935	968	10.6401065
55° 3493429	917	0.6506571	9.9888693	48	0.0111307	9.3604736	967	10.6395264
56° 3498934	916	0.6501066	9.9888403	48	0.0111597	9.3610531	966	10.6389469
57° 3504432	915	0.6495568	9.9888113	48	0.0111887	9.3616319	965	10.6383681
58° 3509922	914	0.6490078	9.9887822	48	0.0112178	9.3622100	964	10.6377900
59° 3515405	913	0.6484595	9.9887531	49	0.0112469	9.3627874	963	10.6372126
60° 3520880		0.6479120	9.9887239		0.0112761	9.3633646	961	10.6366359
Cosine	D10"	Comp. cos.	Sine	D10"	Comp. sin.	Cotang.	D10"	Tangent

Tab. 18.

o

Deg. 77.

Sine	D10"	Comp. sin.	Cosine	D10"	Comp. cos.	Tangent	D10"	Cotangent	D10"
0.9352380	911	0.6457126	9.9887939	49	0.012761	9.3633641	960	10.6366859	960
1.9352634	910	0.6456551	9.9886947	49	0.0128059	9.3639401	959	10.6360599	959
2.9352810	909	0.6456190	9.9886655	49	0.0128443	9.3645155	958	10.6354845	958
3.9352934	908	0.6455736	9.9886363	49	0.0128777	9.3650901	957	10.6349099	957
4.9353010	907	0.6455290	9.9886070	49	0.0129061	9.3656644	956	10.6343359	956
5.9353080	906	0.6454850	9.9885776	49	0.0129295	9.3662377	955	10.6337626	955
6.9353130	905	0.6454418	9.9885482	49	0.0129479	9.3668106	954	10.6331900	954
7.9353160	904	0.6453993	9.9885188	49	0.0129613	9.3673819	953	10.6326181	953
8.9353180	903	0.6453574	9.9884894	49	0.0129697	9.3679532	952	10.6320468	952
9.9353190	902	0.6453164	9.9884600	49	0.0129781	9.3685238	951	10.6314762	951
10.9353190	901	0.6452760	9.9884303	49	0.0129865	9.3690937	950	10.6309063	950
11.9353180	899	0.6452363	9.9884008	49	0.0129949	9.3696629	949	10.6303371	949
12.9353160	898	0.6451973	9.9883715	49	0.0130033	9.3702315	948	10.6297685	948
13.9353130	897	0.6451589	9.9883423	50	0.0130117	9.3707994	947	10.6292006	947
14.9353090	896	0.6451215	9.9883131	50	0.0130201	9.3713667	946	10.6286332	946
15.9353040	895	0.6450846	9.9882840	50	0.0130285	9.3719333	945	10.6280667	945
16.9352980	894	0.6450485	9.9882548	50	0.0130369	9.3724994	944	10.6275008	944
17.9352910	893	0.6450130	9.9882255	50	0.0130453	9.3730645	943	10.6269355	943
18.9352830	892	0.6449783	9.9881962	50	0.0130537	9.3736291	942	10.6263709	942
19.9352740	891	0.6449442	9.9881668	50	0.0130621	9.3741930	941	10.6258070	941
20.9352640	889	0.6449108	9.9881372	50	0.0130705	9.3747563	940	10.6252437	940
21.9352530	888	0.6448781	9.9881079	50	0.0130789	9.3753190	939	10.6246810	939
22.9352410	887	0.6448461	9.9880787	50	0.0130873	9.3758810	938	10.6241190	938
23.9352280	885	0.6448148	9.9880492	50	0.0130957	9.3764423	937	10.6235577	937
24.9352140	884	0.6447842	9.9880198	50	0.0131041	9.3770030	936	10.6229970	936
25.9351990	883	0.6447542	9.9879902	50	0.0131125	9.3775631	935	10.6224369	935
26.9351830	882	0.6447249	9.9879607	50	0.0131209	9.3781225	934	10.6218775	934
27.9351660	881	0.6446964	9.9879313	50	0.0131293	9.3786818	933	10.6213187	933
28.9351480	880	0.6446685	9.9879021	50	0.0131377	9.3792409	932	10.6207606	932
29.9351290	879	0.6446413	9.9878731	50	0.0131461	9.3797994	931	10.6202031	931
30.9351090	877	0.6446147	9.9878443	50	0.0131545	9.3803577	930	10.6196463	930
31.9350880	876	0.6445889	9.9878157	51	0.0131629	9.3809160	929	10.6190900	929
32.9350660	875	0.6445637	9.9877873	51	0.0131713	9.3814745	928	10.6185345	928
33.9350430	874	0.6445392	9.9877590	51	0.0131797	9.3820328	927	10.6179795	927
34.9350190	873	0.6445153	9.9877309	51	0.0131881	9.3825914	926	10.6174252	926
35.9349940	872	0.6444921	9.9877029	51	0.0131965	9.3831501	925	10.6168714	925
36.9349680	871	0.6444696	9.9876748	51	0.0132049	9.3837086	924	10.6163184	924
37.9349410	870	0.6444477	9.9876468	51	0.0132133	9.3842674	923	10.6157660	923
38.9349130	869	0.6444265	9.9876187	51	0.0132217	9.3848264	922	10.6152142	922
39.9348840	868	0.6444059	9.9875906	51	0.0132301	9.3853857	921	10.6146630	921
40.9348540	867	0.6443858	9.9875623	51	0.0132385	9.3859451	920	10.6141124	920
41.9348230	865	0.6443669	9.9875335	51	0.0132469	9.3865046	919	10.6135624	919
42.9347910	864	0.6443483	9.9875048	51	0.0132553	9.3870641	918	10.6130131	918
43.9347580	863	0.6443300	9.9874764	51	0.0132637	9.3876236	917	10.6124644	917
44.9347240	862	0.6443122	9.9874481	51	0.0132721	9.3881831	916	10.6119163	916
45.9346890	861	0.6442949	9.9874200	51	0.0132805	9.3887426	915	10.6113688	915
46.9346530	860	0.6442780	9.9873922	51	0.0132889	9.3893021	914	10.6108219	914
47.9346160	859	0.6442615	9.9873645	52	0.0132973	9.3898616	913	10.6102756	913
48.9345780	858	0.6442454	9.9873370	52	0.0133057	9.3904211	912	10.6097300	912
49.9345390	857	0.6442297	9.9873098	52	0.0133141	9.3909806	911	10.6091849	911
50.9344990	856	0.6442144	9.9872827	52	0.0133225	9.3915401	910	10.6086403	910
51.9344580	855	0.6441995	9.9872558	52	0.0133309	9.3921000	909	10.6080966	909
52.9344160	853	0.6441850	9.9872290	52	0.0133393	9.3926600	908	10.6075534	908
53.9343730	852	0.6441709	9.9872024	52	0.0133477	9.3932200	907	10.6070107	907
54.9343290	851	0.6441572	9.9871760	52	0.0133561	9.3937800	906	10.6064687	906
55.9342840	850	0.6441439	9.9871500	52	0.0133645	9.3943400	905	10.6059273	905
56.9342380	849	0.6441310	9.9871242	52	0.0133729	9.3949000	904	10.6053864	904
57.9341910	848	0.6441185	9.9870988	52	0.0133813	9.3954600	903	10.6048462	903
58.9341430	847	0.6441064	9.9870737	52	0.0133897	9.3960200	902	10.6043065	902
59.9340940	846	0.6440947	9.9870489	52	0.0133981	9.3965800	901	10.6037674	901
60.9340440	845	0.6440834	9.9870244	52	0.0134065	9.3971400	900	10.6032289	900
Cosine	D10"	Comp. cos.	Sine	D10"	Comp. sin	Cotang.	D10"	Tangent	D10"

14 Deg.

Tab. 18.

Sine	D10 ⁰	Comp. sin.	Cosine	Comp. cos.	Tangent	D10 ⁰	Cotang.
09-3836752	844	0-6163248	9-9809041	52	0-0130959	9-3997773	896
19-3841815	845	0-6158185	9-9868735	53	0-0131271	9-3997773	895
29-3846873	846	0-6153122	9-9868410	53	0-0131598	9-3997773	894
39-3851924	847	0-6148076	9-9868094	53	0-0131906	9-3997773	893
49-3856969	848	0-6143031	9-9867778	53	0-0132222	9-3997773	892
59-3862018	849	0-6137982	9-9867461	53	0-0132539	9-3997773	891
69-3867040	850	0-6132968	9-9867144	53	0-0132856	9-3997773	890
79-3872067	851	0-6127933	9-9866827	53	0-0133173	9-3997773	889
89-3877087	852	0-6122897	9-9866509	53	0-0133491	9-3997773	888
99-3882101	853	0-6117899	9-9866191	53	0-0133809	9-3997773	887
109-3887109	854	0-6112891	9-9865872	53	0-0134128	9-3997773	886
119-3892111	855	0-6107889	9-9865553	53	0-0134447	9-3997773	885
129-3897106	856	0-6102894	9-9865233	53	0-0134767	9-3997773	884
139-3902096	857	0-6097904	9-9864913	53	0-0135087	9-3997773	883
149-3907079	858	0-6092921	9-9864593	53	0-0135407	9-3997773	882
159-3912057	859	0-6087943	9-9864273	53	0-0135727	9-3997773	881
169-3917028	860	0-6082970	9-9863952	53	0-0136048	9-3997773	880
179-3921994	861	0-6078000	9-9863630	53	0-0136370	9-3997773	879
189-3926932	862	0-6073048	9-9863308	53	0-0136692	9-3997773	878
199-3931905	863	0-6068093	9-9862986	53	0-0137014	9-3997773	877
209-3936852	864	0-6063148	9-9862663	53	0-0137337	9-3997773	876
219-3941794	865	0-6058200	9-9862340	53	0-0137660	9-3997773	875
229-3946729	866	0-6053271	9-9862017	53	0-0137983	9-3997773	874
239-3951658	867	0-6048342	9-9861693	53	0-0138307	9-3997773	873
249-3956581	868	0-6043419	9-9861369	53	0-0138631	9-3997773	872
259-3961499	869	0-6038501	9-9861045	53	0-0138955	9-3997773	871
269-3966410	870	0-6033590	9-9860720	53	0-0139280	9-3997773	870
279-3971313	871	0-6028682	9-9860394	53	0-0139606	9-3997773	869
289-3976218	872	0-6023785	9-9860069	53	0-0139931	9-3997773	868
299-3981109	873	0-6018891	9-9859742	53	0-0140258	9-3997773	867
309-3985996	874	0-6014004	9-9859416	53	0-0140584	9-3997773	866
319-3990878	875	0-6009122	9-9859088	53	0-0140911	9-3997773	865
329-3995754	876	0-6004244	9-9858760	53	0-0141238	9-3997773	864
339-4000625	877	0-5999373	9-9858431	53	0-0141566	9-3997773	863
349-4005489	878	0-5994511	9-9858101	53	0-0141894	9-3997773	862
359-4010348	879	0-5989652	9-9857771	53	0-0142223	9-3997773	861
369-4015201	880	0-5984799	9-9857444	53	0-0142551	9-3997773	860
379-4020048	881	0-5979952	9-9857114	53	0-0142881	9-3997773	859
389-4024889	882	0-5975111	9-9856789	53	0-0143210	9-3997773	858
399-4029724	883	0-5970276	9-9856460	53	0-0143540	9-3997773	857
409-4034554	884	0-5965446	9-9856129	53	0-0143871	9-3997773	856
419-4039378	885	0-5960622	9-9855796	53	0-0144202	9-3997773	855
429-4044196	886	0-5955804	9-9855467	53	0-0144533	9-3997773	854
439-4049009	887	0-5950991	9-9855135	53	0-0144865	9-3997773	853
449-4053816	888	0-5946184	9-9854803	53	0-0145197	9-3997773	852
459-4058617	889	0-5941383	9-9854471	53	0-0145529	9-3997773	851
469-4063413	890	0-5936587	9-9854138	53	0-0145862	9-3997773	850
479-4068203	891	0-5931797	9-9853803	53	0-0146197	9-3997773	849
489-4072987	892	0-5927013	9-9853471	53	0-0146529	9-3997773	848
499-4077766	893	0-5922234	9-9853138	53	0-0146862	9-3997773	847
509-4082539	894	0-5917461	9-9852803	53	0-0147197	9-3997773	846
519-4087306	895	0-5912694	9-9852468	53	0-0147532	9-3997773	845
529-4092068	896	0-5907930	9-9852133	53	0-0147867	9-3997773	844
539-4096824	897	0-5903176	9-9851798	53	0-0148202	9-3997773	843
549-4101575	898	0-5898423	9-9851462	53	0-0148538	9-3997773	842
559-4106320	899	0-5893680	9-9851125	53	0-0148875	9-3997773	841
569-4111059	900	0-5888941	9-9850789	53	0-0149211	9-3997773	840
579-4115793	901	0-5884207	9-9850452	53	0-0149548	9-3997773	839
589-4120522	902	0-5879478	9-9850114	53	0-0149886	9-3997773	838
599-4125253	903	0-5874755	9-9849776	53	0-0150224	9-3997773	837
609-4129962	904	0-5870038	9-9849438	53	0-0150562	9-3997773	836

Tab. 18.

Des. 75.

15 Deg.

	Sine	D10'	Comp. sin.	Cosine	D10'	Comp. cos.	Tangent	D10'	Cotangent
0	0.4129962	785	0.5870038	0.9849438	56	0.0150562	9.4280525	842	10.5719475
1	0.4131674	784	0.5868326	0.9849099	56	0.0150901	9.4285575	841	10.5714425
2	0.4133381	783	0.5866619	0.9848760	57	0.0151240	9.4290621	840	10.5709375
3	0.4135087	782	0.5864918	0.9848420	57	0.0151580	9.4295661	839	10.5704325
4	0.4136787	781	0.5863229	0.9848081	57	0.0151919	9.4300697	838	10.5699275
5	0.4138486	780	0.5861539	0.9847740	57	0.0152259	9.4305727	837	10.5694225
6	0.4140182	779	0.5859848	0.9847400	57	0.0152600	9.4310753	836	10.5689175
7	0.4141879	778	0.5858157	0.9847059	57	0.0152941	9.4315773	835	10.5684125
8	0.4143575	777	0.5856465	0.9846717	57	0.0153283	9.4320789	834	10.5679075
9	0.4145271	776	0.5854772	0.9846375	57	0.0153625	9.4325799	833	10.5674025
10	0.4146967	775	0.5853079	0.9846033	57	0.0153967	9.4330804	832	10.5668975
11	0.4148661	774	0.5851385	0.9845690	57	0.0154310	9.4335805	831	10.5663925
12	0.4150355	773	0.5849692	0.9845347	57	0.0154653	9.4340800	830	10.5658875
13	0.4152049	772	0.5848000	0.9845004	57	0.0154996	9.4345791	829	10.5653825
14	0.4153743	771	0.5846307	0.9844661	57	0.0155340	9.4350776	828	10.5648775
15	0.4155437	770	0.5844615	0.9844316	57	0.0155684	9.4355757	827	10.5643725
16	0.4157131	769	0.5842922	0.9843971	57	0.0156029	9.4360733	826	10.5638675
17	0.4158825	768	0.5841230	0.9843626	57	0.0156373	9.4365704	825	10.5633625
18	0.4160519	767	0.5839537	0.9843281	58	0.0156719	9.4370670	824	10.5628575
19	0.4162213	766	0.5837844	0.9842935	58	0.0157063	9.4375631	823	10.5623525
20	0.4163907	765	0.5836151	0.9842589	58	0.0157411	9.4380587	822	10.5618475
21	0.4165601	764	0.5834458	0.9842242	58	0.0157758	9.4385538	821	10.5613425
22	0.4167295	763	0.5832765	0.9841895	58	0.0158105	9.4390485	820	10.5608375
23	0.4168989	762	0.5831072	0.9841548	58	0.0158452	9.4395426	819	10.5603325
24	0.4170683	761	0.5829379	0.9841200	58	0.0158800	9.4400363	818	10.5598275
25	0.4172377	760	0.5827686	0.9840852	58	0.0159146	9.4405295	817	10.5593225
26	0.4174071	759	0.5825993	0.9840505	58	0.0159493	9.4410222	816	10.5588175
27	0.4175765	758	0.5824300	0.9840157	58	0.0159840	9.4415145	815	10.5583125
28	0.4177459	757	0.5822607	0.9839810	58	0.0160187	9.4420062	814	10.5578075
29	0.4179153	756	0.5820914	0.9839462	58	0.0160535	9.4424975	813	10.5573025
30	0.4180847	755	0.5819221	0.9839115	58	0.0160885	9.4429883	812	10.5567975
31	0.4182541	754	0.5817528	0.9838767	59	0.0161233	9.4434786	811	10.5562925
32	0.4184235	753	0.5815835	0.9838420	59	0.0161581	9.4439683	810	10.5557875
33	0.4185929	752	0.5814142	0.9838072	59	0.0161929	9.4444579	809	10.5552825
34	0.4187623	751	0.5812449	0.9837725	59	0.0162277	9.4449476	808	10.5547775
35	0.4189317	750	0.5810756	0.9837377	59	0.0162625	9.4454372	807	10.5542725
36	0.4191011	749	0.5809063	0.9837030	59	0.0162973	9.4459269	806	10.5537675
37	0.4192705	748	0.5807370	0.9836682	59	0.0163321	9.4464166	805	10.5532625
38	0.4194399	747	0.5805677	0.9836335	59	0.0163669	9.4469063	804	10.5527575
39	0.4196093	746	0.5803984	0.9835987	59	0.0164017	9.4473960	803	10.5522525
40	0.4197787	745	0.5802291	0.9835640	59	0.0164365	9.4478857	802	10.5517475
41	0.4199481	744	0.5800598	0.9835292	59	0.0164713	9.4483754	801	10.5512425
42	0.4201175	743	0.5798905	0.9834945	59	0.0165061	9.4488651	800	10.5507375
43	0.4202869	742	0.5797212	0.9834597	59	0.0165409	9.4493548	799	10.5502325
44	0.4204563	741	0.5795519	0.9834250	59	0.0165757	9.4498445	798	10.5497275
45	0.4206257	740	0.5793826	0.9833902	59	0.0166105	9.4503342	797	10.5492225
46	0.4207951	739	0.5792133	0.9833555	59	0.0166453	9.4508239	796	10.5487175
47	0.4209645	738	0.5790440	0.9833207	59	0.0166801	9.4513136	795	10.5482125
48	0.4211339	737	0.5788747	0.9832860	60	0.0167149	9.4518033	794	10.5477075
49	0.4213033	736	0.5787054	0.9832512	60	0.0167497	9.4522930	793	10.5472025
50	0.4214727	735	0.5785361	0.9832165	60	0.0167845	9.4527827	792	10.5466975
51	0.4216421	734	0.5783668	0.9831817	60	0.0168193	9.4532724	791	10.5461925
52	0.4218115	733	0.5781975	0.9831470	60	0.0168541	9.4537621	790	10.5456875
53	0.4219809	732	0.5780282	0.9831122	60	0.0168889	9.4542518	789	10.5451825
54	0.4221503	731	0.5778589	0.9830775	60	0.0169237	9.4547415	788	10.5446775
55	0.4223197	730	0.5776896	0.9830427	60	0.0169585	9.4552312	787	10.5441725
56	0.4224891	729	0.5775203	0.9830080	60	0.0169933	9.4557209	786	10.5436675
57	0.4226585	728	0.5773510	0.9829732	60	0.0170281	9.4562106	785	10.5431625
58	0.4228279	727	0.5771817	0.9829385	60	0.0170629	9.4567003	784	10.5426575
59	0.4229973	726	0.5770124	0.9829037	60	0.0170977	9.4571900	783	10.5421525
60	0.4231667	725	0.5768431	0.9828690	60	0.0171325	9.4576797	782	10.5416475

Tab. 18.

Deg. 74.

16 Deg.

Tab. 18.

Sine	D10"	Comp. sin.	Cosine.	D10"	Comp. cos.	Tangent	D10"	Cotangent
0° 4403381	734	0.5586619	9.9828116	60	0.0171584	9.4574451	734	10.5425549
1° 4407784	733	0.5592216	9.9828054	60	0.0171946	9.4574451	733	10.5425549
2° 4412182	732	0.5597818	9.9827991	60	0.0172308	9.4574451	732	10.5425549
3° 4416576	731	0.5603424	9.9827928	61	0.0172672	9.4574451	731	10.5425549
4° 4420965	731	0.5609035	9.9827865	61	0.0173036	9.4574451	731	10.5425549
5° 4425349	730	0.5614651	9.9827800	61	0.0173400	9.4574451	730	10.5425549
6° 4429728	730	0.5620272	9.9827736	61	0.0173764	9.4574451	730	10.5425549
7° 4434103	728	0.5625897	9.9827671	61	0.0174128	9.4574451	728	10.5425549
8° 4438472	727	0.5631528	9.9827606	61	0.0174494	9.4574451	727	10.5425549
9° 4442837	727	0.5637163	9.9827540	61	0.0174860	9.4574451	727	10.5425549
10° 4447197	726	0.5642803	9.9827474	61	0.0175226	9.4574451	726	10.5425549
11° 4451553	725	0.5648447	9.9827408	61	0.0175592	9.4574451	725	10.5425549
12° 4455904	724	0.5654096	9.9827343	61	0.0175959	9.4574451	724	10.5425549
13° 4460250	724	0.5659750	9.9827277	61	0.0176326	9.4574451	724	10.5425549
14° 4464591	723	0.5665409	9.9827212	61	0.0176694	9.4574451	723	10.5425549
15° 4468927	723	0.5671073	9.9827146	61	0.0177062	9.4574451	723	10.5425549
16° 4473259	721	0.5676741	9.9827080	62	0.0177431	9.4574451	721	10.5425549
17° 4477586	720	0.5682414	9.9827014	62	0.0177799	9.4574451	720	10.5425549
18° 4481909	720	0.5688091	9.9826948	62	0.0178169	9.4574451	720	10.5425549
19° 4486227	719	0.5693773	9.9826882	62	0.0178538	9.4574451	719	10.5425549
20° 4490540	718	0.5699460	9.9826816	62	0.0178908	9.4574451	718	10.5425549
21° 4494849	717	0.5705151	9.9826750	62	0.0179279	9.4574451	717	10.5425549
22° 4499153	716	0.5710847	9.9826684	62	0.0179649	9.4574451	716	10.5425549
23° 4503452	716	0.5716548	9.9826618	62	0.0180021	9.4574451	716	10.5425549
24° 4507747	715	0.5722253	9.9826552	62	0.0180392	9.4574451	715	10.5425549
25° 4512037	714	0.5727963	9.9826486	62	0.0180764	9.4574451	714	10.5425549
26° 4516322	713	0.5733678	9.9826420	62	0.0181137	9.4574451	713	10.5425549
27° 4520603	713	0.5739397	9.9826354	62	0.0181510	9.4574451	713	10.5425549
28° 4524879	712	0.5745121	9.9826288	62	0.0181883	9.4574451	712	10.5425549
29° 4529151	711	0.5750849	9.9826222	62	0.0182256	9.4574451	711	10.5425549
30° 4533418	710	0.5756582	9.9826156	62	0.0182630	9.4574451	710	10.5425549
31° 4537681	710	0.5762319	9.9826090	62	0.0183003	9.4574451	710	10.5425549
32° 4541939	709	0.5768061	9.9826024	62	0.0183380	9.4574451	709	10.5425549
33° 4546192	708	0.5773808	9.9825958	62	0.0183753	9.4574451	708	10.5425549
34° 4550441	707	0.5779559	9.9825892	63	0.0184130	9.4574451	707	10.5425549
35° 4554686	707	0.5785314	9.9825826	63	0.0184506	9.4574451	707	10.5425549
36° 4558926	706	0.5791074	9.9825760	63	0.0184883	9.4574451	706	10.5425549
37° 4563161	705	0.5796839	9.9825694	63	0.0185260	9.4574451	705	10.5425549
38° 4567392	704	0.5802608	9.9825628	63	0.0185637	9.4574451	704	10.5425549
39° 4571618	704	0.5808382	9.9825562	63	0.0186014	9.4574451	704	10.5425549
40° 4575840	703	0.5814160	9.9825496	63	0.0186392	9.4574451	703	10.5425549
41° 4580058	702	0.5819942	9.9825430	63	0.0186771	9.4574451	702	10.5425549
42° 4584271	701	0.5825729	9.9825364	63	0.0187150	9.4574451	701	10.5425549
43° 4588480	700	0.5831520	9.9825298	63	0.0187529	9.4574451	700	10.5425549
44° 4592684	700	0.5837316	9.9825232	63	0.0187909	9.4574451	700	10.5425549
45° 4596884	699	0.5843116	9.9825166	63	0.0188289	9.4574451	699	10.5425549
46° 4601079	699	0.5848921	9.9825100	63	0.0188669	9.4574451	699	10.5425549
47° 4605270	698	0.5854730	9.9825034	63	0.0189050	9.4574451	698	10.5425549
48° 4609456	697	0.5860543	9.9824968	64	0.0189431	9.4574451	697	10.5425549
49° 4613639	696	0.5866362	9.9824902	64	0.0189813	9.4574451	696	10.5425549
50° 4617816	696	0.5872184	9.9824836	64	0.0190195	9.4574451	696	10.5425549
51° 4621989	695	0.5878011	9.9824770	64	0.0190577	9.4574451	695	10.5425549
52° 4626158	694	0.5883842	9.9824704	64	0.0190960	9.4574451	694	10.5425549
53° 4630323	693	0.5889677	9.9824638	64	0.0191343	9.4574451	693	10.5425549
54° 4634483	693	0.5895517	9.9824572	64	0.0191727	9.4574451	693	10.5425549
55° 4638639	692	0.5901361	9.9824506	64	0.0192111	9.4574451	692	10.5425549
56° 4642790	691	0.5907210	9.9824440	64	0.0192495	9.4574451	691	10.5425549
57° 4646938	690	0.5913062	9.9824374	64	0.0192880	9.4574451	690	10.5425549
58° 4651081	690	0.5918918	9.9824308	64	0.0193265	9.4574451	690	10.5425549
59° 4655219	689	0.5924777	9.9824242	64	0.0193651	9.4574451	689	10.5425549
60° 4659353	689	0.5930647	9.9824176	64	0.0194037	9.4574451	689	10.5425549
Cosine	D10"	Comp. cos.	Sine	D10"	Comp. sin	Cotangent	D10"	Tangent

Tab. 18.

Tab. 18.

17 Deg.

Sine	D10	Comp. sin.	Cosine	D10	Comp. cos.	Tangent	D10	Cotangent
0.9849535	64	0.0150467	9.9805963	64	0.0194037	9.4833380	753	10.5146610
0.9849535	64	0.0150467	9.9805963	64	0.0194423	9.4857907	752	10.5142093
0.9849535	64	0.0150467	9.9805963	64	0.0194810	9.4882419	751	10.5137581
0.9849535	64	0.0150467	9.9805963	64	0.0195197	9.4886928	750	10.5133072
0.9849535	64	0.0150467	9.9805963	64	0.0195583	9.4871433	749	10.5128567
0.9849535	64	0.0150467	9.9805963	64	0.0195969	9.4875933	748	10.5124067
0.9849535	64	0.0150467	9.9805963	64	0.0196356	9.4880430	747	10.5119570
0.9849535	64	0.0150467	9.9805963	64	0.0196742	9.4884924	746	10.5115076
0.9849535	64	0.0150467	9.9805963	64	0.0197129	9.4889413	745	10.5110582
0.9849535	64	0.0150467	9.9805963	64	0.0197515	9.4893908	744	10.5106092
0.9849535	64	0.0150467	9.9805963	64	0.0197902	9.4898380	743	10.5101620
0.9849535	64	0.0150467	9.9805963	64	0.0198288	9.4902858	742	10.5097142
0.9849535	64	0.0150467	9.9805963	64	0.0198675	9.4907332	741	10.5092668
0.9849535	64	0.0150467	9.9805963	64	0.0199061	9.4911802	740	10.5088198
0.9849535	64	0.0150467	9.9805963	64	0.0199448	9.4916269	739	10.5083731
0.9849535	64	0.0150467	9.9805963	64	0.0199834	9.4920731	738	10.5079269
0.9849535	64	0.0150467	9.9805963	64	0.0200221	9.4925190	737	10.5074810
0.9849535	64	0.0150467	9.9805963	64	0.0200607	9.4929646	736	10.5070354
0.9849535	64	0.0150467	9.9805963	64	0.0200994	9.4934097	735	10.5065903
0.9849535	64	0.0150467	9.9805963	64	0.0201380	9.4938545	734	10.5061455
0.9849535	64	0.0150467	9.9805963	64	0.0201767	9.4942988	733	10.5057012
0.9849535	64	0.0150467	9.9805963	64	0.0202153	9.4947429	732	10.5052571
0.9849535	64	0.0150467	9.9805963	64	0.0202540	9.4951865	731	10.5048135
0.9849535	64	0.0150467	9.9805963	64	0.0202926	9.4956298	730	10.5043702
0.9849535	64	0.0150467	9.9805963	64	0.0203313	9.4960727	729	10.5039273
0.9849535	64	0.0150467	9.9805963	64	0.0203699	9.4965152	728	10.5034848
0.9849535	64	0.0150467	9.9805963	64	0.0204086	9.4969574	727	10.5030420
0.9849535	64	0.0150467	9.9805963	64	0.0204472	9.4973991	726	10.5026009
0.9849535	64	0.0150467	9.9805963	64	0.0204859	9.4978406	725	10.5021594
0.9849535	64	0.0150467	9.9805963	64	0.0205245	9.4982816	724	10.5017184
0.9849535	64	0.0150467	9.9805963	64	0.0205632	9.4987223	723	10.5012777
0.9849535	64	0.0150467	9.9805963	64	0.0206018	9.4991626	722	10.5008374
0.9849535	64	0.0150467	9.9805963	64	0.0206405	9.4996029	721	10.5003974
0.9849535	64	0.0150467	9.9805963	64	0.0206791	9.5000422	720	10.4999578
0.9849535	64	0.0150467	9.9805963	64	0.0207178	9.5004814	719	10.4995186
0.9849535	64	0.0150467	9.9805963	64	0.0207564	9.5009203	718	10.4990797
0.9849535	64	0.0150467	9.9805963	64	0.0207951	9.5013588	717	10.4986413
0.9849535	64	0.0150467	9.9805963	64	0.0208337	9.5017969	716	10.4982031
0.9849535	64	0.0150467	9.9805963	64	0.0208724	9.5022347	715	10.4977653
0.9849535	64	0.0150467	9.9805963	64	0.0209110	9.5026721	714	10.4973279
0.9849535	64	0.0150467	9.9805963	64	0.0209497	9.5031091	713	10.4968908
0.9849535	64	0.0150467	9.9805963	64	0.0209883	9.5035459	712	10.4964541
0.9849535	64	0.0150467	9.9805963	64	0.0210270	9.5039822	711	10.4960178
0.9849535	64	0.0150467	9.9805963	64	0.0210656	9.5044182	710	10.4955818
0.9849535	64	0.0150467	9.9805963	64	0.0211043	9.5048538	709	10.4951462
0.9849535	64	0.0150467	9.9805963	64	0.0211429	9.5052891	708	10.4947109
0.9849535	64	0.0150467	9.9805963	64	0.0211816	9.5057240	707	10.4942760
0.9849535	64	0.0150467	9.9805963	64	0.0212202	9.5061586	706	10.4938414
0.9849535	64	0.0150467	9.9805963	64	0.0212589	9.5065928	705	10.4934072
0.9849535	64	0.0150467	9.9805963	64	0.0212975	9.5070267	704	10.4929733
0.9849535	64	0.0150467	9.9805963	64	0.0213362	9.5074602	703	10.4925398
0.9849535	64	0.0150467	9.9805963	64	0.0213748	9.5078933	702	10.4921067
0.9849535	64	0.0150467	9.9805963	64	0.0214135	9.5083261	701	10.4916739
0.9849535	64	0.0150467	9.9805963	64	0.0214521	9.5087588	700	10.4912414
0.9849535	64	0.0150467	9.9805963	64	0.0214908	9.5091907	699	10.4908093
0.9849535	64	0.0150467	9.9805963	64	0.0215294	9.5096224	698	10.4903776
0.9849535	64	0.0150467	9.9805963	64	0.0215681	9.5100539	697	10.4899461
0.9849535	64	0.0150467	9.9805963	64	0.0216067	9.5104849	696	10.4895151
0.9849535	64	0.0150467	9.9805963	64	0.0216454	9.5109156	695	10.4890846
0.9849535	64	0.0150467	9.9805963	64	0.0216840	9.5113460	694	10.4886540
0.9849535	64	0.0150467	9.9805963	64	0.0217227	9.5117760	693	10.4882240

Tab. 18.

Deg. 72.

18 Deg.

Sine	D10'	Comp. sin.	Cosine	D10'	Comp. sin.	Tangent	D10'	Cotangent
0.94899824	648	0.5180176	0.9782069	68	0.0217859	5.117778	709	10.4672240
1.94902710	647	0.5096290	0.9781653	68	0.0218347	5.118222	710	10.4677952
2.94907592	646	0.5092408	0.9781241	69	0.0218739	5.118666	711	10.4683664
3.94912471	645	0.5088529	0.9780830	69	0.0219130	5.119110	712	10.4689376
4.94917345	645	0.5084646	0.9780418	69	0.0219522	5.119554	713	10.4695088
5.94922216	644	0.5080764	0.9780006	69	0.0219913	5.120000	714	10.4700800
6.94927083	644	0.5076877	0.9779593	69	0.0220305	5.120444	715	10.4706512
7.94931948	643	0.5072995	0.9779180	69	0.0220697	5.120888	716	10.4712224
8.94936806	643	0.5069114	0.9778766	69	0.0221089	5.121332	717	10.4717936
9.94941661	642	0.5065239	0.9778353	69	0.0221481	5.121776	718	10.4723648
10.94946513	641	0.5061357	0.9777938	69	0.0221873	5.122220	719	10.4729360
11.94951361	641	0.5057473	0.9777523	69	0.0222265	5.122664	720	10.4735072
12.94956203	640	0.5053595	0.9777109	69	0.0222657	5.123108	721	10.4740784
13.94961046	639	0.5049714	0.9776695	69	0.0223049	5.123552	722	10.4746496
14.94965883	639	0.5045831	0.9776280	69	0.0223441	5.123996	723	10.4752208
15.94970716	638	0.5041948	0.9775866	69	0.0223833	5.124440	724	10.4757920
16.94975545	637	0.5038065	0.9775451	69	0.0224225	5.124884	725	10.4763632
17.94980370	637	0.5034182	0.9775036	70	0.0224617	5.125328	726	10.4769344
18.94985192	636	0.5030299	0.9774621	70	0.0225009	5.125772	727	10.4775056
19.94990006	636	0.5026416	0.9774206	70	0.0225401	5.126216	728	10.4780768
20.94994824	635	0.5022533	0.9773791	70	0.0225793	5.126660	729	10.4786480
21.94999635	635	0.5018650	0.9773376	70	0.0226185	5.127104	730	10.4792192
22.94994442	634	0.5014767	0.9772961	70	0.0226577	5.127548	731	10.4797904
23.94989245	633	0.5010884	0.9772546	70	0.0226969	5.127992	732	10.4803616
24.94984045	632	0.5006999	0.9772131	70	0.0227361	5.128436	733	10.4809328
25.94978840	632	0.5003116	0.9771716	70	0.0227753	5.128880	734	10.4815040
26.94973633	631	0.5000000	0.9771301	70	0.0228145	5.129324	735	10.4820752
27.94968421	631	0.4995883	0.9770886	70	0.0228537	5.129768	736	10.4826464
28.94963206	630	0.4991766	0.9770471	70	0.0228929	5.130212	737	10.4832176
29.94957991	629	0.4987649	0.9769956	70	0.0229321	5.130656	738	10.4837888
30.94952776	629	0.4983532	0.9769541	70	0.0229713	5.131100	739	10.4843600
31.94947561	628	0.4979415	0.9769126	70	0.0230105	5.131544	740	10.4849312
32.94942342	628	0.4975298	0.9768711	71	0.0230497	5.131988	741	10.4855024
33.94937123	627	0.4971181	0.9768296	71	0.0230889	5.132432	742	10.4860736
34.94931904	626	0.4967064	0.9767881	71	0.0231281	5.132876	743	10.4866448
35.94926685	626	0.4962947	0.9767466	71	0.0231673	5.133320	744	10.4872160
36.94921466	625	0.4958830	0.9767051	71	0.0232065	5.133764	745	10.4877872
37.94916247	625	0.4954713	0.9766636	71	0.0232457	5.134208	746	10.4883584
38.94911028	624	0.4950596	0.9766221	71	0.0232849	5.134652	747	10.4889296
39.94905809	623	0.4946479	0.9765806	71	0.0233241	5.135096	748	10.4895008
40.94900590	623	0.4942362	0.9765391	71	0.0233633	5.135540	749	10.4900720
41.94895371	622	0.4938245	0.9764976	71	0.0234025	5.135984	750	10.4906432
42.94890152	622	0.4934128	0.9764561	71	0.0234417	5.136428	751	10.4912144
43.94884933	621	0.4930011	0.9764146	71	0.0234809	5.136872	752	10.4917856
44.94879714	620	0.4925894	0.9763731	71	0.0235201	5.137316	753	10.4923568
45.94874495	620	0.4921777	0.9763316	71	0.0235593	5.137760	754	10.4929280
46.94869276	619	0.4917660	0.9762901	71	0.0235985	5.138204	755	10.4934992
47.94864057	619	0.4913543	0.9762486	71	0.0236377	5.138648	756	10.4940704
48.94858838	618	0.4909426	0.9762071	71	0.0236769	5.139092	757	10.4946416
49.94853619	618	0.4905309	0.9761656	71	0.0237161	5.139536	758	10.4952128
50.94848400	617	0.4901192	0.9761241	71	0.0237553	5.139980	759	10.4957840
51.94843181	616	0.4897075	0.9760826	71	0.0237945	5.140424	760	10.4963552
52.94837962	616	0.4892958	0.9760411	71	0.0238337	5.140868	761	10.4969264
53.94832743	615	0.4888841	0.9759996	71	0.0238729	5.141312	762	10.4974976
54.94827524	615	0.4884724	0.9759581	71	0.0239121	5.141756	763	10.4980688
55.94822305	614	0.4880607	0.9759166	71	0.0239513	5.142200	764	10.4986400
56.94817086	613	0.4876490	0.9758751	71	0.0239905	5.142644	765	10.4992112
57.94811867	613	0.4872373	0.9758336	71	0.0240297	5.143088	766	10.4997824
58.94806648	612	0.4868256	0.9757921	71	0.0240689	5.143532	767	10.5003536
59.94801429	612	0.4864139	0.9757506	71	0.0241081	5.143976	768	10.5009248
60.94796210	612	0.4860022	0.9757091	71	0.0241473	5.144420	769	10.5014960

Tab. 18.

Deg. 71.

Sine.	D10'	Comp. sin.	Cosine	D10'	Comp. cos.	Tangent	D10'	Cotangent
1 0-5124119	611	0-4873581	9-9756701	73	0-0243299	9-5369718	684	10-4630281
2 0-5130086	611	0-4869914	9-9756265	73	0-0243735	9-5373821	683	10-4626179
3 0-5133726	610	0-4866256	9-9755830	73	0-0244170	9-5377920	683	10-4622080
4 0-5137410	609	0-4862599	9-9755394	73	0-0244606	9-5382017	683	10-4617983
5 0-5141067	609	0-4858943	9-9754957	73	0-0245042	9-5386110	682	10-4613890
6 0-5144721	608	0-4855287	9-9754521	73	0-0245479	9-5390200	682	10-4609800
7 0-5148371	608	0-4851629	9-9754084	73	0-0245917	9-5394287	681	10-4605712
8 0-5152017	607	0-4847983	9-9753646	73	0-0246354	9-5398371	681	10-4601629
9 0-5155660	607	0-4844340	9-9753208	73	0-0246792	9-5402453	680	10-4597547
10 0-5159300	606	0-4840700	9-9752769	73	0-0247231	9-5406531	680	10-4593469
11 0-5162936	605	0-4837060	9-9752330	73	0-0247670	9-5410606	679	10-4589394
12 0-5166569	606	0-4833431	9-9751891	73	0-0248109	9-5414678	679	10-4585322
13 0-5170198	604	0-4829802	9-9751451	73	0-0248549	9-5418747	678	10-4581253
14 0-5173824	604	0-4826176	9-9751011	73	0-0248989	9-5422813	677	10-4577187
15 0-5177447	603	0-4822559	9-9750570	73	0-0249430	9-5426877	677	10-4573123
16 0-5181066	603	0-4818934	9-9750129	73	0-0249871	9-5430937	676	10-4569063
17 0-5184689	602	0-4815318	9-9749688	74	0-0250312	9-5434994	676	10-4565006
18 0-5188295	601	0-4811705	9-9749246	74	0-0250753	9-5439048	675	10-4560952
19 0-5191904	601	0-4808096	9-9748804	74	0-0251195	9-5443101	675	10-4556900
20 0-5195510	600	0-4804490	9-9748361	74	0-0251639	9-5447148	674	10-4552852
21 0-5199112	600	0-4800888	9-9747918	74	0-0252082	9-5451193	674	10-4548807
22 0-5202711	599	0-4797289	9-9747475	74	0-0252525	9-5455236	673	10-4544764
23 0-5206307	599	0-4793693	9-9747031	74	0-0252969	9-5459276	673	10-4540724
24 0-5209899	598	0-4790101	9-9746587	74	0-0253413	9-5463312	672	10-4536688
25 0-5213488	598	0-4786512	9-9746142	74	0-0253858	9-5467346	672	10-4532654
26 0-5217074	597	0-4782926	9-9745697	74	0-0254303	9-5471377	671	10-4528623
27 0-5220656	596	0-4779344	9-9745252	74	0-0254748	9-5475405	671	10-4524595
28 0-5224235	596	0-4775765	9-9744806	74	0-0255194	9-5479431	670	10-4520570
29 0-5227811	595	0-4772189	9-9744359	74	0-0255641	9-5483452	670	10-4516548
30 0-5231383	595	0-4768617	9-9743913	74	0-0256087	9-5487471	669	10-4512529
31 0-5234953	594	0-4765047	9-9743466	75	0-0256534	9-5491487	669	10-4508513
32 0-5238518	594	0-4761482	9-9743018	75	0-0256982	9-5495500	668	10-4504500
33 0-5242081	593	0-4757919	9-9742570	75	0-0257430	9-5499511	668	10-4500489
34 0-5245640	593	0-4754360	9-9742122	75	0-0257878	9-5503519	667	10-4496481
35 0-5249196	592	0-4750804	9-9741673	75	0-0258327	9-5507523	667	10-4492477
36 0-5252749	591	0-4747251	9-9741224	75	0-0258776	9-5511525	666	10-4488475
37 0-5256298	591	0-4743702	9-9740774	75	0-0259226	9-5515524	666	10-4484476
38 0-5259844	590	0-4740156	9-9740324	75	0-0259676	9-5519521	665	10-4480479
39 0-5263387	590	0-4736613	9-9739873	75	0-0260127	9-5523514	665	10-4476486
40 0-5266927	589	0-4733075	9-9739423	75	0-0260578	9-5527504	664	10-4472496
41 0-5270463	589	0-4729532	9-9738971	75	0-0261029	9-5531492	664	10-4468508
42 0-5273997	588	0-4726003	9-9738519	75	0-0261481	9-5535477	664	10-4464523
43 0-5277526	588	0-4722477	9-9738067	75	0-0261933	9-5539459	663	10-4460539
44 0-5281053	587	0-4718947	9-9737615	75	0-0262385	9-5543438	663	10-4456562
45 0-5284577	587	0-4715423	9-9737162	75	0-0262837	9-5547415	662	10-4452585
46 0-5288099	586	0-4711903	9-9736709	76	0-0263289	9-5551388	662	10-4448612
47 0-5291614	585	0-4708386	9-9736253	76	0-0263741	9-5555359	661	10-4444641
48 0-5295128	585	0-4704872	9-9735801	76	0-0264193	9-5559327	661	10-4440673
49 0-5298638	584	0-4701362	9-9735346	76	0-0264645	9-5563292	661	10-4436708
50 0-5302146	583	0-4697854	9-9734891	76	0-0265097	9-5567255	660	10-4432745
51 0-5305650	583	0-4694350	9-9734435	76	0-0265550	9-5571214	659	10-4428786
52 0-5309151	583	0-4690849	9-9733980	76	0-0266002	9-5575171	659	10-4424829
53 0-5312649	582	0-4687351	9-9733523	76	0-0266454	9-5579125	659	10-4420875
54 0-5316143	582	0-4683857	9-9733067	76	0-0266906	9-5583077	658	10-4416923
55 0-5319635	581	0-4680363	9-9732610	76	0-0267358	9-5587025	658	10-4412975
56 0-5323123	581	0-4676873	9-9732152	76	0-0267809	9-5590971	657	10-4409029
57 0-5326608	580	0-4673389	9-9731694	76	0-0268261	9-5594914	657	10-4405086
58 0-5330090	580	0-4669910	9-9731236	76	0-0268713	9-5598854	656	10-4401146
59 0-5333569	579	0-4666431	9-9730777	76	0-0269165	9-5602792	656	10-4397208
60 0-5337044	578	0-4662956	9-9730318	77	0-0269617	9-5606727	655	10-4393273
61 0-5340517	578	0-4659483	9-9729858	77	0-0270069	9-5610659	655	10-4389341
Cosine.	D10'	Comp. sin.	Sine	D10'	Comp. sin.	Cotang.	D10'	Tangent

20 Deg.

Tab. 18.

Sine	D10"	Comp. sin.	Cosine	D10"	Comp. sin.	Tangent	D10"	Cotangent
0 9-5340517	578	0-4659189	9-9729836	77	0-0270142	9-5610656	635	10-4389344
1 9-5343586	578	0-4656014	9-9729398	77	0-0270602	9-5615538	635	10-4386272
2 9-5346745	577	0-4652848	9-9728938	77	0-0271062	9-5620419	635	10-4383200
3 9-5349915	577	0-4649682	9-9728477	77	0-0271522	9-5625300	635	10-4380128
4 9-5353085	576	0-4646516	9-9728016	77	0-0271982	9-5630181	635	10-4377056
5 9-5356255	576	0-4643350	9-9727554	77	0-0272442	9-5635062	635	10-4373984
6 9-5359425	575	0-4640184	9-9727093	77	0-0272902	9-5639943	635	10-4370912
7 9-5362595	574	0-4637018	9-9726632	77	0-0273362	9-5644824	635	10-4367840
8 9-5365765	574	0-4633852	9-9726171	77	0-0273822	9-5649705	635	10-4364768
9 9-5368935	573	0-4630686	9-9725710	77	0-0274282	9-5654586	635	10-4361696
10 9-5372105	573	0-4627520	9-9725249	77	0-0274742	9-5659467	635	10-4358624
11 9-5375275	572	0-4624354	9-9724788	77	0-0275202	9-5664348	635	10-4355552
12 9-5378445	572	0-4621188	9-9724327	77	0-0275662	9-5669229	635	10-4352480
13 9-5381615	571	0-4618022	9-9723866	77	0-0276122	9-5674110	635	10-4349408
14 9-5384785	571	0-4614856	9-9723405	77	0-0276582	9-5678991	635	10-4346336
15 9-5387955	570	0-4611690	9-9722944	77	0-0277042	9-5683872	635	10-4343264
16 9-5391125	570	0-4608524	9-9722483	77	0-0277502	9-5688753	635	10-4340192
17 9-5394295	569	0-4605358	9-9722022	77	0-0277962	9-5693634	635	10-4337120
18 9-5397465	569	0-4602192	9-9721561	77	0-0278422	9-5698515	635	10-4334048
19 9-5400635	568	0-4599026	9-9721100	77	0-0278882	9-5703396	635	10-4330976
20 9-5403805	568	0-4595860	9-9720639	77	0-0279342	9-5708277	635	10-4327904
21 9-5406975	567	0-4592694	9-9720178	77	0-0279802	9-5713158	635	10-4324832
22 9-5410145	567	0-4589528	9-9719717	77	0-0280262	9-5718039	635	10-4321760
23 9-5413315	566	0-4586362	9-9719256	77	0-0280722	9-5722920	635	10-4318688
24 9-5416485	566	0-4583196	9-9718795	77	0-0281182	9-5727801	635	10-4315616
25 9-5419655	565	0-4580030	9-9718334	77	0-0281642	9-5732682	635	10-4312544
26 9-5422825	565	0-4576864	9-9717873	77	0-0282102	9-5737563	635	10-4309472
27 9-5425995	564	0-4573698	9-9717412	77	0-0282562	9-5742444	635	10-4306400
28 9-5429165	564	0-4570532	9-9716951	77	0-0283022	9-5747325	635	10-4303328
29 9-5432335	563	0-4567366	9-9716490	77	0-0283482	9-5752206	635	10-4300256
30 9-5435505	563	0-4564200	9-9716029	77	0-0283942	9-5757087	635	10-4297184
31 9-5438675	562	0-4561034	9-9715568	77	0-0284402	9-5761968	635	10-4294112
32 9-5441845	562	0-4557868	9-9715107	77	0-0284862	9-5766849	635	10-4291040
33 9-5445015	561	0-4554702	9-9714646	77	0-0285322	9-5771730	635	10-4287968
34 9-5448185	561	0-4551536	9-9714185	77	0-0285782	9-5776611	635	10-4284896
35 9-5451355	560	0-4548370	9-9713724	77	0-0286242	9-5781492	635	10-4281824
36 9-5454525	560	0-4545204	9-9713263	77	0-0286702	9-5786373	635	10-4278752
37 9-5457695	559	0-4542038	9-9712802	77	0-0287162	9-5791254	635	10-4275680
38 9-5460865	559	0-4538872	9-9712341	77	0-0287622	9-5796135	635	10-4272608
39 9-5464035	558	0-4535706	9-9711880	77	0-0288082	9-5801016	635	10-4269536
40 9-5467205	558	0-4532540	9-9711419	77	0-0288542	9-5805897	635	10-4266464
41 9-5470375	557	0-4529374	9-9710958	77	0-0289002	9-5810778	635	10-4263392
42 9-5473545	557	0-4526208	9-9710497	77	0-0289462	9-5815659	635	10-4260320
43 9-5476715	556	0-4523042	9-9710036	77	0-0289922	9-5820540	635	10-4257248
44 9-5479885	556	0-4519876	9-9709575	77	0-0290382	9-5825421	635	10-4254176
45 9-5483055	555	0-4516710	9-9709114	77	0-0290842	9-5830302	635	10-4251104
46 9-5486225	555	0-4513544	9-9708653	77	0-0291302	9-5835183	635	10-4248032
47 9-5489395	554	0-4510378	9-9708192	77	0-0291762	9-5840064	635	10-4244960
48 9-5492565	554	0-4507212	9-9707731	77	0-0292222	9-5844945	635	10-4241888
49 9-5495735	553	0-4504046	9-9707270	77	0-0292682	9-5849826	635	10-4238816
50 9-5498905	553	0-4500880	9-9706809	77	0-0293142	9-5854707	635	10-4235744
51 9-5502075	552	0-4497714	9-9706348	77	0-0293602	9-5859588	635	10-4232672
52 9-5505245	552	0-4494548	9-9705887	77	0-0294062	9-5864469	635	10-4229600
53 9-5508415	551	0-4491382	9-9705426	77	0-0294522	9-5869350	635	10-4226528
54 9-5511585	551	0-4488216	9-9704965	77	0-0294982	9-5874231	635	10-4223456
55 9-5514755	550	0-4485050	9-9704504	77	0-0295442	9-5879112	635	10-4220384
56 9-5517925	550	0-4481884	9-9704043	77	0-0295902	9-5883993	635	10-4217312
57 9-5521095	549	0-4478718	9-9703582	77	0-0296362	9-5888874	635	10-4214240
58 9-5524265	549	0-4475552	9-9703121	77	0-0296822	9-5893755	635	10-4211168
59 9-5527435	549	0-4472386	9-9702660	77	0-0297282	9-5898636	635	10-4208096
60 9-5530605	548	0-4469220	9-9702199	77	0-0297742	9-5903517	635	10-4205024
61 9-5533775	548	0-4466054	9-9701738	77	0-0298202	9-5908398	635	10-4201952
62 9-5536945	547	0-4462888	9-9701277	77	0-0298662	9-5913279	635	10-4198880
63 9-5540115	547	0-4459722	9-9700816	77	0-0299122	9-5918160	635	10-4195808
64 9-5543285	546	0-4456556	9-9700355	77	0-0299582	9-5923041	635	10-4192736
65 9-5546455	546	0-4453390	9-9700000	77	0-0300042	9-5927922	635	10-4189664
66 9-5549625	545	0-4450224	9-9699539	77	0-0300502	9-5932803	635	10-4186592
67 9-5552795	545	0-4447058	9-9699078	77	0-0300962	9-5937684	635	10-4183520
68 9-5555965	544	0-4443892	9-9698617	77	0-0301422	9-5942565	635	10-4180448
69 9-5559135	544	0-4440726	9-9698156	77	0-0301882	9-5947446	635	10-4177376
70 9-5562305	543	0-4437560	9-9697695	77	0-0302342	9-5952327	635	10-4174304
71 9-5565475	543	0-4434394	9-9697234	77	0-0302802	9-5957208	635	10-4171232
72 9-5568645	542	0-4431228	9-9696773	77	0-0303262	9-5962089	635	10-4168160
73 9-5571815	542	0-4428062	9-9696312	77	0-0303722	9-5966970	635	10-4165088
74 9-5574985	541	0-4424896	9-9695851	77	0-0304182	9-5971851	635	10-4162016
75 9-5578155	541	0-4421730	9-9695390	77	0-0304642	9-5976732	635	10-4158944
76 9-5581325	540	0-4418564	9-9694929	77	0-0305102	9-5981613	635	10-4155872
77 9-5584495	540	0-4415398	9-9694468	77	0-0305562	9-5986494	635	10-4152800
78 9-5587665	539	0-4412232	9-9694007	77	0-0306022	9-5991375	635	10-4149728
79 9-5590835	539	0-4409066	9-9693546	77	0-0306482	9-5996256	635	10-4146656
80 9-5594005	538	0-4405900	9-9693085	77	0-0306942	9-6001137	635	10-4143584
81 9-5597175	538	0-4402734	9-9692624	77	0-0307402	9-6006018	635	10-4140512
82 9-5600345	537	0-4399568	9-9692163	77	0-0307862	9-6010899	635	10-4137440
83 9-5603515	537	0-4396402	9-9691702	77	0-0308322	9-6015780	635	10-4134368
84 9-5606685	536	0-4393236	9-9691241	77	0-0308782	9-6020661	635	10-4131296
85 9-5609855	536	0-4390070	9-9690780	77	0-0309242	9-6025542	635	10-4128224
86 9-5613025	535	0-4386904	9-9690319	77	0-0309702	9-6030423	635	10-4125152
87 9-5616195	535	0-4383738	9-9689858	77	0-0310162	9-6035304	635	10-4122080
88 9-5619365	534	0-4380572	9-9689397	77	0-0310622	9-6040185	635	10-4119008
89 9-5622535	534	0-4377406	9-9688936	77	0-0311082	9-6045066	635	10-4115936
90 9-5625705	533	0-4374240	9-9688475	77	0-0311542	9-6049947	635	10-4112864
91 9-5628875	533	0-4371074	9-9688014	77	0-0312002	9-6054828	635	10-4109792
92 9-5632045	532	0-4367908	9-9687553	77	0-0312462	9-6059709	635	10-4106720
93 9-5635215	532	0-4364742	9-9687092	77	0-0312922	9-6064590	635	10-4103648
94 9-5638385	531	0-4361576	9-9686631	77	0-0313382	9-6069471	635	10-4100576
95 9-5641555	531	0-4358410	9-9686170	77	0-0313842	9-6074352	635	10-4097504
96 9-5644725	530	0-4355244	9-9685709	77	0-0314302	9-6079233	635	10-4094432
97 9-5647895	530	0-4352078	9-9685248	77	0-0314762	9-6084114	635	10-4091360
98 9-5651065	529	0-4348912	9-9684787	77	0-0315222	9-6088995	635	10-4088288
99 9-5654235	529	0-4345746	9-9684326	77	0-0315682	9-6093876	635	10-4085216
100 9-5657405	528	0-4342580	9-9683865	77	0-0316142	9-6098757	635	10-4082144

Cosine	D10"	Comp. cos.	Sine	D10"	Comp. sin.	Cotang.	D10"	Tangent
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Tab. 18.

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Dep. 69.

Sine	D10	Comp. sin.	Cosine	D10	Comp. cos.	Tangent	D10	Cotangent
0.554922	568	0.4456708	0.9701317	80	0.0298483	0.5841774	629	0.4158226
1.0.554961	568	0.4456419	0.9701037	81	0.0298968	0.5843540	629	0.4158451
2.0.554986	567	0.4456132	0.9700747	81	0.0299453	0.5845321	628	0.4158679
3.0.555015	567	0.4455843	0.9700461	81	0.0299938	0.5847109	628	0.4158907
4.0.555043	566	0.4455555	0.9699957	81	0.0300423	0.5848895	627	0.4159134
5.0.555071	566	0.4455267	0.9699660	81	0.0300908	0.5850682	627	0.4159362
6.0.555098	565	0.4454979	0.9699360	81	0.0301393	0.5852468	626	0.4159589
7.0.555125	565	0.4454691	0.9699061	81	0.0301878	0.5854254	626	0.4159817
8.0.555152	564	0.4454403	0.9698762	81	0.0302363	0.5856040	625	0.4160044
9.0.555179	564	0.4454115	0.9698462	81	0.0302848	0.5857826	625	0.4160272
10.0.555206	563	0.4453827	0.9698163	81	0.0303333	0.5859612	625	0.4160499
11.0.555233	563	0.4453539	0.9697863	81	0.0303818	0.5861398	624	0.4160727
12.0.555260	562	0.4453251	0.9697564	81	0.0304303	0.5863184	624	0.4160954
13.0.555287	562	0.4452963	0.9697264	81	0.0304788	0.5864970	623	0.4161182
14.0.555314	561	0.4452675	0.9696965	81	0.0305273	0.5866756	623	0.4161409
15.0.555341	561	0.4452387	0.9696665	81	0.0305758	0.5868542	622	0.4161637
16.0.555368	560	0.4452099	0.9696366	81	0.0306243	0.5870328	622	0.4161864
17.0.555395	560	0.4451811	0.9696066	81	0.0306728	0.5872114	621	0.4162092
18.0.555422	559	0.4451523	0.9695767	81	0.0307213	0.5873900	621	0.4162319
19.0.555449	559	0.4451235	0.9695467	81	0.0307698	0.5875686	620	0.4162547
20.0.555476	558	0.4450947	0.9695168	81	0.0308183	0.5877472	620	0.4162774
21.0.555503	558	0.4450659	0.9694868	81	0.0308668	0.5879258	619	0.4163002
22.0.555530	557	0.4450371	0.9694569	81	0.0309153	0.5881044	619	0.4163229
23.0.555557	557	0.4450083	0.9694269	81	0.0309638	0.5882830	618	0.4163457
24.0.555584	556	0.4449795	0.9693970	81	0.0310123	0.5884616	618	0.4163684
25.0.555611	556	0.4449507	0.9693670	81	0.0310608	0.5886402	617	0.4163912
26.0.555638	555	0.4449219	0.9693371	81	0.0311093	0.5888188	617	0.4164139
27.0.555665	555	0.4448931	0.9693071	81	0.0311578	0.5889974	616	0.4164367
28.0.555692	554	0.4448643	0.9692772	81	0.0312063	0.5891760	616	0.4164594
29.0.555719	554	0.4448355	0.9692472	81	0.0312548	0.5893546	615	0.4164822
30.0.555746	553	0.4448067	0.9692173	81	0.0313033	0.5895332	615	0.4165049
31.0.555773	553	0.4447779	0.9691873	81	0.0313518	0.5897118	614	0.4165277
32.0.555800	552	0.4447491	0.9691574	81	0.0314003	0.5898904	614	0.4165504
33.0.555827	552	0.4447203	0.9691274	81	0.0314488	0.5900690	613	0.4165732
34.0.555854	551	0.4446915	0.9690975	81	0.0314973	0.5902476	613	0.4165959
35.0.555881	551	0.4446627	0.9690675	81	0.0315458	0.5904262	612	0.4166187
36.0.555908	550	0.4446339	0.9690376	81	0.0315943	0.5906048	612	0.4166414
37.0.555935	550	0.4446051	0.9690076	81	0.0316428	0.5907834	611	0.4166642
38.0.555962	549	0.4445763	0.9689777	81	0.0316913	0.5909620	611	0.4166869
39.0.555989	549	0.4445475	0.9689477	81	0.0317398	0.5911406	610	0.4167097
40.0.556016	548	0.4445187	0.9689178	81	0.0317883	0.5913192	610	0.4167324
41.0.556043	548	0.4444899	0.9688878	81	0.0318368	0.5914978	609	0.4167552
42.0.556070	547	0.4444611	0.9688579	81	0.0318853	0.5916764	609	0.4167779
43.0.556097	547	0.4444323	0.9688279	81	0.0319338	0.5918550	608	0.4168007
44.0.556124	546	0.4444035	0.9687980	81	0.0319823	0.5920336	608	0.4168234
45.0.556151	546	0.4443747	0.9687680	81	0.0320308	0.5922122	607	0.4168462
46.0.556178	545	0.4443459	0.9687381	81	0.0320793	0.5923908	607	0.4168689
47.0.556205	545	0.4443171	0.9687081	81	0.0321278	0.5925694	606	0.4168917
48.0.556232	544	0.4442883	0.9686782	81	0.0321763	0.5927480	606	0.4169144
49.0.556259	544	0.4442595	0.9686482	81	0.0322248	0.5929266	605	0.4169372
50.0.556286	543	0.4442307	0.9686183	81	0.0322733	0.5931052	605	0.4169599
51.0.556313	543	0.4442019	0.9685883	81	0.0323218	0.5932838	604	0.4169827
52.0.556340	542	0.4441731	0.9685584	81	0.0323703	0.5934624	604	0.4170054
53.0.556367	542	0.4441443	0.9685284	81	0.0324188	0.5936410	603	0.4170282
54.0.556394	541	0.4441155	0.9684985	81	0.0324673	0.5938196	603	0.4170509
55.0.556421	541	0.4440867	0.9684685	81	0.0325158	0.5939982	602	0.4170737
56.0.556448	540	0.4440579	0.9684386	81	0.0325643	0.5941768	602	0.4170964
57.0.556475	540	0.4440291	0.9684086	81	0.0326128	0.5943554	601	0.4171192
58.0.556502	539	0.4440003	0.9683787	81	0.0326613	0.5945340	601	0.4171419
59.0.556529	539	0.4439715	0.9683487	81	0.0327098	0.5947126	600	0.4171647
60.0.556556	538	0.4439427	0.9683188	81	0.0327583	0.5948912	600	0.4171874
61.0.556583	538	0.4439139	0.9682888	81	0.0328068	0.5950698	599	0.4172102
62.0.556610	537	0.4438851	0.9682589	81	0.0328553	0.5952484	599	0.4172329
63.0.556637	537	0.4438563	0.9682289	81	0.0329038	0.5954270	598	0.4172557
64.0.556664	536	0.4438275	0.9681990	81	0.0329523	0.5956056	598	0.4172784
65.0.556691	536	0.4437987	0.9681690	81	0.0330008	0.5957842	597	0.4173012
66.0.556718	535	0.4437699	0.9681391	81	0.0330493	0.5959628	597	0.4173239
67.0.556745	535	0.4437411	0.9681091	81	0.0330978	0.5961414	596	0.4173467
68.0.556772	534	0.4437123	0.9680792	81	0.0331463	0.5963200	596	0.4173694
69.0.556799	534	0.4436835	0.9680492	81	0.0331948	0.5964986	595	0.4173922
70.0.556826	533	0.4436547	0.9680193	81	0.0332433	0.5966772	595	0.4174149
71.0.556853	533	0.4436259	0.9679893	81	0.0332918	0.5968558	594	0.4174377
72.0.556880	532	0.4435971	0.9679594	81	0.0333403	0.5970344	594	0.4174604
73.0.556907	532	0.4435683	0.9679294	81	0.0333888	0.5972130	593	0.4174832
74.0.556934	531	0.4435395	0.9678995	81	0.0334373	0.5973916	593	0.4175059
75.0.556961	531	0.4435107	0.9678695	81	0.0334858	0.5975702	592	0.4175287
76.0.556988	530	0.4434819	0.9678396	81	0.0335343	0.5977488	592	0.4175514
77.0.557015	530	0.4434531	0.9678096	81	0.0335828	0.5979274	591	0.4175742
78.0.557042	529	0.4434243	0.9677797	81	0.0336313	0.5981060	591	0.4175969
79.0.557069	529	0.4433955	0.9677497	81	0.0336798	0.5982846	590	0.4176197
80.0.557096	528	0.4433667	0.9677198	81	0.0337283	0.5984632	590	0.4176424
81.0.557123	528	0.4433379	0.9676898	81	0.0337768	0.5986418	589	0.4176652
82.0.557150	527	0.4433091	0.9676599	81	0.0338253	0.5988204	589	0.4176879
83.0.557177	527	0.4432803	0.9676299	81	0.0338738	0.5989990	588	0.4177107
84.0.557204	526	0.4432515	0.9676000	81	0.0339223	0.5991776	588	0.4177334
85.0.557231	526	0.4432227	0.9675700	81	0.0339708	0.5993562	587	0.4177562
86.0.557258	525	0.4431939	0.9675401	81	0.0340193	0.5995348	587	0.4177789
87.0.557285	525	0.4431651	0.9675101	81	0.0340678	0.5997134	586	0.4178017
88.0.557312	524	0.4431363	0.9674802	81	0.0341163	0.5998920	586	0.4178244
89.0.557339	524	0.4431075	0.9674502	81	0.0341648	0.6000706	585	0.4178472
90.0.557366	523	0.4430787	0.9674203	81	0.0342133	0.6002492	585	0.4178699
91.0.557393	523	0.4430499	0.9673903	81	0.0342618	0.6004278	584	0.4178927
92.0.557420	522	0.4430211	0.9673604	81	0.0343103	0.6006064	584	0.4179154
93.0.557447	522	0.4429923	0.9673304	81	0.0343588	0.6007850	583	0.4179382
94.0.557474	521	0.4429635	0.9673005	81	0.0344073	0.6009636	583	0.4179609
95.0.557501	521	0.4429347	0.9672705	81	0.0344558	0.6011422	582	0.4179837
96.0.557528	520	0.4429059	0.9672406	81	0.0345043	0.6013208	582	0.4180064
97.0.557555	520	0.4428771	0.9672106	81	0.0345528	0.6014994	581	0.4180292
98.0.557582	519	0.4428483	0.9671807	81	0.0346013	0.6016780	581	0.4180519
99.0.557609	519	0.4428195	0.9671507	81	0.0346498	0.6018566	580	0.4180747
100.0.557636	518	0.4427907	0.9671208	81	0.0346983	0.6020352	580	0.4180974

22 Deg.

Tab. 18.

Sine	D10	Comp. sin.	Cosine	D10	Comp. cos.	Tangent	D10	Cotangent
0 9 573545	521	0 426428	9 967165	82	0 032851	9 606409	600	10 393591
1 9 573880	520	0 426112	9 967114	85	0 032832	9 606732	600	10 393268
2 9 574203	520	0 425797	9 967063	85	0 032813	9 607056	600	10 392945
3 9 574519	519	0 425482	9 967012	85	0 032794	9 607380	600	10 392622
4 9 574830	519	0 425166	9 966961	85	0 032775	9 607704	600	10 392299
5 9 575136	519	0 424851	9 966910	85	0 032756	9 608028	600	10 391976
6 9 575448	518	0 424535	9 966859	85	0 032737	9 608352	600	10 391653
7 9 575757	518	0 424220	9 966808	85	0 032718	9 608676	600	10 391330
8 9 576063	517	0 423904	9 966757	85	0 032699	9 609000	600	10 390997
9 9 576370	517	0 423589	9 966706	85	0 032680	9 609324	600	10 390674
10 9 576682	516	0 423273	9 966655	85	0 032661	9 609648	600	10 390351
11 9 576991	516	0 422958	9 966604	85	0 032642	9 610000	600	10 389997
12 9 577308	516	0 422642	9 966553	85	0 032623	9 610324	600	10 389674
13 9 577618	515	0 422327	9 966502	85	0 032604	9 610648	600	10 389351
14 9 577927	515	0 422011	9 966451	85	0 032585	9 610972	600	10 389028
15 9 578236	514	0 421696	9 966400	85	0 032566	9 611296	600	10 388705
16 9 578545	514	0 421380	9 966349	85	0 032547	9 611620	600	10 388382
17 9 578853	513	0 421065	9 966298	85	0 032528	9 611944	600	10 388059
18 9 579161	513	0 420749	9 966247	85	0 032509	9 612268	600	10 387736
19 9 579469	513	0 420434	9 966196	85	0 032490	9 612592	600	10 387413
20 9 579777	512	0 420118	9 966145	85	0 032471	9 612916	600	10 387090
21 9 580084	512	0 419803	9 966094	85	0 032452	9 613240	600	10 386767
22 9 580391	511	0 419487	9 966043	85	0 032433	9 613564	600	10 386444
23 9 580698	511	0 419172	9 965992	85	0 032414	9 613888	600	10 386121
24 9 581005	511	0 418856	9 965941	85	0 032395	9 614212	600	10 385798
25 9 581311	510	0 418541	9 965890	85	0 032376	9 614536	600	10 385475
26 9 581617	510	0 418225	9 965839	85	0 032357	9 614860	600	10 385152
27 9 581923	509	0 417910	9 965788	85	0 032338	9 615184	600	10 384829
28 9 582229	509	0 417594	9 965737	85	0 032319	9 615508	600	10 384506
29 9 582535	509	0 417279	9 965686	85	0 032300	9 615832	600	10 384183
30 9 582839	508	0 416963	9 965635	85	0 032281	9 616156	600	10 383860
31 9 583144	508	0 416648	9 965584	85	0 032262	9 616480	600	10 383537
32 9 583449	507	0 416332	9 965533	85	0 032243	9 616804	600	10 383214
33 9 583753	507	0 416017	9 965482	85	0 032224	9 617128	600	10 382891
34 9 584057	506	0 415701	9 965431	85	0 032205	9 617452	600	10 382568
35 9 584361	506	0 415386	9 965380	85	0 032186	9 617776	600	10 382245
36 9 584665	506	0 415070	9 965329	85	0 032167	9 618100	600	10 381922
37 9 584968	505	0 414755	9 965278	85	0 032148	9 618424	600	10 381599
38 9 585271	505	0 414439	9 965227	85	0 032129	9 618748	600	10 381276
39 9 585574	504	0 414124	9 965176	85	0 032110	9 619072	600	10 380953
40 9 585877	504	0 413808	9 965125	85	0 032091	9 619396	600	10 380630
41 9 586179	503	0 413493	9 965074	85	0 032072	9 619720	600	10 380307
42 9 586481	503	0 413177	9 965023	85	0 032053	9 620044	600	10 379984
43 9 586783	503	0 412862	9 964972	85	0 032034	9 620368	600	10 379661
44 9 587085	502	0 412546	9 964921	85	0 032015	9 620692	600	10 379338
45 9 587386	502	0 412231	9 964870	85	0 031996	9 621016	600	10 379015
46 9 587687	501	0 411915	9 964819	85	0 031977	9 621340	600	10 378692
47 9 587988	501	0 411600	9 964768	85	0 031958	9 621664	600	10 378369
48 9 588289	500	0 411284	9 964717	85	0 031939	9 621988	600	10 378046
49 9 588589	500	0 410969	9 964666	85	0 031920	9 622312	600	10 377723
50 9 588889	500	0 410653	9 964615	85	0 031901	9 622636	600	10 377400
51 9 589189	499	0 410338	9 964564	85	0 031882	9 622960	600	10 377077
52 9 589489	499	0 410022	9 964513	85	0 031863	9 623284	600	10 376754
53 9 589788	499	0 409707	9 964462	85	0 031844	9 623608	600	10 376431
54 9 590088	498	0 409391	9 964411	85	0 031825	9 623932	600	10 376108
55 9 590386	498	0 409076	9 964360	85	0 031806	9 624256	600	10 375785
56 9 590685	497	0 408760	9 964309	85	0 031787	9 624580	600	10 375462
57 9 590984	497	0 408445	9 964258	85	0 031768	9 624904	600	10 375139
58 9 591282	497	0 408129	9 964207	85	0 031749	9 625228	600	10 374816
59 9 591580	496	0 407814	9 964156	85	0 031730	9 625552	600	10 374493
60 9 591878	496	0 407498	9 964105	85	0 031711	9 625876	600	10 374170

Tab. 18.

Deg. 67.

23 Deg.

Sine	D10	Comp. sin.	Cosine	D10	Comp. cos.	Tangent	D10	Cotangent
0.95918790	496	0.4081220	0.9640261	89	0.0359739	9.6378519	585	10.3721481
1.95921755	496	0.4081220	0.9639724	89	0.0360276	9.6382031	585	10.3717969
2.95924720	495	0.4081220	0.9639187	89	0.0360813	9.6385544	585	10.3714456
3.95927685	495	0.4081220	0.9638650	89	0.0361350	9.6389057	585	10.3710943
4.95930650	494	0.4081220	0.9638112	90	0.0361887	9.6392569	584	10.3707430
5.95933611	494	0.4081220	0.9637575	90	0.0362424	9.6396082	584	10.3703917
6.95936574	493	0.4081220	0.9637038	90	0.0362961	9.6399595	583	10.3700404
7.95939535	493	0.4081220	0.9636500	90	0.0363498	9.6403108	583	10.3696891
8.95942512	493	0.4081220	0.9635963	90	0.0364035	9.6406621	583	10.3693378
9.95945469	492	0.4081220	0.9635426	90	0.0364572	9.6410134	582	10.3689865
10.95948422	492	0.4081220	0.9634889	90	0.0365109	9.6413647	582	10.3686352
11.95951375	491	0.4081220	0.9634352	90	0.0365646	9.6417160	582	10.3682839
12.95954322	491	0.4081220	0.9633815	90	0.0366183	9.6420673	581	10.3679326
13.95957278	491	0.4081220	0.9633278	90	0.0366720	9.6424186	581	10.3675813
14.95960212	490	0.4081220	0.9632741	90	0.0367257	9.6427699	581	10.3672300
15.95963154	490	0.4081220	0.9632204	90	0.0367794	9.6431212	580	10.3668787
16.95966092	489	0.4081220	0.9631667	90	0.0368331	9.6434725	580	10.3665274
17.95969030	489	0.4081220	0.9631130	91	0.0368868	9.6438238	579	10.3661761
18.95971965	489	0.4081220	0.9630593	91	0.0369405	9.6441751	579	10.3658248
19.95974897	488	0.4081220	0.9630056	91	0.0369942	9.6445264	579	10.3654735
20.95977827	488	0.4081220	0.9629519	91	0.0370479	9.6448777	579	10.3651222
21.95980751	487	0.4081220	0.9628982	91	0.0371016	9.6452290	578	10.3647709
22.95983679	487	0.4081220	0.9628445	91	0.0371553	9.6455803	578	10.3644196
23.95986602	487	0.4081220	0.9627908	91	0.0372090	9.6459316	578	10.3640683
24.95989522	486	0.4081220	0.9627371	91	0.0372627	9.6462829	577	10.3637170
25.95992441	486	0.4081220	0.9626834	91	0.0373164	9.6466342	577	10.3633657
26.95995357	485	0.4081220	0.9626297	91	0.0373701	9.6469855	577	10.3630144
27.95998273	485	0.4081220	0.9625760	91	0.0374238	9.6473368	577	10.3626631
28.95999181	484	0.4081220	0.9625223	91	0.0374775	9.6476881	576	10.3623118
29.95999094	484	0.4081220	0.9624686	91	0.0375312	9.6480394	576	10.3619605
30.95998907	484	0.4081220	0.9624149	92	0.0375849	9.6483907	576	10.3616092
31.95998710	484	0.4081220	0.9623612	92	0.0376386	9.6487420	575	10.3612579
32.95998513	483	0.4081220	0.9623075	92	0.0376923	9.6490933	575	10.3609066
33.95998316	483	0.4081220	0.9622538	92	0.0377460	9.6494446	575	10.3605553
34.95998119	482	0.4081220	0.9621999	92	0.0378000	9.6497959	574	10.3602040
35.95997922	482	0.4081220	0.9621462	92	0.0378537	9.6501472	574	10.3598527
36.95997725	481	0.4081220	0.9620925	92	0.0379074	9.6504985	574	10.3595014
37.95997528	481	0.4081220	0.9620388	92	0.0379611	9.6508498	573	10.3591501
38.95997331	481	0.4081220	0.9619851	92	0.0380148	9.6512011	573	10.3587988
39.95997134	481	0.4081220	0.9619314	92	0.0380685	9.6515524	573	10.3584475
40.95996937	480	0.4081220	0.9618777	92	0.0381222	9.6519037	572	10.3580962
41.95996740	480	0.4081220	0.9618240	92	0.0381759	9.6522550	572	10.3577449
42.95996543	479	0.4081220	0.9617703	92	0.0382296	9.6526063	572	10.3573936
43.95996346	479	0.4081220	0.9617166	92	0.0382833	9.6529576	571	10.3570423
44.95996149	479	0.4081220	0.9616629	92	0.0383370	9.6533089	571	10.3566910
45.95995952	478	0.4081220	0.9616092	93	0.0383907	9.6536602	571	10.3563397
46.95995755	478	0.4081220	0.9615555	93	0.0384444	9.6540115	571	10.3559884
47.95995558	478	0.4081220	0.9615018	93	0.0384981	9.6543628	570	10.3556371
48.95995361	477	0.4081220	0.9614481	93	0.0385518	9.6547141	570	10.3552858
49.95995164	477	0.4081220	0.9613944	93	0.0386055	9.6550654	570	10.3549345
50.95994967	476	0.4081220	0.9613407	93	0.0386592	9.6554167	569	10.3545832
51.95994770	476	0.4081220	0.9612870	93	0.0387129	9.6557680	569	10.3542319
52.95994573	475	0.4081220	0.9612333	93	0.0387666	9.6561193	569	10.3538806
53.95994376	475	0.4081220	0.9611796	93	0.0388203	9.6564706	568	10.3535293
54.95994179	475	0.4081220	0.9611259	93	0.0388740	9.6568219	568	10.3531780
55.95993982	474	0.4081220	0.9610722	93	0.0389277	9.6571732	568	10.3528267
56.95993785	474	0.4081220	0.9610185	93	0.0389814	9.6575245	567	10.3524754
57.95993588	474	0.4081220	0.9609648	93	0.0390351	9.6578758	567	10.3521241
58.95993391	473	0.4081220	0.9609111	94	0.0390888	9.6582271	567	10.3517728
59.95993194	473	0.4081220	0.9608574	94	0.0391425	9.6585784	567	10.3514215
60.95992997	473	0.4081220	0.9608037	94	0.0391962	9.6589297	567	10.3510702

Tab. 18.

Deg. 66.

24 Deg.

Tab. 18.

Sine	D10	Comp. No.	Cosine	D10	Comp. No.	Tangent	D10	Cotangent
0.96093133	473	0.8906867	0.9607302	94	0.0392698	0.6385831	566	10.3541100
1.9.6095969	472	0.8904931	0.9606739	94	0.0393821	0.6489231	565	10.3540710
2.9.6098808	472	0.8901192	0.9606176	94	0.0394824	0.6487872	565	10.3540732
3.9.6101635	472	0.8898363	0.9605612	94	0.0395888	0.6486512	565	10.3540754
4.9.6104465	471	0.8895533	0.9605048	94	0.0396952	0.6485152	565	10.3540776
5.9.6107293	471	0.8892703	0.9604484	94	0.0398016	0.6483792	565	10.3540798
6.9.6110118	470	0.8889873	0.9603919	94	0.0399080	0.6482432	565	10.3540820
7.4.6112941	470	0.8887043	0.9603354	94	0.0400144	0.6481072	565	10.3540842
8.9.6115762	470	0.8884213	0.9602788	94	0.0401208	0.6479712	564	10.3540864
9.9.6118580	469	0.8881383	0.9602222	94	0.0402272	0.6478352	564	10.3540886
10.9.6121397	469	0.8878553	0.9601655	94	0.0403336	0.6476992	564	10.3540908
11.9.6124211	469	0.8875723	0.9601088	94	0.0404400	0.6475632	564	10.3540930
12.9.6127023	468	0.8872893	0.9600522	94	0.0405464	0.6474272	564	10.3540952
13.9.6129833	468	0.8870063	0.9599955	94	0.0406528	0.6472912	564	10.3540974
14.9.6132641	467	0.8867233	0.9599388	94	0.0407592	0.6471552	564	10.3540996
15.9.6135446	467	0.8864403	0.9598822	94	0.0408656	0.6470192	564	10.3541018
16.9.6138250	467	0.8861573	0.9598255	94	0.0409720	0.6468832	564	10.3541040
17.9.6141051	466	0.8858743	0.9597688	94	0.0410784	0.6467472	564	10.3541062
18.9.6143850	466	0.8855913	0.9597122	94	0.0411848	0.6466112	564	10.3541084
19.9.6146647	466	0.8853083	0.9596555	94	0.0412912	0.6464752	564	10.3541106
20.9.6149441	465	0.8850253	0.9595988	94	0.0413976	0.6463392	564	10.3541128
21.9.6152234	465	0.8847423	0.9595422	94	0.0415040	0.6462032	564	10.3541150
22.9.6155023	465	0.8844593	0.9594855	94	0.0416104	0.6460672	564	10.3541172
23.9.6157812	464	0.8841763	0.9594288	94	0.0417168	0.6459312	564	10.3541194
24.9.6160599	464	0.8838933	0.9593722	94	0.0418232	0.6457952	564	10.3541216
25.9.6163388	464	0.8836103	0.9593155	94	0.0419296	0.6456592	564	10.3541238
26.9.6166164	463	0.8833273	0.9592588	94	0.0420360	0.6455232	564	10.3541260
27.9.6168941	463	0.8830443	0.9592022	94	0.0421424	0.6453872	564	10.3541282
28.9.6171721	462	0.8827613	0.9591455	94	0.0422488	0.6452512	564	10.3541304
29.9.6174490	462	0.8824783	0.9590888	94	0.0423552	0.6451152	564	10.3541326
30.9.6177271	462	0.8821953	0.9590322	94	0.0424616	0.6449792	564	10.3541348
31.9.6180041	461	0.8819123	0.9589755	94	0.0425680	0.6448432	564	10.3541370
32.9.6182809	461	0.8816293	0.9589188	94	0.0426744	0.6447072	564	10.3541392
33.9.6185576	461	0.8813463	0.9588622	94	0.0427808	0.6445712	564	10.3541414
34.9.6188341	460	0.8810633	0.9588055	94	0.0428872	0.6444352	564	10.3541436
35.9.6191103	460	0.8807803	0.9587488	94	0.0429936	0.6442992	564	10.3541458
36.9.6193868	459	0.8804973	0.9586922	94	0.0430999	0.6441632	564	10.3541480
37.9.6196622	459	0.8802143	0.9586355	94	0.0432063	0.6440272	564	10.3541502
38.9.6199378	458	0.8799313	0.9585788	94	0.0433127	0.6438912	564	10.3541524
39.9.6202132	458	0.8796483	0.9585222	94	0.0434191	0.6437552	564	10.3541546
40.9.6204884	458	0.8793653	0.9584655	94	0.0435255	0.6436192	564	10.3541568
41.9.6207637	458	0.8790823	0.9584088	94	0.0436319	0.6434832	564	10.3541590
42.9.6210382	458	0.8787993	0.9583522	94	0.0437383	0.6433472	564	10.3541612
43.9.6213127	457	0.8785163	0.9582955	94	0.0438447	0.6432112	564	10.3541634
44.9.6215871	457	0.8782333	0.9582388	94	0.0439511	0.6430752	564	10.3541656
45.9.6218619	457	0.8779503	0.9581822	94	0.0440575	0.6429392	564	10.3541678
46.9.6221351	456	0.8776673	0.9581255	94	0.0441639	0.6428032	564	10.3541700
47.9.6224088	456	0.8773843	0.9580688	94	0.0442703	0.6426672	564	10.3541722
48.9.6226824	455	0.8771013	0.9580122	94	0.0443767	0.6425312	564	10.3541744
49.9.6229557	455	0.8768183	0.9579555	94	0.0444831	0.6423952	564	10.3541766
50.9.6232287	455	0.8765353	0.9578988	94	0.0445895	0.6422592	564	10.3541788
51.9.6235016	454	0.8762523	0.9578422	94	0.0446959	0.6421232	564	10.3541810
52.9.6237743	454	0.8759693	0.9577855	94	0.0448023	0.6419872	564	10.3541832
53.9.6240468	454	0.8756863	0.9577288	94	0.0449087	0.6418512	564	10.3541854
54.9.6243190	453	0.8754033	0.9576722	94	0.0450151	0.6417152	564	10.3541876
55.9.6245911	453	0.8751203	0.9576155	94	0.0451215	0.6415792	564	10.3541898
56.9.6248639	453	0.8748373	0.9575588	94	0.0452279	0.6414432	564	10.3541920
57.9.6251360	452	0.8745543	0.9575022	94	0.0453343	0.6413072	564	10.3541942
58.9.6254060	452	0.8742713	0.9574455	94	0.0454407	0.6411712	564	10.3541964
59.9.6256772	452	0.8739883	0.9573888	94	0.0455471	0.6410352	564	10.3541986
60.9.6259483	452	0.8737053	0.9573322	94	0.0456535	0.6408992	564	10.3542008

Tab. 18.

Deg. 65.

25. Deg.

Tab. 13:

[illegible]

7-24-18.

DEC. 64

96 Deg.

Tab. 15.

Sine	Dio	Comp. sin.	Cosine	Dio	Comp. sin.	Tangent	Dio	Cotangent
0° 0' 0" 0.00000	431	0.3555580	0.9536602	103	0.0463338	0.0000000	534	10.8181818
1° 0' 0" 0.00000	431	0.3555580	0.9535985	103	0.0464013	0.0000000	534	10.8181818
2° 0' 0" 0.00000	431	0.3557640	0.9535369	103	0.0464687	0.0000000	534	10.8181818
3° 0' 0" 0.00000	431	0.3559700	0.9534753	103	0.0465362	0.0000000	534	10.8181818
4° 0' 0" 0.00000	431	0.3561760	0.9534137	103	0.0466036	0.0000000	534	10.8181818
5° 0' 0" 0.00000	431	0.3563820	0.9533521	103	0.0466711	0.0000000	534	10.8181818
6° 0' 0" 0.00000	431	0.3565880	0.9532905	103	0.0467385	0.0000000	534	10.8181818
7° 0' 0" 0.00000	431	0.3567940	0.9532289	103	0.0468059	0.0000000	534	10.8181818
8° 0' 0" 0.00000	431	0.3570000	0.9531673	103	0.0468734	0.0000000	534	10.8181818
9° 0' 0" 0.00000	431	0.3572060	0.9531057	103	0.0469408	0.0000000	534	10.8181818
10° 0' 0" 0.00000	431	0.3574120	0.9530441	103	0.0470082	0.0000000	534	10.8181818
11° 0' 0" 0.00000	431	0.3576180	0.9529825	103	0.0470757	0.0000000	534	10.8181818
12° 0' 0" 0.00000	431	0.3578240	0.9529209	103	0.0471431	0.0000000	534	10.8181818
13° 0' 0" 0.00000	431	0.3580300	0.9528593	103	0.0472105	0.0000000	534	10.8181818
14° 0' 0" 0.00000	431	0.3582360	0.9527977	103	0.0472779	0.0000000	534	10.8181818
15° 0' 0" 0.00000	431	0.3584420	0.9527361	103	0.0473453	0.0000000	534	10.8181818
16° 0' 0" 0.00000	431	0.3586480	0.9526745	103	0.0474127	0.0000000	534	10.8181818
17° 0' 0" 0.00000	431	0.3588540	0.9526129	103	0.0474801	0.0000000	534	10.8181818
18° 0' 0" 0.00000	431	0.3590600	0.9525513	103	0.0475475	0.0000000	534	10.8181818
19° 0' 0" 0.00000	431	0.3592660	0.9524897	103	0.0476149	0.0000000	534	10.8181818
20° 0' 0" 0.00000	431	0.3594720	0.9524281	103	0.0476823	0.0000000	534	10.8181818
21° 0' 0" 0.00000	431	0.3596780	0.9523665	103	0.0477497	0.0000000	534	10.8181818
22° 0' 0" 0.00000	431	0.3598840	0.9523049	103	0.0478171	0.0000000	534	10.8181818
23° 0' 0" 0.00000	431	0.3600900	0.9522433	103	0.0478845	0.0000000	534	10.8181818
24° 0' 0" 0.00000	431	0.3602960	0.9521817	103	0.0479519	0.0000000	534	10.8181818
25° 0' 0" 0.00000	431	0.3605020	0.9521201	103	0.0480193	0.0000000	534	10.8181818
26° 0' 0" 0.00000	431	0.3607080	0.9520585	103	0.0480867	0.0000000	534	10.8181818
27° 0' 0" 0.00000	431	0.3609140	0.9519969	103	0.0481541	0.0000000	534	10.8181818
28° 0' 0" 0.00000	431	0.3611200	0.9519353	103	0.0482215	0.0000000	534	10.8181818
29° 0' 0" 0.00000	431	0.3613260	0.9518737	103	0.0482889	0.0000000	534	10.8181818
30° 0' 0" 0.00000	431	0.3615320	0.9518121	103	0.0483563	0.0000000	534	10.8181818
31° 0' 0" 0.00000	431	0.3617380	0.9517505	103	0.0484237	0.0000000	534	10.8181818
32° 0' 0" 0.00000	431	0.3619440	0.9516889	103	0.0484911	0.0000000	534	10.8181818
33° 0' 0" 0.00000	431	0.3621500	0.9516273	103	0.0485585	0.0000000	534	10.8181818
34° 0' 0" 0.00000	431	0.3623560	0.9515657	103	0.0486259	0.0000000	534	10.8181818
35° 0' 0" 0.00000	431	0.3625620	0.9515041	103	0.0486933	0.0000000	534	10.8181818
36° 0' 0" 0.00000	431	0.3627680	0.9514425	103	0.0487607	0.0000000	534	10.8181818
37° 0' 0" 0.00000	431	0.3629740	0.9513809	103	0.0488281	0.0000000	534	10.8181818
38° 0' 0" 0.00000	431	0.3631800	0.9513193	103	0.0488955	0.0000000	534	10.8181818
39° 0' 0" 0.00000	431	0.3633860	0.9512577	103	0.0489629	0.0000000	534	10.8181818
40° 0' 0" 0.00000	431	0.3635920	0.9511961	103	0.0490303	0.0000000	534	10.8181818
41° 0' 0" 0.00000	431	0.3637980	0.9511345	103	0.0490977	0.0000000	534	10.8181818
42° 0' 0" 0.00000	431	0.3640040	0.9510729	103	0.0491651	0.0000000	534	10.8181818
43° 0' 0" 0.00000	431	0.3642100	0.9510113	103	0.0492325	0.0000000	534	10.8181818
44° 0' 0" 0.00000	431	0.3644160	0.9509497	103	0.0492999	0.0000000	534	10.8181818
45° 0' 0" 0.00000	431	0.3646220	0.9508881	103	0.0493673	0.0000000	534	10.8181818
46° 0' 0" 0.00000	431	0.3648280	0.9508265	103	0.0494347	0.0000000	534	10.8181818
47° 0' 0" 0.00000	431	0.3650340	0.9507649	103	0.0495021	0.0000000	534	10.8181818
48° 0' 0" 0.00000	431	0.3652400	0.9507033	103	0.0495695	0.0000000	534	10.8181818
49° 0' 0" 0.00000	431	0.3654460	0.9506417	103	0.0496369	0.0000000	534	10.8181818
50° 0' 0" 0.00000	431	0.3656520	0.9505801	103	0.0497043	0.0000000	534	10.8181818
51° 0' 0" 0.00000	431	0.3658580	0.9505185	103	0.0497717	0.0000000	534	10.8181818
52° 0' 0" 0.00000	431	0.3660640	0.9504569	103	0.0498391	0.0000000	534	10.8181818
53° 0' 0" 0.00000	431	0.3662700	0.9503953	103	0.0499065	0.0000000	534	10.8181818
54° 0' 0" 0.00000	431	0.3664760	0.9503337	103	0.0499739	0.0000000	534	10.8181818
55° 0' 0" 0.00000	431	0.3666820	0.9502721	103	0.0500413	0.0000000	534	10.8181818
56° 0' 0" 0.00000	431	0.3668880	0.9502105	103	0.0501087	0.0000000	534	10.8181818
57° 0' 0" 0.00000	431	0.3670940	0.9501489	103	0.0501761	0.0000000	534	10.8181818
58° 0' 0" 0.00000	431	0.3673000	0.9500873	103	0.0502435	0.0000000	534	10.8181818
59° 0' 0" 0.00000	431	0.3675060	0.9500257	103	0.0503109	0.0000000	534	10.8181818
60° 0' 0" 0.00000	431	0.3677120	0.9499641	103	0.0503783	0.0000000	534	10.8181818

Tab. 15.

Deg. 63.

Sine.		Comp. sin.	Cosine	Dio	Comp. cos.	Tangent	Dio	Cotangent
0.6570368	413	0.3429332	0.9498809	107	0.0501191	7071659	520	10.2928571
1.9.6572946	413	0.3427054	0.9498173	107	0.0501835	7074781	520	10.2925219
2.9.6575423	413	0.3424476	0.9497537	107	0.0502479	7077902	520	10.2922098
3.9.6577898	412	0.3421898	0.9496901	108	0.0503123	7081022	520	10.2918978
4.9.6580371	412	0.3419320	0.9496263	108	0.0503767	7084141	519	10.2915859
5.9.6582842	412	0.3416742	0.9495625	108	0.0504411	7087258	519	10.2912742
6.9.6585312	411	0.3414164	0.9494988	108	0.0505055	7090374	519	10.2909626
7.9.6587780	411	0.3411586	0.9494349	108	0.0505699	7093488	519	10.2906512
8.9.6590246	411	0.3409008	0.9493715	108	0.0506343	7096601	519	10.2903399
9.9.6592710	410	0.3406430	0.9493077	108	0.0506987	7099713	518	10.2900287
10.9.6595173	410	0.3403852	0.9492439	108	0.0507631	7102824	518	10.2897176
11.9.6597633	410	0.3401274	0.9491799	108	0.0508275	7105937	518	10.2894067
12.9.6600093	409	0.3398696	0.9491161	108	0.0508919	7109041	518	10.2890959
13.9.6602510	409	0.3396118	0.9490520	108	0.0509563	7112148	518	10.2887852
14.9.6604905	409	0.3393540	0.9489875	108	0.0510207	7115254	518	10.2884746
15.9.6607459	408	0.3390962	0.9489230	108	0.0510851	7118358	517	10.2881642
16.9.6609911	408	0.3388384	0.9488585	108	0.0511495	7121461	517	10.2878539
17.9.6612361	408	0.3385806	0.9487939	109	0.0512139	7124562	517	10.2875438
18.9.6614810	408	0.3383228	0.9487291	109	0.0512783	7127662	517	10.2872338
19.9.6617257	407	0.3380650	0.9486645	109	0.0513427	7130761	516	10.2869239
20.9.6619702	407	0.3378072	0.9486000	109	0.0514071	7133859	516	10.2866141
21.9.6622145	407	0.3375494	0.9485353	109	0.0514715	7136956	516	10.2863044
22.9.6624586	407	0.3372916	0.9484707	109	0.0515359	7140051	516	10.2859949
23.9.6627026	406	0.3370338	0.9484060	109	0.0516003	7143145	515	10.2856855
24.9.6629464	406	0.3367760	0.9483412	109	0.0516647	7146237	515	10.2853763
25.9.6631900	406	0.3365182	0.9482765	109	0.0517291	7149329	515	10.2850671
26.9.6634335	406	0.3362604	0.9482116	109	0.0517935	7152419	515	10.2847581
27.9.6636768	405	0.3360026	0.9481466	109	0.0518579	7155508	515	10.2844492
28.9.6639199	405	0.3357448	0.9480815	109	0.0519223	7158595	515	10.2841405
29.9.6641628	405	0.3354870	0.9480163	110	0.0520005	7161682	514	10.2838318
30.9.6644056	404	0.3352292	0.9479509	110	0.0520711	7164767	514	10.2835233
31.9.6646482	404	0.3349714	0.9478853	110	0.0521369	7167851	514	10.2832149
32.9.6648906	404	0.3347136	0.9478197	110	0.0522027	7170933	514	10.2829067
33.9.6651329	403	0.3344558	0.9477539	110	0.0522686	7174014	514	10.2825986
34.9.6653749	403	0.3341980	0.9476881	110	0.0523344	7177094	513	10.2822906
35.9.6656166	403	0.3339402	0.9476222	110	0.0524003	7180173	513	10.2819827
36.9.6658586	402	0.3336824	0.9475562	110	0.0524661	7183251	513	10.2816749
37.9.6661001	402	0.3334246	0.9474901	110	0.0525320	7186327	513	10.2813673
38.9.6663413	402	0.3331668	0.9474241	110	0.0525978	7189402	512	10.2810598
39.9.6665823	401	0.3329090	0.9473579	110	0.0526637	7192476	512	10.2807524
40.9.6668235	401	0.3326512	0.9472916	110	0.0527295	7195549	512	10.2804451
41.9.6670644	401	0.3323934	0.9472252	110	0.0527953	7198620	512	10.2801380
42.9.6673051	401	0.3321356	0.9471586	111	0.0528611	7201690	511	10.2798310
43.9.6675459	401	0.3318778	0.9470920	111	0.0529269	7204759	511	10.2795241
44.9.6677863	400	0.3316200	0.9470253	111	0.0529927	7207827	511	10.2792173
45.9.6680265	400	0.3313622	0.9469587	111	0.0530585	7210894	511	10.2789107
46.9.6682665	400	0.3311044	0.9468920	111	0.0531243	7213958	511	10.2786042
47.9.6685063	399	0.3308466	0.9468252	111	0.0531901	7217022	510	10.2782978
48.9.6687451	399	0.3305888	0.9467584	111	0.0532559	7220085	510	10.2779915
49.9.6689856	399	0.3303310	0.9466916	111	0.0533217	7223147	510	10.2776853
50.9.6692230	399	0.3300732	0.9466248	111	0.0533875	7226207	510	10.2773793
51.9.6694642	398	0.3298154	0.9465579	111	0.0534533	7229266	509	10.2770734
52.9.6697032	398	0.3295576	0.9464910	111	0.0535191	7232324	509	10.2767676
53.9.6699420	398	0.3293000	0.9464240	111	0.0535849	7235381	509	10.2764619
54.9.6701807	397	0.3290422	0.9463571	111	0.0536507	7238436	509	10.2761564
55.9.6704192	397	0.3287844	0.9462900	112	0.0537165	7241490	509	10.2758510
56.9.6706576	397	0.3285266	0.9462229	112	0.0537823	7244543	509	10.2755457
57.9.6708958	397	0.3282688	0.9461557	112	0.0538481	7247595	509	10.2752405
58.9.6711338	396	0.3280110	0.9460885	112	0.0539139	7250646	508	10.2749354
59.9.6713716	396	0.3277532	0.9460212	112	0.0539797	7253695	508	10.2746305
60.9.6716092	396	0.3274954	0.9459539	112	0.0540455	7256744	508	10.2743256
Sine.	Dio	Comp. cos.	Sine.	Dio	Comp. sin.	Cotang.	Dio	Tangent

28 Deg.

Tab. 18.

Sine	D10'	Comp. sin.	Cosine	D10'	Comp. cos.	Tangent	D10'	Cotangent
0° 6' 16093	596	0.3283507	0.9454349	112	0.0540651	9.7286744	508	10.2743533
1° 6' 18168	395	0.3281589	0.9458677	113	0.0541325	9.7287981	508	10.2740909
2° 6' 20041	395	0.3279158	0.9458005	112	0.0541995	9.7289237	507	10.2737163
3° 6' 21813	395	0.3276787	0.9457332	112	0.0542664	9.7290491	507	10.2733419
4° 6' 23583	395	0.3274417	0.9456659	112	0.0543334	9.7291745	507	10.2729675
5° 6' 25232	394	0.3272046	0.9455985	112	0.0544003	9.7292997	507	10.2725931
6° 6' 26919	394	0.3269681	0.9455310	112	0.0544670	9.7294250	507	10.2722187
7° 6' 28584	394	0.3267316	0.9454636	112	0.0545336	9.7295503	506	10.2718443
8° 6' 30247	394	0.3264958	0.9453960	112	0.0546004	9.7296756	506	10.2714699
9° 6' 31909	393	0.3262591	0.9453285	113	0.0546671	9.7298009	506	10.2710955
10° 6' 33789	393	0.3260224	0.9452609	113	0.0547339	9.7299262	506	10.2707211
11° 6' 35428	393	0.3257857	0.9451933	113	0.0548006	9.7300515	506	10.2703467
12° 6' 37148	393	0.3255490	0.9451257	113	0.0548674	9.7301768	505	10.2700000
13° 6' 38868	392	0.3253123	0.9450581	113	0.0549342	9.7303021	505	10.2696533
14° 6' 40591	392	0.3250756	0.9449905	113	0.0550010	9.7304274	505	10.2693066
15° 6' 42315	392	0.3248389	0.9449229	113	0.0550678	9.7305527	505	10.2689599
16° 6' 44040	391	0.3246022	0.9448553	113	0.0551346	9.7306780	505	10.2686132
17° 6' 45765	391	0.3243655	0.9447877	113	0.0552014	9.7308033	504	10.2682665
18° 6' 47490	391	0.3241288	0.9447201	113	0.0552682	9.7309286	504	10.2679198
19° 6' 49215	391	0.3238921	0.9446525	113	0.0553350	9.7310539	504	10.2675731
20° 6' 50940	390	0.3236554	0.9445849	114	0.0554018	9.7311792	504	10.2672264
21° 6' 52665	390	0.3234187	0.9445173	114	0.0554686	9.7313045	504	10.2668797
22° 6' 54390	390	0.3231820	0.9444497	114	0.0555354	9.7314298	503	10.2665330
23° 6' 56115	390	0.3229453	0.9443821	114	0.0556022	9.7315551	503	10.2661863
24° 6' 57840	389	0.3227086	0.9443145	114	0.0556690	9.7316804	503	10.2658396
25° 6' 59565	389	0.3224719	0.9442469	114	0.0557358	9.7318057	503	10.2654929
26° 6' 61290	389	0.3222352	0.9441793	114	0.0558026	9.7319310	503	10.2651462
27° 6' 63015	388	0.3220000	0.9441117	114	0.0558694	9.7320563	502	10.2647995
28° 6' 64740	388	0.3217633	0.9440441	114	0.0559362	9.7321816	502	10.2644528
29° 6' 66465	388	0.3215266	0.9439765	114	0.0560030	9.7323069	502	10.2641061
30° 6' 68190	388	0.3212899	0.9439089	114	0.0560698	9.7324322	502	10.2637594
31° 6' 69915	387	0.3210532	0.9438413	114	0.0561366	9.7325575	502	10.2634127
32° 6' 71640	387	0.3208165	0.9437737	114	0.0562034	9.7326828	501	10.2630660
33° 6' 73365	387	0.3205798	0.9437061	114	0.0562702	9.7328081	501	10.2627193
34° 6' 75090	387	0.3203431	0.9436385	115	0.0563370	9.7329334	501	10.2623726
35° 6' 76815	386	0.3201064	0.9435709	115	0.0564038	9.7330587	501	10.2620259
36° 6' 78540	386	0.3198697	0.9435033	115	0.0564706	9.7331840	501	10.2616792
37° 6' 80265	386	0.3196330	0.9434357	115	0.0565374	9.7333093	500	10.2613325
38° 6' 81990	385	0.3193963	0.9433681	115	0.0566042	9.7334346	500	10.2609858
39° 6' 83715	385	0.3191596	0.9433005	115	0.0566710	9.7335599	500	10.2606391
40° 6' 85440	385	0.3189229	0.9432329	115	0.0567378	9.7336852	500	10.2602924
41° 6' 87165	385	0.3186862	0.9431653	115	0.0568046	9.7338105	500	10.2600000
42° 6' 88890	384	0.3184495	0.9430977	115	0.0568714	9.7339358	500	10.2596533
43° 6' 90615	384	0.3182128	0.9430301	115	0.0569382	9.7340611	499	10.2593066
44° 6' 92340	384	0.3179761	0.9429625	115	0.0570050	9.7341864	499	10.2589599
45° 6' 94065	384	0.3177394	0.9428949	116	0.0570718	9.7343117	499	10.2586132
46° 6' 95790	383	0.3175027	0.9428273	116	0.0571386	9.7344370	499	10.2582665
47° 6' 97515	383	0.3172660	0.9427597	116	0.0572054	9.7345623	499	10.2579198
48° 6' 99240	383	0.3170293	0.9426921	116	0.0572722	9.7346876	499	10.2575731
49° 6' 10000	383	0.3167926	0.9426245	116	0.0573390	9.7348129	499	10.2572264
50° 6' 10100	382	0.3165559	0.9425569	116	0.0574058	9.7349382	498	10.2568797
51° 6' 10200	382	0.3163192	0.9424893	116	0.0574726	9.7350635	498	10.2565330
52° 6' 10300	382	0.3160825	0.9424217	116	0.0575394	9.7351888	498	10.2561863
53° 6' 10400	382	0.3158458	0.9423541	116	0.0576062	9.7353141	498	10.2558396
54° 6' 10500	381	0.3156091	0.9422865	116	0.0576730	9.7354394	497	10.2554929
55° 6' 10600	381	0.3153724	0.9422189	116	0.0577398	9.7355647	497	10.2551462
56° 6' 10700	381	0.3151357	0.9421513	116	0.0578066	9.7356900	497	10.2547995
57° 6' 10800	380	0.3148990	0.9420837	116	0.0578734	9.7358153	497	10.2544528
58° 6' 10900	380	0.3146623	0.9420161	116	0.0579402	9.7359406	497	10.2541061
59° 6' 11000	380	0.3144256	0.9419485	117	0.0580070	9.7360659	497	10.2537594
60° 6' 11100	380	0.3141889	0.9418809	117	0.0580738	9.7361912	497	10.2534127

Tab. 18.

q

Deg. 61.

29 Deg.

Sine	D10	Comp. sin.	Cosine	D10	Comp. cos.	Tangent	D10	Cotangent
0 9-6853712	380	0-314288	9-941879	117	0-0581807	9-7437520	496	10-2562480
1 9-6853791	379	0-3142809	9-941749	117	0-0582508	9-7440499	496	10-2559501
2 9-6860267	379	0-3132783	9-941679	117	0-0583209	9-7443476	496	10-2556524
3 9-6862542	379	0-3132758	9-941609	117	0-0583910	9-7446453	496	10-2553547
4 9-6864817	379	0-3132684	9-941538	117	0-0584612	9-7449428	496	10-2550572
5 9-6867088	378	0-3132612	9-941468	117	0-0585315	9-7452403	495	10-2547597
6 9-6869359	378	0-3130641	9-941398	117	0-0586018	9-7455376	495	10-2544624
7 9-6871628	378	0-3128372	9-941327	117	0-0586721	9-7458349	495	10-2541651
8 9-6873895	378	0-3126105	9-941257	117	0-0587425	9-7461320	495	10-2538680
9 9-6876161	377	0-3123839	9-941187	117	0-0588129	9-7464290	495	10-2535710
10 9-6878425	377	0-3121575	9-941116	117	0-0588834	9-7467259	495	10-2532741
11 9-6880688	377	0-3119312	9-941046	118	0-0589539	9-7470227	495	10-2529773
12 9-6882949	377	0-3117051	9-940975	118	0-0590245	9-7473194	494	10-2526806
13 9-6885209	376	0-3114791	9-940904	118	0-0590952	9-7476161	494	10-2523840
14 9-6887467	376	0-3112533	9-940834	118	0-0591658	9-7479125	494	10-2520873
15 9-6889725	376	0-3110277	9-940763	118	0-0592366	9-7482089	494	10-2517911
16 9-6891978	376	0-3108027	9-940692	118	0-0593073	9-7485052	494	10-2514948
17 9-6894232	375	0-3105767	9-940621	118	0-0593781	9-7488015	493	10-2511987
18 9-6896484	375	0-3103516	9-940550	118	0-0594489	9-7490978	493	10-2509026
19 9-6898734	375	0-3101266	9-940480	118	0-0595199	9-7493934	493	10-2506066
20 9-6900983	375	0-3099017	9-940409	118	0-0595909	9-7496892	493	10-2503108
21 9-6903231	374	0-3096769	9-940338	118	0-0596619	9-7499850	493	10-2500150
22 9-6905476	374	0-3094524	9-940267	118	0-0597330	9-7502806	493	10-2497194
23 9-6907721	374	0-3092279	9-940195	118	0-0598041	9-7505762	492	10-2494238
24 9-6909964	374	0-3090036	9-940124	119	0-0598752	9-7508716	492	10-2491284
25 9-6912205	374	0-3087792	9-940053	119	0-0599465	9-7511669	492	10-2488331
26 9-6914445	373	0-3085555	9-939982	119	0-0600177	9-7514622	492	10-2485378
27 9-6916683	373	0-3083317	9-939911	119	0-0600889	9-7517573	492	10-2482427
28 9-6918919	373	0-3081081	9-939839	119	0-0601604	9-7520523	491	10-2479477
29 9-6921155	373	0-3078845	9-939768	119	0-0602318	9-7523472	491	10-2476528
30 9-6923388	372	0-3076612	9-939696	119	0-0603032	9-7526420	491	10-2473580
31 9-6925620	372	0-3074380	9-939625	119	0-0603747	9-7529368	491	10-2470632
32 9-6927851	371	0-3072149	9-939553	119	0-0604463	9-7532314	491	10-2467686
33 9-6930080	371	0-3069920	9-939482	119	0-0605179	9-7535259	491	10-2464741
34 9-6932308	371	0-3067692	9-939410	119	0-0605895	9-7538203	490	10-2461797
35 9-6934534	371	0-3065466	9-939338	119	0-0606612	9-7541146	490	10-2458854
36 9-6936758	371	0-3063242	9-939267	120	0-0607329	9-7544088	490	10-2455912
37 9-6938981	370	0-3061019	9-939195	120	0-0608045	9-7547029	490	10-2452971
38 9-6941205	370	0-3058797	9-939123	120	0-0608765	9-7550069	490	10-2450031
39 9-6943428	370	0-3056577	9-939051	120	0-0609485	9-7553008	490	10-2447092
40 9-6945642	369	0-3054358	9-938979	120	0-0610204	9-7555946	489	10-2444154
41 9-6947859	369	0-3052141	9-938907	120	0-0610924	9-7558883	489	10-2441217
42 9-6950074	369	0-3049926	9-938835	120	0-0611644	9-7561818	489	10-2438282
43 9-6952288	369	0-3047712	9-938763	120	0-0612365	9-7564753	489	10-2435347
44 9-6954501	369	0-3045499	9-938691	120	0-0613086	9-7567687	489	10-2432413
45 9-6956712	368	0-3043288	9-938619	120	0-0613808	9-7570620	489	10-2429480
46 9-6958922	368	0-3041078	9-938547	120	0-0614530	9-7573552	488	10-2426546
47 9-6961136	368	0-3038870	9-938474	120	0-0615252	9-7576483	488	10-2423617
48 9-6963346	367	0-3036664	9-938402	121	0-0615976	9-7579413	488	10-2420687
49 9-6965551	367	0-3034459	9-938330	121	0-0616700	9-7582342	488	10-2417758
50 9-6967745	367	0-3032255	9-938257	121	0-0617424	9-7585270	488	10-2414830
51 9-6969947	367	0-3030053	9-938185	121	0-0618149	9-7588200	488	10-2411904
52 9-6972148	366	0-3027852	9-938112	121	0-0618874	9-7591122	487	10-2408978
53 9-6974347	366	0-3025653	9-938040	121	0-0619600	9-7594047	487	10-2406053
54 9-6976543	366	0-3023456	9-937967	121	0-0620326	9-7596971	487	10-2403129
55 9-6978741	366	0-3021259	9-937894	121	0-0621053	9-7599894	487	10-2400206
56 9-6980936	365	0-3019064	9-937820	121	0-0621780	9-7602816	487	10-2397284
57 9-6983129	365	0-3016871	9-937747	121	0-0622508	9-7605737	486	10-2394363
58 9-6985321	365	0-3014679	9-937674	121	0-0623236	9-7608657	486	10-2391443
59 9-6987511	365	0-3012489	9-937602	121	0-0623965	9-7611576	486	10-2388524
60 9-6989700	365	0-3010300	9-937530	121	0-0624694	9-7614494	486	10-2385606
Cotang.	D10	Comp. cos.	Sine	D10	Comp. sin.	Cotang.	D10	Tangent

Tab. 18.

Deg. 60.

30 Deg.

Tab. 18.

Sine	D10'	Comp. Sin.	Cosine.	D10'	Comp. cos.	Tangent	D10'	Cotangent
09° 69 59 700	364	0.3010300	9.9375306	121	0.0624694	9.7614384	486	10.2385606
19° 69 59 1867	364	0.3008113	9.9374577	122	0.0625423	9.7613711	486	10.2385689
29° 69 59 4075	364	0.3005927	9.9373847	122	0.0626153	9.7613027	486	10.2379775
39° 69 59 258	364	0.3003742	9.9373116	122	0.0626884	9.7612342	486	10.2373861
49° 69 59 441	364	0.3001556	9.9372385	122	0.0627615	9.7611656	486	10.2367947
59° 69 59 0622	363	0.2999370	9.9371653	122	0.0628346	9.7610970	485	10.2371033
69° 69 58 2802	363	0.2997185	9.9370921	122	0.0629077	9.7610285	485	10.2365119
79° 69 57 4981	363	0.2995000	9.9370189	122	0.0629808	9.7609600	485	10.2359205
89° 69 57 1158	363	0.2992814	9.9369456	122	0.0630539	9.7608915	485	10.2353291
99° 69 56 334	362	0.2990629	9.9368722	122	0.0631270	9.7608230	485	10.2347377
109° 69 55 5508	362	0.2988443	9.9367988	122	0.0632001	9.7607545	484	10.2341463
119° 69 55 1681	362	0.2986258	9.9367254	122	0.0632732	9.7606860	484	10.2335549
129° 69 54 3852	362	0.2984072	9.9366520	122	0.0633463	9.7606175	484	10.2329635
139° 69 53 6022	361	0.2981887	9.9365786	123	0.0634194	9.7605490	484	10.2323721
149° 69 52 8190	361	0.2979701	9.9365052	123	0.0634925	9.7604805	484	10.2317807
159° 69 52 0357	361	0.2977516	9.9364318	123	0.0635656	9.7604120	484	10.2311893
169° 69 51 2523	361	0.2975330	9.9363584	123	0.0636387	9.7603435	484	10.2305979
179° 69 50 4688	360	0.2973145	9.9362850	123	0.0637118	9.7602750	483	10.2299965
189° 69 50 0849	360	0.2970959	9.9362116	123	0.0637849	9.7602065	483	10.2293951
199° 69 49 3011	360	0.2968774	9.9361382	123	0.0638580	9.7601380	483	10.2287937
209° 69 48 5170	360	0.2966588	9.9360648	123	0.0639311	9.7600695	483	10.2281923
219° 69 48 1329	359	0.2964403	9.9359914	123	0.0640042	9.7600010	483	10.2275909
229° 69 47 3486	359	0.2962217	9.9359180	123	0.0640773	9.7599325	483	10.2269895
239° 69 46 5641	359	0.2960032	9.9358446	123	0.0641504	9.7598640	482	10.2263881
249° 69 46 1795	359	0.2957846	9.9357712	123	0.0642235	9.7597955	482	10.2257867
259° 69 45 3947	359	0.2955661	9.9356978	124	0.0642966	9.7597270	482	10.2251853
269° 69 44 6099	358	0.2953475	9.9356244	124	0.0643697	9.7596585	482	10.2245839
279° 69 43 8248	358	0.2951290	9.9355510	124	0.0644428	9.7595900	482	10.2239825
289° 69 43 0397	358	0.2949104	9.9354776	124	0.0645159	9.7595215	482	10.2233811
299° 69 42 2543	358	0.2946919	9.9354042	124	0.0645890	9.7594530	481	10.2227797
309° 69 41 4689	357	0.2944733	9.9353308	124	0.0646621	9.7593845	481	10.2221783
319° 69 40 6833	357	0.2942548	9.9352574	124	0.0647352	9.7593160	481	10.2215769
329° 69 39 8975	357	0.2940362	9.9351840	124	0.0648083	9.7592475	481	10.2209755
339° 69 39 1116	357	0.2938177	9.9351106	124	0.0648814	9.7591790	481	10.2203741
349° 69 38 3256	356	0.2935991	9.9350372	124	0.0649545	9.7591105	481	10.2197727
359° 69 37 5394	356	0.2933806	9.9349638	124	0.0650276	9.7590420	481	10.2191713
369° 69 37 1531	356	0.2931620	9.9348904	124	0.0651007	9.7589735	480	10.2185699
379° 69 36 3667	356	0.2929435	9.9348170	125	0.0651738	9.7589050	480	10.2179685
389° 69 35 5801	355	0.2927249	9.9347436	125	0.0652469	9.7588365	480	10.2173671
399° 69 35 1933	355	0.2925064	9.9346702	125	0.0653200	9.7587680	480	10.2167657
409° 69 34 4064	355	0.2922878	9.9345968	125	0.0653931	9.7587000	480	10.2161643
419° 69 33 6194	355	0.2920693	9.9345234	125	0.0654662	9.7586315	479	10.2155629
429° 69 32 8322	354	0.2918507	9.9344500	125	0.0655393	9.7585630	479	10.2149615
439° 69 32 0450	354	0.2916322	9.9343766	125	0.0656124	9.7584945	479	10.2143601
449° 69 31 2575	354	0.2914136	9.9343032	125	0.0656855	9.7584260	479	10.2137587
459° 69 30 4699	354	0.2911951	9.9342298	125	0.0657586	9.7583575	479	10.2131573
469° 69 30 0822	353	0.2909765	9.9341564	125	0.0658317	9.7582890	479	10.2125559
479° 69 29 2943	353	0.2907580	9.9340830	125	0.0659048	9.7582205	479	10.2119545
489° 69 28 5068	353	0.2905394	9.9340096	126	0.0659779	9.7581520	478	10.2113531
499° 69 28 1182	353	0.2903209	9.9339362	126	0.0660510	9.7580835	478	10.2107517
509° 69 27 3299	353	0.2901023	9.9338628	126	0.0661241	9.7580150	478	10.2101503
519° 69 26 5413	352	0.2898838	9.9337894	126	0.0661972	9.7579465	478	10.2095489
529° 69 26 1529	352	0.2896652	9.9337160	126	0.0662703	9.7578780	478	10.2089475
539° 69 25 3642	352	0.2894467	9.9336426	126	0.0663434	9.7578095	478	10.2083461
549° 69 24 5753	352	0.2892281	9.9335692	126	0.0664165	9.7577410	478	10.2077447
559° 69 24 1863	352	0.2890096	9.9334958	126	0.0664896	9.7576725	477	10.2071433
569° 69 23 3972	351	0.2887910	9.9334224	126	0.0665627	9.7576040	477	10.2065419
579° 69 22 6080	351	0.2885725	9.9333490	126	0.0666358	9.7575355	477	10.2059405
589° 69 21 8186	351	0.2883539	9.9332756	126	0.0667089	9.7574670	477	10.2053391
599° 69 21 0290	350	0.2881354	9.9332022	126	0.0667820	9.7573985	477	10.2047377
609° 69 20 2393	350	0.2879168	9.9331288	126	0.0668551	9.7573300	477	10.2041363

Tab. 18.

30 Deg.

Sine	D10"	Comp. sine	Cosine	D10"	Comp. cos.	Tangent	D10"	Cotangent
0-9-7118393	350	0-2881607	9-9330558	126	0-0669344	9-7877737	477	10-2212268
1-9-7120493	350	0-2879503	9-9329897	127	0-0670108	9-7780599	477	10-2209401
2-9-7122596	350	0-2877404	9-9329137	127	0-0670863	9-7793439	477	10-2206541
3-9-7124695	350	0-2875305	9-9328376	127	0-0671624	9-7796318	476	10-2203682
4-9-7126792	349	0-2873208	9-9327616	127	0-0672384	9-7799177	476	10-2200823
5-9-7128899	349	0-2871111	9-9326854	127	0-0673146	9-7802034	476	10-2197966
6-9-7130983	349	0-2869017	9-9326092	127	0-0673908	9-7804891	476	10-2195109
7-9-7133077	349	0-2866923	9-9325330	127	0-0674670	9-7807747	476	10-2192253
8-9-7135169	348	0-2864831	9-9324567	127	0-0675433	9-7810602	476	10-2189398
9-9-7137260	348	0-2862740	9-9323804	127	0-0676196	9-7813456	476	10-2186544
10-9-7139349	348	0-2860651	9-9323040	127	0-0676960	9-7816309	475	10-2183691
11-9-7141437	348	0-2858563	9-9322276	127	0-0677724	9-7819162	475	10-2180838
12-9-7143524	347	0-2856476	9-9321511	127	0-0678489	9-7822015	475	10-2177987
13-9-7145609	347	0-2854391	9-9320746	127	0-0679254	9-7824864	475	10-2175136
14-9-7147693	347	0-2852307	9-9319980	128	0-0680020	9-7827713	475	10-2172287
15-9-7149776	347	0-2850224	9-9319213	128	0-0680787	9-7830562	475	10-2169438
16-9-7151827	347	0-2848143	9-9318447	128	0-0681553	9-7833410	475	10-2166590
17-9-7153937	347	0-2846063	9-9317679	128	0-0682321	9-7836258	474	10-2163742
18-9-7156015	346	0-2843983	9-9316911	128	0-0683089	9-7839108	474	10-2160896
19-9-7158092	346	0-2841908	9-9316143	128	0-0683857	9-7841949	474	10-2158051
20-9-7160168	346	0-2839832	9-9315374	128	0-0684626	9-7844794	474	10-2155206
21-9-7162243	345	0-2837757	9-9314605	128	0-0685395	9-7847638	474	10-2152362
22-9-7164316	345	0-2835684	9-9313835	128	0-0686165	9-7850481	474	10-2149519
23-9-7166387	345	0-2833613	9-9313065	128	0-0686935	9-7853323	473	10-2146677
24-9-7168458	345	0-2831542	9-9312294	129	0-0687706	9-7856164	473	10-2143836
25-9-7170526	345	0-2829474	9-9311522	129	0-0688478	9-7859004	473	10-2140996
26-9-7172594	344	0-2827406	9-9310750	129	0-0689250	9-7861844	473	10-2138156
27-9-7174660	344	0-2825340	9-9309978	129	0-0690022	9-7864682	473	10-2135318
28-9-7176725	344	0-2823275	9-9309205	129	0-0690795	9-7867520	473	10-2132480
29-9-7178789	344	0-2821211	9-9308432	129	0-0691566	9-7870357	473	10-2129643
30-9-7180851	343	0-2819149	9-9307658	129	0-0692342	9-7873193	472	10-2126807
31-9-7182912	343	0-2817088	9-9306883	129	0-0693117	9-7876028	472	10-2123972
32-9-7184971	343	0-2815029	9-9306109	129	0-0693891	9-7878863	472	10-2121137
33-9-7187030	343	0-2812970	9-9305333	129	0-0694667	9-7881696	472	10-2118304
34-9-7189086	343	0-2810914	9-9304557	129	0-0695443	9-7884529	472	10-2115471
35-9-7191142	342	0-2808858	9-9303781	129	0-0696219	9-7887361	472	10-2112639
36-9-7193196	342	0-2806804	9-9303004	129	0-0696996	9-7890192	472	10-2109808
37-9-7195249	342	0-2804751	9-9302226	130	0-0697774	9-7893023	472	10-2106977
38-9-7197300	342	0-2802700	9-9301448	130	0-0698552	9-7895852	471	10-2104148
39-9-7199350	341	0-2800650	9-9300670	130	0-0699330	9-7898681	471	10-2101319
40-9-7201399	341	0-2798601	9-9299891	130	0-0700109	9-7901508	471	10-2098492
41-9-7203447	341	0-2796553	9-9299112	130	0-0700888	9-7904335	471	10-2095665
42-9-7205493	341	0-2794507	9-9298332	130	0-0701668	9-7907161	471	10-2092839
43-9-7207538	340	0-2792462	9-9297551	130	0-0702449	9-7909987	471	10-2090013
44-9-7209581	340	0-2790419	9-9296770	130	0-0703230	9-7912811	471	10-2087189
45-9-7211622	340	0-2788377	9-9295989	130	0-0704011	9-7915635	471	10-2084365
46-9-7213664	340	0-2786336	9-9295207	130	0-0704793	9-7918458	470	10-2081542
47-9-7215704	340	0-2784296	9-9294424	130	0-0705576	9-7921280	470	10-2078720
48-9-7217742	339	0-2782258	9-9293641	131	0-0706359	9-7924101	470	10-2075899
49-9-7219779	339	0-2780221	9-9292857	131	0-0707143	9-7926921	470	10-2073079
50-9-7221814	339	0-2778186	9-9292073	131	0-0707927	9-7929741	470	10-2070259
51-9-7223848	339	0-2776152	9-9291289	131	0-0708711	9-7932560	470	10-2067440
52-9-7225881	339	0-2774119	9-9290504	131	0-0709496	9-7935378	469	10-2064622
53-9-7227915	338	0-2772087	9-9289718	131	0-0710282	9-7938195	469	10-2061805
54-9-7229943	338	0-2770057	9-9288932	131	0-0711068	9-7941011	469	10-2058989
55-9-7231972	338	0-2768028	9-9288145	131	0-0711855	9-7943827	469	10-2056173
56-9-7234000	338	0-2766000	9-9287358	131	0-0712642	9-7946641	469	10-2053359
57-9-7236028	337	0-2763974	9-9286571	131	0-0713429	9-7949455	469	10-2050545
58-9-7238051	337	0-2761949	9-9285783	131	0-0714217	9-7952268	469	10-2047732
59-9-7240075	337	0-2759925	9-9284994	131	0-0715006	9-7955081	468	10-2044919
60-9-7242097	337	0-2757903	9-9284205	131	0-0715795	9-7957892	468	10-2042108
Comp. sine	D10"	Comp. cos.	Sine	D10"	Comp. sin.	Cotang.	D10"	Tangent

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Tab. 18.

Sine	D10"	Comp. sin.	Cosine	D10"	Comp. cos.	Tangent	D10"	Cotangent	D10"
0-7242097	337	0-2757903	9-9284205	132	0-0715795	9-7957892	468	10-2042108	60
19-7244118	337	0-2755882	9-9283415	132	0-0716583	9-7960703	468	10-2039297	59
29-7246138	336	0-2753862	9-9282625	132	0-0717375	9-7963513	468	10-2036387	58
39-7248156	336	0-2751844	9-9281834	132	0-0718166	9-7966322	468	10-2033478	57
49-7250174	336	0-2749826	9-9281043	132	0-0718957	9-7969130	468	10-2030570	56
59-7252189	336	0-2747811	9-9280251	132	0-0719749	9-7971938	468	10-2027662	55
69-7254204	335	0-2745796	9-9279459	132	0-0720541	9-7974745	468	10-2024755	54
79-7256217	335	0-2743783	9-9278666	132	0-0721334	9-7977551	468	10-2021849	53
89-7258229	335	0-2741771	9-9277873	132	0-0722127	9-7980356	467	10-2018944	52
99-7260240	335	0-2739760	9-9277079	132	0-0722921	9-7983160	467	10-2016040	51
10-7262249	335	0-2737751	9-9276283	132	0-0723715	9-7985964	467	10-2013136	50
11-7264257	334	0-2735743	9-9275496	132	0-0724510	9-7988767	467	10-2010233	49
12-7266264	334	0-2733736	9-9274693	133	0-0725305	9-7991569	467	10-2007331	48
13-7268269	334	0-2731731	9-9273899	133	0-0726101	9-7994370	467	10-2004430	47
14-7270273	334	0-2729727	9-9273103	133	0-0726897	9-7997170	467	10-2001530	46
15-7272276	334	0-2727724	9-9272306	133	0-0727694	9-7999970	467	10-1998630	45
16-7274278	333	0-2725722	9-9271509	133	0-0728491	9-8002769	466	10-1995731	44
17-7276278	333	0-2723722	9-9270711	133	0-0729289	9-8005567	466	10-1992833	43
18-7278277	333	0-2721723	9-9269913	133	0-0730087	9-8008365	466	10-1989934	42
19-7280275	333	0-2719725	9-9269114	133	0-0730886	9-8011161	466	10-1987036	41
20-7282271	333	0-2717729	9-9268314	133	0-0731686	9-8013957	466	10-1984138	40
21-7284267	332	0-2715733	9-9267514	133	0-0732486	9-8016752	466	10-1981240	39
22-7286260	332	0-2713740	9-9266714	133	0-0733286	9-8019546	466	10-1978343	38
23-7288253	332	0-2711747	9-9265913	133	0-0734087	9-8022340	465	10-1975446	37
24-7290244	332	0-2709756	9-9265112	134	0-0734888	9-8025135	465	10-1972549	36
25-7292234	331	0-2707766	9-9264310	134	0-0735690	9-8027925	465	10-1969652	35
26-7294223	331	0-2705777	9-9263507	134	0-0736493	9-8030716	465	10-1966755	34
27-7296211	331	0-2703789	9-9262704	134	0-0737296	9-8033506	465	10-1963858	33
28-7298197	331	0-2701803	9-9261901	134	0-0738099	9-8036296	465	10-1960961	32
29-7300182	331	0-2699818	9-9261096	134	0-0738904	9-8039083	465	10-1958064	31
30-7302165	330	0-2697835	9-9260292	134	0-0739708	9-8041873	465	10-1955167	30
31-7304148	330	0-2695852	9-9259487	134	0-0740513	9-8044661	464	10-1952270	29
32-7306129	330	0-2693871	9-9258681	134	0-0741319	9-8047447	464	10-1949373	28
33-7308109	330	0-2691891	9-9257875	134	0-0742125	9-8050233	464	10-1946476	27
34-7310087	329	0-2689913	9-9257069	134	0-0742931	9-8053019	464	10-1943579	26
35-7312064	329	0-2687936	9-9256261	134	0-0743739	9-8055803	464	10-1940682	25
36-7314040	329	0-2685960	9-9255454	135	0-0744546	9-8058587	463	10-1937785	24
37-7316015	329	0-2683985	9-9254646	135	0-0745354	9-8061370	464	10-1934888	23
38-7317989	329	0-2682011	9-9253837	135	0-0746163	9-8064152	463	10-1931991	22
39-7319961	328	0-2680039	9-9253028	135	0-0746972	9-8066933	463	10-1929094	21
40-7321932	328	0-2678066	9-9252218	135	0-0747782	9-8069714	463	10-1926197	20
41-7323905	328	0-2676095	9-9251408	135	0-0748592	9-8072494	463	10-1923300	19
42-7325870	328	0-2674130	9-9250597	135	0-0749403	9-8075273	463	10-1920403	18
43-7327837	328	0-2672162	9-9249786	135	0-0750214	9-8078052	463	10-1917506	17
44-7329803	327	0-2670197	9-9248974	135	0-0751026	9-8080839	463	10-1914609	16
45-7331768	327	0-2668232	9-9248161	135	0-0751839	9-8083626	463	10-1911712	15
46-7333731	327	0-2666269	9-9247349	135	0-0752651	9-8086413	462	10-1908815	14
47-7335693	327	0-2664307	9-9246535	136	0-0753463	9-8089193	462	10-1905918	13
48-7337654	327	0-2662346	9-9245721	136	0-0754279	9-8091973	462	10-1903021	12
49-7339614	326	0-2660386	9-9244907	136	0-0755093	9-8094757	462	10-1900124	11
50-7341572	326	0-2658428	9-9244092	136	0-0755908	9-8097540	462	10-1897227	10
51-7343529	326	0-2656471	9-9243277	136	0-0756723	9-8100323	462	10-1894330	9
52-7345485	326	0-2654515	9-9242461	136	0-0757539	9-8103105	462	10-1891433	8
53-7347440	325	0-2652560	9-9241644	136	0-0758356	9-8105887	462	10-1888536	7
54-7349393	325	0-2650607	9-9240827	136	0-0759173	9-8108669	462	10-1885639	6
55-7351345	325	0-2648655	9-9240010	136	0-0759990	9-8111451	461	10-1882742	5
56-7353296	325	0-2646704	9-9239193	136	0-0760809	9-8114233	461	10-1879845	4
57-7355246	325	0-2644754	9-9238373	136	0-0761627	9-8117015	461	10-1876948	3
58-7357195	324	0-2642805	9-9237554	137	0-0762446	9-8119796	461	10-1874051	2
59-7359142	324	0-2640855	9-9236734	137	0-0763266	9-8122578	461	10-1871154	1
60-7361088	324	0-2638912	9-9235914	137	0-0764086	9-8125359	461	10-1868257	0
Cosine	D10"	Comp. cos.	Sine	D10"	Comp. sin.	Cotang.	D10"	Tangent	D10"

Tab. 18.

Deg. 57.

Sine	D10	Compos.	Cosine	D10	Comp. cos.	Tangent	D10	Cotangent
0° 7361088	324	0-2658812	9-9235914	137	0-0764086	9-8125174	461	10-1874825
1° 7363032	324	0-2636968	9-9235093	137	0-0764907	9-8127939	461	10-1872067
2° 7364976	324	0-2635924	9-9234272	137	0-0765728	9-8130704	461	10-1869309
3° 7366918	323	0-2635082	9-9233450	137	0-0766550	9-8133468	461	10-1866552
4° 7368859	323	0-2634141	9-9232628	137	0-0767372	9-8136231	460	10-1863793
5° 7370799	323	0-2633200	9-9231805	137	0-0768193	9-8138993	460	10-1861035
6° 7372737	323	0-2632263	9-9230982	137	0-0769015	9-8141755	460	10-1858275
7° 7374675	323	0-2631325	9-9230158	137	0-0769837	9-8144516	460	10-1855516
8° 7376611	322	0-2630388	9-9229334	137	0-0770658	9-8147277	460	10-1852757
9° 7378546	322	0-2629454	9-9228509	137	0-0771479	9-8150036	460	10-1850000
10° 7380479	322	0-2619581	9-9227684	138	0-0772301	9-8152795	460	10-1847243
11° 7382412	322	0-2617588	9-9226858	138	0-0773142	9-8155554	459	10-1844486
12° 7384345	322	0-2615587	9-9226032	138	0-0773969	9-8158311	459	10-1841729
13° 7386278	321	0-2613737	9-9225205	138	0-0774795	9-8161068	459	10-1838972
14° 7388201	321	0-2611799	9-9224377	138	0-0775623	9-8163824	459	10-1836215
15° 7390129	321	0-2609871	9-9223549	138	0-0776451	9-8166580	459	10-1833458
16° 7392055	321	0-2607943	9-9222721	138	0-0777279	9-8169335	459	10-1830701
17° 7393980	321	0-2606020	9-9221891	138	0-0778109	9-8172089	459	10-1827944
18° 7395904	320	0-2604096	9-9221062	138	0-0778938	9-8174842	459	10-1825187
19° 7397827	320	0-2602173	9-9220232	138	0-0779768	9-8177595	459	10-1822430
20° 7399748	320	0-2600252	9-9219401	138	0-0780599	9-8180347	458	10-1819673
21° 7401668	320	0-2598332	9-9218570	139	0-0781430	9-8183098	458	10-1816916
22° 7403587	320	0-2596413	9-9217738	139	0-0782262	9-8185849	458	10-1814159
23° 7405505	319	0-2594492	9-9216906	139	0-0783094	9-8188599	458	10-1811401
24° 7407421	319	0-2592571	9-9216073	139	0-0783927	9-8191348	458	10-1808643
25° 7409337	319	0-2590653	9-9215240	139	0-0784760	9-8194095	458	10-1805886
26° 7411251	319	0-2588734	9-9214406	139	0-0785594	9-8196844	458	10-1803129
27° 7413164	319	0-2586816	9-9213572	139	0-0786428	9-8199592	458	10-1800372
28° 7415075	318	0-2584895	9-9212737	139	0-0787262	9-8202338	458	10-1797615
29° 7416986	318	0-2583014	9-9211902	139	0-0788098	9-8205084	457	10-1794858
30° 7418895	318	0-2581105	9-9211066	139	0-0788934	9-8207829	457	10-1792101
31° 7420803	318	0-2579197	9-9210229	139	0-0789771	9-8210574	457	10-1789344
32° 7422710	318	0-2577289	9-9209393	140	0-0790607	9-8213317	457	10-1786587
33° 7424616	317	0-2575384	9-9208553	140	0-0791445	9-8216060	457	10-1783830
34° 7426520	317	0-2573480	9-9207717	140	0-0792283	9-8218803	457	10-1781073
35° 7428422	317	0-2571577	9-9206878	140	0-0793122	9-8221545	457	10-1778316
36° 7430323	317	0-2569675	9-9206039	140	0-0793961	9-8224286	457	10-1775559
37° 7432223	317	0-2567774	9-9205200	140	0-0794800	9-8227026	457	10-1772802
38° 7434123	316	0-2565874	9-9204360	140	0-0795640	9-8229766	456	10-1770045
39° 7436023	316	0-2563976	9-9203519	140	0-0796481	9-8232505	456	10-1767288
40° 7437921	316	0-2562079	9-9202678	140	0-0797322	9-8235244	456	10-1764531
41° 7439818	316	0-2560183	9-9201836	140	0-0798164	9-8237981	456	10-1761774
42° 7441712	316	0-2558288	9-9200994	140	0-0799006	9-8240719	456	10-1759017
43° 7443606	315	0-2556393	9-9200151	140	0-0799849	9-8243455	456	10-1756260
44° 7445498	315	0-2554498	9-9199308	141	0-0800692	9-8246191	456	10-1753503
45° 7447390	315	0-2552610	9-9198464	141	0-0801536	9-8248926	456	10-1750746
46° 7449280	315	0-2550720	9-9197619	141	0-0802381	9-8251660	456	10-1747989
47° 7451169	315	0-2548831	9-9196775	141	0-0803225	9-8254394	455	10-1745232
48° 7453055	314	0-2546944	9-9195929	141	0-0804071	9-8257127	455	10-1742475
49° 7454943	314	0-2545057	9-9195083	141	0-0804917	9-8259860	455	10-1739718
50° 7456828	314	0-2543172	9-9194237	141	0-0805762	9-8262592	455	10-1736961
51° 7458712	314	0-2541288	9-9193390	141	0-0806610	9-8265323	455	10-1734204
52° 7460595	313	0-2539403	9-9192542	141	0-0807458	9-8268053	455	10-1731447
53° 7462477	313	0-2537523	9-9191694	141	0-0808306	9-8270783	455	10-1728690
54° 7464358	313	0-2535642	9-9190846	141	0-0809155	9-8273513	455	10-1725933
55° 7466237	313	0-2533763	9-9189996	142	0-0810004	9-8276241	455	10-1723176
56° 7468115	313	0-2531885	9-9189146	142	0-0810854	9-8278969	455	10-1720419
57° 7469992	313	0-2530008	9-9188296	142	0-0811704	9-8281696	454	10-1717662
58° 7471868	312	0-2528132	9-9187445	142	0-0812555	9-8284423	454	10-1714905
59° 7473743	312	0-2526257	9-9186594	142	0-0813406	9-8287149	454	10-1712148
60° 7475617	312	0-2524383	9-9185742	142	0-0814258	9-8289874	454	10-1709391
Cosine	D10	Comp. cos.	Sine	D10	Comp. sin.	Cotang.	D10	Tangent

34 Deg.

Tab. 18.

Sine	D10	Comp. sin	Cosine	D10	Comp. cos.	Tangent	D10	Cotangent
1 74715617	312	0.2524353	9.9185742	144	0.0814356	9.8229877	434	10.1710126
2 7477480	312	0.2525211	9.9184890	142	0.0815110	9.8229529	434	10.1707401
3 7479360	312	0.2526064	9.9184037	142	0.0815863	9.8229182	434	10.1704677
4 7481239	311	0.2518750	9.9183783	142	0.0816617	9.8228834	434	10.1701953
5 7483099	311	0.2516904	9.9182829	142	0.0817671	9.8228487	434	10.1699229
6 7484967	311	0.2515033	9.9181475	142	0.0818525	9.8228140	434	10.1696505
7 7486833	311	0.2513167	9.9180630	142	0.0819380	9.8227793	434	10.1693781
8 7488698	311	0.2511309	9.9179764	142	0.0820235	9.8227446	434	10.1691057
9 7490562	311	0.2509438	9.9178908	143	0.0821090	9.8227100	433	10.1688333
10 7492425	310	0.2507575	9.9178051	143	0.0821944	9.8226753	433	10.1685609
11 7494287	310	0.2505713	9.9177194	143	0.0822806	9.8226407	433	10.1682885
12 7496148	310	0.2503852	9.9176336	143	0.0823664	9.8226060	433	10.1680161
13 7498007	310	0.2501989	9.9175478	143	0.0824522	9.8225714	433	10.1677437
14 7499866	309	0.2500124	9.9174621	143	0.0825381	9.8225367	433	10.1674713
15 7501723	309	0.2498277	9.9173760	143	0.0826240	9.8225020	433	10.1671989
16 7503579	309	0.2496421	9.9172900	143	0.0827100	9.8224673	433	10.1669265
17 7505434	309	0.2494566	9.9172040	143	0.0827960	9.8224326	433	10.1666541
18 7507287	309	0.2492713	9.9171179	143	0.0828821	9.8223979	433	10.1663817
19 7509140	308	0.2490860	9.9170317	144	0.0829683	9.8223632	433	10.1661093
20 7510991	308	0.2489009	9.9169455	144	0.0830545	9.8223285	433	10.1658369
21 7512842	308	0.2487158	9.9168593	144	0.0831407	9.8222938	433	10.1655645
22 7514691	308	0.2485309	9.9167730	144	0.0832270	9.8222591	433	10.1652921
23 7516538	308	0.2483462	9.9166866	144	0.0833134	9.8222244	433	10.1650197
24 7518385	308	0.2481615	9.9166002	144	0.0833998	9.8221897	433	10.1647473
25 7520231	307	0.2479769	9.9165137	144	0.0834863	9.8221550	433	10.1644749
26 7522077	307	0.2477925	9.9164272	144	0.0835728	9.8221203	433	10.1642025
27 7523919	307	0.2476081	9.9163406	144	0.0836594	9.8220856	433	10.1639301
28 7525761	307	0.2474239	9.9162539	144	0.0837461	9.8220509	433	10.1636577
29 7527602	307	0.2472398	9.9161673	145	0.0838327	9.8220162	433	10.1633853
30 7529442	306	0.2470558	9.9160805	145	0.0839195	9.8219815	433	10.1631129
31 7531280	306	0.2468720	9.9159937	145	0.0840063	9.8219468	433	10.1628405
32 7533118	306	0.2466882	9.9159069	145	0.0840931	9.8219121	433	10.1625681
33 7534954	306	0.2465046	9.9158200	145	0.0841800	9.8218774	433	10.1622957
34 7536790	306	0.2463210	9.9157330	145	0.0842670	9.8218427	433	10.1620233
35 7538624	305	0.2461376	9.9156460	145	0.0843540	9.8218080	433	10.1617509
36 7540467	305	0.2459543	9.9155589	145	0.0844411	9.8217733	433	10.1614785
37 7542308	305	0.2457712	9.9154718	145	0.0845282	9.8217386	433	10.1612061
38 7544149	305	0.2455881	9.9153846	145	0.0846154	9.8217039	433	10.1609337
39 7545989	305	0.2454051	9.9152974	145	0.0847026	9.8216692	433	10.1606613
40 7547827	304	0.2452223	9.9152101	145	0.0847899	9.8216345	433	10.1603889
41 7549664	304	0.2450396	9.9151228	146	0.0848772	9.8215998	433	10.1601165
42 7551502	304	0.2448569	9.9150354	146	0.0849646	9.8215651	433	10.1598441
43 7553336	304	0.2446744	9.9149479	146	0.0850521	9.8215304	433	10.1595717
44 7555170	304	0.2444920	9.9148604	146	0.0851396	9.8214957	433	10.1592993
45 7556999	304	0.2443098	9.9147729	146	0.0852271	9.8214610	433	10.1590269
46 7558824	303	0.2441276	9.9146852	146	0.0853148	9.8214263	433	10.1587545
47 7560644	303	0.2439456	9.9145976	146	0.0854024	9.8213916	433	10.1584821
48 7562462	303	0.2437636	9.9145100	146	0.0854901	9.8213569	433	10.1582097
49 7564284	303	0.2435818	9.9144221	146	0.0855779	9.8213222	433	10.1579373
50 7566099	303	0.2434001	9.9143342	146	0.0856658	9.8212875	433	10.1576649
51 7567915	302	0.2432185	9.9142464	147	0.0857536	9.8212528	433	10.1573925
52 7569730	302	0.2430370	9.9141584	147	0.0858416	9.8212181	433	10.1571201
53 7571545	302	0.2428556	9.9140704	147	0.0859296	9.8211834	433	10.1568477
54 7573360	302	0.2426744	9.9139823	147	0.0860176	9.8211487	433	10.1565753
55 7575176	302	0.2424932	9.9138943	147	0.0861057	9.8211140	433	10.1563029
56 7576991	301	0.2423122	9.9138061	147	0.0861939	9.8210793	433	10.1560305
57 7578805	301	0.2421313	9.9137179	147	0.0862824	9.8210446	433	10.1557581
58 7580619	301	0.2419505	9.9136296	147	0.0863704	9.8210100	433	10.1554857
59 7582432	301	0.2417698	9.9135413	147	0.0864587	9.8209753	433	10.1552133
60 7584246	301	0.2415892	9.9134530	147	0.0865470	9.8209406	433	10.1549409
61 7586059	301	0.2414087	9.9133645	147	0.0866355	9.8209059	433	10.1546685
Cosine	D10	Comp. sin	Sine	D10	Comp. sin	Cotang.	D10	Tangent

Tab. 18.

Deg. 34.

35 Deg.

Sine	D10"	Comp. sin.	Cosine	D10"	Comp. cos.	Tangent	D10"	Cotangent
0° 7585913	301	0-2414087	9-9133645	147	0-0866355	9-8452268	448	10-1547732
1° 7587717	300	0-2412283	9-9132760	147	0-0867240	9-8454956	448	10-1545044
2° 7589519	300	0-2410481	9-9131875	148	0-0868125	9-8457644	448	10-1542356
3° 7591321	300	0-2408679	9-9130989	148	0-0869011	9-8460332	448	10-1539668
4° 7593121	300	0-2406879	9-9130102	148	0-0869898	9-8463018	448	10-1536982
5° 7594920	300	0-2405080	9-9129215	148	0-0870785	9-8465705	447	10-1534295
6° 7596718	299	0-2403282	9-9128328	148	0-0871672	9-8468390	447	10-1531618
7° 7598515	299	0-2401485	9-9127440	148	0-0872560	9-8471075	447	10-1528925
8° 7600311	299	0-2399689	9-9126551	148	0-0873448	9-8473760	447	10-1526210
9° 7602109	299	0-2397894	9-9125662	148	0-0874338	9-8476444	447	10-1523556
10° 7603899	299	0-2396101	9-9124772	148	0-0875228	9-8479127	447	10-1520873
11° 7605692	298	0-2394306	9-9123882	148	0-0876118	9-8481810	447	10-1518190
12° 7607483	298	0-2392511	9-9122991	149	0-0877009	9-8484492	447	10-1515508
13° 7609274	298	0-2390716	9-9122099	149	0-0877901	9-8487174	447	10-1512826
14° 7611063	298	0-2388923	9-9121207	149	0-0878793	9-8489855	447	10-1510145
15° 7612851	298	0-2387129	9-9120315	149	0-0879685	9-8492536	447	10-1507464
16° 7614638	298	0-2385336	9-9119422	149	0-0880578	9-8495216	447	10-1504784
17° 7616424	297	0-2383542	9-9118528	149	0-0881472	9-8497896	446	10-1502104
18° 7618208	297	0-2381749	9-9117634	149	0-0882366	9-8500575	446	10-1499425
19° 7619992	297	0-2380008	9-9116739	149	0-0883261	9-8503253	446	10-1496747
20° 7621775	297	0-2378225	9-9115844	149	0-0884156	9-8505931	446	10-1494069
21° 7623556	297	0-2376444	9-9114948	149	0-0885052	9-8508608	446	10-1491392
22° 7625337	296	0-2374663	9-9114051	149	0-0885949	9-8511285	446	10-1488715
23° 7627116	296	0-2372882	9-9113155	150	0-0886845	9-8513961	446	10-1486039
24° 7628894	296	0-2371106	9-9112257	150	0-0887743	9-8516637	446	10-1483363
25° 7630671	296	0-2369329	9-9111359	150	0-0888641	9-8519312	446	10-1480688
26° 7632447	296	0-2367553	9-9110460	150	0-0889540	9-8521987	446	10-1478013
27° 7634222	296	0-2365778	9-9109561	150	0-0890439	9-8524661	446	10-1475339
28° 7635996	295	0-2364004	9-9108661	150	0-0891339	9-8527335	445	10-1472665
29° 7637769	295	0-2362231	9-9107761	150	0-0892239	9-8530008	445	10-1469992
30° 7639540	295	0-2360460	9-9106860	150	0-0893140	9-8532680	445	10-1467320
31° 7641311	295	0-2358689	9-9105959	150	0-0894041	9-8535352	445	10-1464648
32° 7643080	295	0-2356920	9-9105057	150	0-0894943	9-8538023	445	10-1461977
33° 7644849	294	0-2355151	9-9104155	150	0-0895845	9-8540694	445	10-1459306
34° 7646616	294	0-2353384	9-9103251	151	0-0896749	9-8543365	445	10-1456635
35° 7648382	294	0-2351618	9-9102348	151	0-0897652	9-8546034	445	10-1453966
36° 7650147	294	0-2349853	9-9101444	151	0-0898556	9-8548704	445	10-1451296
37° 7651911	294	0-2348089	9-9100539	151	0-0899461	9-8551372	445	10-1448628
38° 7653674	294	0-2346326	9-9099634	151	0-0900366	9-8554041	445	10-1445959
39° 7655436	293	0-2344564	9-9098728	151	0-0901272	9-8556708	444	10-1443292
40° 7657197	293	0-2342803	9-9097821	151	0-0902179	9-8559376	444	10-1440624
41° 7658957	293	0-2341043	9-9096915	151	0-0903085	9-8562042	444	10-1437958
42° 7660715	293	0-2339285	9-9096007	151	0-0903995	9-8564708	444	10-1435292
43° 7662473	293	0-2337527	9-9095099	151	0-0904901	9-8567374	444	10-1432626
44° 7664229	293	0-2335771	9-9094190	151	0-0905810	9-8570039	444	10-1429961
45° 7665985	292	0-2334015	9-9093281	152	0-0906719	9-8572704	444	10-1427296
46° 7667739	292	0-2332261	9-9092371	152	0-0907625	9-8575368	444	10-1424632
47° 7669492	292	0-2330508	9-9091461	152	0-0908539	9-8578031	444	10-1421968
48° 7671244	292	0-2328756	9-9090550	152	0-0909450	9-8580694	444	10-1419306
49° 7672996	292	0-2327004	9-9089639	152	0-0910361	9-8583357	444	10-1416643
50° 7674746	291	0-2325254	9-9088727	152	0-0911273	9-8586019	443	10-1413981
51° 7676494	291	0-2323506	9-9087814	152	0-0912186	9-8588680	443	10-1411320
52° 7678242	291	0-2321758	9-9086901	152	0-0913099	9-8591341	443	10-1408659
53° 7679988	291	0-2320011	9-9085988	152	0-0914012	9-8594002	443	10-1405998
54° 7681735	291	0-2318263	9-9085075	152	0-0914927	9-8596661	443	10-1403339
55° 7683480	290	0-2316516	9-9084159	153	0-0915841	9-8599321	443	10-1400679
56° 7685222	290	0-2314777	9-9083243	153	0-0916757	9-8601980	443	10-1398020
57° 7686966	290	0-2313034	9-9082327	153	0-0917673	9-8604638	443	10-1395362
58° 7688707	290	0-2311293	9-9081411	153	0-0918589	9-8607296	443	10-1392704
59° 7690448	290	0-2309552	9-9080494	153	0-0919506	9-8609954	443	10-1390046
60° 7692187	290	0-2307813	9-9079576	153	0-0920424	9-8612610	443	10-1387390
Cosine	D10"	Comp. cos.	Sine	D10"	Comp. sin.	Cotang.	D10"	Tangent

Tab.

Deg. 54.

86 Deg.

Tab. 18.

Sine	D10'	Comp. sin.	Cosine	D10'	Comp. cos.	Tangent	D10'	Cotangent
0.7692187	280	0.2307813	0.9079376	153	0.0920454	0.8612610	443	10.1367390
1.7693925	289	0.2306075	0.9078658	153	0.0921342	0.8613267	443	10.1367339
2.7695663	289	0.2304338	0.9077740	153	0.0922260	0.8613923	443	10.1367277
3.7697398	289	0.2302602	0.9076820	153	0.0923180	0.8614579	443	10.1367215
4.7699134	289	0.2300866	0.9075901	153	0.0924099	0.8615233	443	10.1367153
5.7700868	289	0.2299132	0.9074980	153	0.0925020	0.8615887	443	10.1367091
6.7702601	288	0.2297399	0.9074059	153	0.0925941	0.8616541	443	10.1367029
7.7704332	288	0.2295668	0.9073138	154	0.0926862	0.8617195	443	10.1366967
8.7706063	288	0.2293937	0.9072216	154	0.0927784	0.8617848	443	10.1366905
9.7707793	288	0.2292207	0.9071293	154	0.0928707	0.8618500	443	10.1366843
10.7709522	288	0.2290478	0.9070370	154	0.0929630	0.8619152	442	10.1366781
11.7711249	288	0.2288751	0.9069446	154	0.0930554	0.8619803	442	10.1366719
12.7712976	288	0.2287024	0.9068522	154	0.0931478	0.8620454	442	10.1366657
13.7714702	287	0.2285298	0.9067597	154	0.0932403	0.8621105	442	10.1366595
14.7716426	287	0.2283574	0.9066671	154	0.0933329	0.8621755	441	10.1366533
15.7718150	287	0.2281850	0.9065743	154	0.0934255	0.8622404	441	10.1366471
16.7719874	287	0.2280128	0.9064819	154	0.0935181	0.8623053	441	10.1366409
17.7721593	287	0.2278407	0.9063892	155	0.0936108	0.8623702	441	10.1366347
18.7723314	286	0.2276686	0.9062964	155	0.0937036	0.8624350	441	10.1366285
19.7725033	286	0.2274967	0.9062036	155	0.0937964	0.8624997	441	10.1366223
20.7726751	286	0.2273249	0.9061107	155	0.0938893	0.8625644	441	10.1366161
21.7728468	286	0.2271532	0.9060177	155	0.0939822	0.8626291	441	10.1366099
22.7730185	286	0.2269815	0.9059247	155	0.0940753	0.8626937	441	10.1366037
23.7731900	286	0.2268100	0.9058317	155	0.0941683	0.8627583	441	10.1365975
24.7733614	285	0.2266386	0.9057386	155	0.0942614	0.8628229	441	10.1365913
25.7735327	285	0.2264673	0.9056454	155	0.0943546	0.8628875	441	10.1365851
26.7737039	285	0.2262961	0.9055522	155	0.0944478	0.8629521	441	10.1365789
27.7738749	285	0.2261251	0.9054589	155	0.0945411	0.8630166	440	10.1365727
28.7740459	285	0.2259541	0.9053656	156	0.0946344	0.8630812	440	10.1365665
29.7742168	285	0.2257832	0.9052722	156	0.0947278	0.8631456	440	10.1365603
30.7743876	284	0.2256124	0.9051787	156	0.0948213	0.8632100	440	10.1365541
31.7745583	284	0.2254417	0.9050852	156	0.0949148	0.8632743	440	10.1365479
32.7747288	284	0.2252712	0.9049916	156	0.0950084	0.8633387	440	10.1365417
33.7748994	284	0.2251007	0.9048980	156	0.0951020	0.8634030	440	10.1365355
34.7750697	284	0.2249303	0.9048043	156	0.0951957	0.8634673	440	10.1365293
35.7752399	284	0.2247600	0.9047106	156	0.0952894	0.8635316	440	10.1365231
36.7754101	283	0.2245899	0.9046168	156	0.0953832	0.8635959	440	10.1365169
37.7755801	283	0.2244199	0.9045230	156	0.0954770	0.8636602	440	10.1365107
38.7757501	283	0.2242498	0.9044291	157	0.0955709	0.8637245	440	10.1365045
39.7759199	283	0.2240798	0.9043351	157	0.0956649	0.8637888	440	10.1364983
40.7760897	283	0.2239098	0.9042411	157	0.0957589	0.8638531	439	10.1364921
41.7762593	283	0.2237397	0.9041470	157	0.0958530	0.8639174	439	10.1364859
42.7764289	282	0.2235697	0.9040529	157	0.0959471	0.8639817	439	10.1364797
43.7765983	282	0.2233997	0.9039587	157	0.0960413	0.8640460	439	10.1364735
44.7767676	282	0.2232297	0.9038644	157	0.0961356	0.8641103	439	10.1364673
45.7769369	282	0.2230597	0.9037701	157	0.0962299	0.8641746	439	10.1364611
46.7771060	282	0.2228897	0.9036758	157	0.0963243	0.8642389	439	10.1364549
47.7772750	282	0.2227197	0.9035813	157	0.0964187	0.8643032	439	10.1364487
48.7774439	281	0.2225497	0.9034868	157	0.0965132	0.8643675	439	10.1364425
49.7776128	281	0.2223797	0.9033923	158	0.0966077	0.8644318	439	10.1364363
50.7777815	281	0.2222097	0.9032977	158	0.0967023	0.8644961	439	10.1364301
51.7779501	281	0.2220397	0.9032031	158	0.0967969	0.8645604	439	10.1364239
52.7781186	281	0.2218697	0.9031084	158	0.0968916	0.8646247	439	10.1364177
53.7782870	280	0.2216997	0.9030136	158	0.0969864	0.8646890	438	10.1364115
54.7784553	280	0.2215297	0.9029188	158	0.0970812	0.8647533	438	10.1364053
55.7786235	280	0.2213597	0.9028239	158	0.0971761	0.8648176	438	10.1363991
56.7787916	280	0.2211897	0.9027289	158	0.0972711	0.8648819	438	10.1363929
57.7789596	280	0.2210197	0.9026339	158	0.0973661	0.8649462	438	10.1363867
58.7791275	280	0.2208497	0.9025389	158	0.0974611	0.8650105	438	10.1363805
59.7792953	280	0.2206797	0.9024438	159	0.0975561	0.8650748	438	10.1363743
60.7794630	279	0.2205097	0.9023486		0.0976514	0.8651391	438	10.1363681
Cosine	D10'	Comp. cos.	Sine	D10'	Comp. sin.	Cotang.	D10'	Tangent

Tab. 18.

r

Deg. 83.

37 Dec.

Tab. 16.

Sine	D10'	Comp. sin.	Cosine	D10'	Comp. cos.	Tangent	D10'	Cotangent
9 779440	279	0-2205379	9-9075435	159	0-0877114	9-8771134	438	10-1225536
10 7795308	279	0-2205569	9-9075534	159	0-0877466	9-8773772	438	10-1225628
11 7796176	279	0-2205901	9-9075815	159	0-0877841	9-8776400	438	10-1225690
12 7796955	279	0-2206315	9-9076228	159	0-0878372	9-8779027	438	10-1225973
13 7797632	279	0-2196677	9-9076677	159	0-0879038	9-8781654	438	10-1215358
14 7798328	279	0-2197000	9-9018719	159	0-0881283	9-8784281	438	10-1215719
15 7799000	278	0-2188329	9-9017764	159	0-0882236	9-8786907	438	10-1213099
16 7799671	278	0-2188661	9-9016808	159	0-0883192	9-8789533	438	10-1210467
17 7800341	278	0-2189000	9-9015852	159	0-0884148	9-8792158	437	10-1207842
18 7801010	278	0-2189323	9-9014895	159	0-0885103	9-8794782	437	10-1205218
19 7801677	278	0-2188656	9-9013938	160	0-0886062	9-8797407	437	10-1202593
20 7802341	277	0-2188990	9-9012980	160	0-0887030	9-8800031	437	10-1199969
21 7803008	277	0-2189323	9-9012024	160	0-0887997	9-8802654	437	10-1197346
22 7803673	277	0-2189656	9-9011067	160	0-0888958	9-8805277	437	10-1194723
23 7804340	277	0-2189989	9-9010110	160	0-0889920	9-8807900	437	10-1192100
24 7805005	277	0-2188329	9-9009152	160	0-0890885	9-8810522	437	10-1189478
25 7805673	277	0-2188661	9-9008195	160	0-0891849	9-8813144	437	10-1186854
26 7806341	276	0-2177016	9-9007239	160	0-0892811	9-8815765	437	10-1184233
27 7807008	276	0-2175357	9-9006282	160	0-0893773	9-8818386	437	10-1181614
28 7807677	276	0-2173699	9-9005324	160	0-0894736	9-8821007	437	10-1178992
29 7808344	276	0-2172042	9-9004367	161	0-0895699	9-8823627	437	10-1176374
30 7809011	276	0-2170386	9-9003410	161	0-0896663	9-8826246	436	10-1173750
31 7809678	275	0-2168732	9-9002452	161	0-0897627	9-8828866	436	10-1171124
32 7810345	275	0-2167079	9-9001495	161	0-0898592	9-8831484	436	10-1168516
33 7811012	275	0-2165425	9-9000537	161	0-0899556	9-8834107	436	10-1165897
34 7811679	275	0-2163772	9-8999580	161	0-0900521	9-8836721	436	10-1163279
35 7812346	275	0-2162119	9-8998623	161	0-0901486	9-8839338	436	10-1160662
36 7813013	275	0-2160467	9-8997666	161	0-0902450	9-8841956	436	10-1158044
37 7813680	275	0-2158813	9-8996709	161	0-0903415	9-8844572	436	10-1155428
38 7814347	274	0-2157160	9-8995752	161	0-0904380	9-8847189	436	10-1152811
39 7815014	274	0-2155507	9-8994795	162	0-0905345	9-8849805	436	10-1150195
40 7815681	274	0-2153853	9-8993838	162	0-0906310	9-8852420	436	10-1147580
41 7816348	274	0-2152200	9-8992881	162	0-0907275	9-8855035	436	10-1144963
42 7817015	274	0-2150546	9-8991924	162	0-0908240	9-8857650	436	10-1142350
43 7817682	274	0-2148893	9-8990967	162	0-0909205	9-8860264	436	10-1139736
44 7818349	273	0-2147239	9-8989010	162	0-0910170	9-8862878	436	10-1137122
45 7819016	273	0-2145586	9-8988053	162	0-0911135	9-8865492	435	10-1134508
46 7819683	273	0-2143933	9-8987096	162	0-0912100	9-8868105	435	10-1131895
47 7820350	273	0-2142280	9-8986139	162	0-0913065	9-8870718	435	10-1129282
48 7821017	273	0-2140627	9-8985182	162	0-0914030	9-8873330	435	10-1126670
49 7821684	273	0-2138974	9-8984225	163	0-0915000	9-8875942	435	10-1124058
50 7822351	272	0-2137321	9-8983268	163	0-0916000	9-8878554	435	10-1121446
51 7823018	272	0-2135668	9-8982311	163	0-0917000	9-8881166	435	10-1118833
52 7823685	272	0-2134015	9-8981354	163	0-0918000	9-8883778	435	10-1116221
53 7824352	272	0-2132362	9-8980397	163	0-0919000	9-8886386	435	10-1113614
54 7825019	272	0-2130709	9-8979440	163	0-0920000	9-8888996	435	10-1111004
55 7825686	272	0-2129056	9-8978483	163	0-0921000	9-8891605	435	10-1108393
56 7826353	271	0-2127403	9-8977526	163	0-0922000	9-8894214	435	10-1105786
57 7827020	271	0-2125750	9-8976569	163	0-0923000	9-8896823	435	10-1103179
58 7827687	271	0-2124097	9-8975612	163	0-0924000	9-8899432	435	10-1100568
59 7828354	271	0-2122444	9-8974655	163	0-0925000	9-8902040	434	10-1097960
60 7829021	271	0-2120791	9-8973698	164	0-0926000	9-8904647	434	10-1095353
61 7829688	271	0-2119138	9-8972741	164	0-0927000	9-8907254	434	10-1092746
62 7830355	271	0-2117485	9-8971784	164	0-0928000	9-8909861	434	10-1090139
63 7831022	271	0-2115832	9-8970827	164	0-0929000	9-8912468	434	10-1087532
64 7831689	270	0-2114179	9-8969870	164	0-0930000	9-8915074	434	10-1084926
65 7832356	270	0-2112526	9-8968913	164	0-0931000	9-8917679	434	10-1082321
66 7833023	270	0-2110873	9-8967956	164	0-0932000	9-8920285	434	10-1079715
67 7833690	270	0-2109220	9-8967000	164	0-0933000	9-8922890	434	10-1077110
68 7834357	270	0-2107567	9-8966043	164	0-0934000	9-8925494	434	10-1074506
69 7835024	270	0-2105914	9-8965086	164	0-0935000	9-8928098	434	10-1071902

Tan.

Deg. 52.

38 Deg.

Sine	D10	Comp. sin.	Cosine	D10	Comp. cos.	Cotangent	D10	Tangent
0.9783440	269	0.2106550	0.8193450	164	0.1834550	0.5438390	434	10.7015070
0.9783503	269	0.2106487	0.8193513	164	0.1834613	0.5438327	434	10.7015133
0.9783566	269	0.2106424	0.8193576	164	0.1834676	0.5438264	434	10.7015196
0.9783629	269	0.2106361	0.8193639	164	0.1834739	0.5438201	434	10.7015259
0.9783692	269	0.2106298	0.8193702	164	0.1834802	0.5438138	434	10.7015322
0.9783755	269	0.2106235	0.8193765	164	0.1834865	0.5438075	434	10.7015385
0.9783818	269	0.2106172	0.8193828	164	0.1834928	0.5438012	434	10.7015448
0.9783881	269	0.2106109	0.8193891	164	0.1834991	0.5437949	434	10.7015511
0.9783944	269	0.2106046	0.8193954	164	0.1835054	0.5437886	434	10.7015574
0.9784007	269	0.2105983	0.8194017	164	0.1835117	0.5437823	434	10.7015637
0.9784070	269	0.2105920	0.8194080	164	0.1835180	0.5437760	434	10.7015700
0.9784133	269	0.2105857	0.8194143	164	0.1835243	0.5437697	434	10.7015763
0.9784196	269	0.2105794	0.8194206	164	0.1835306	0.5437634	434	10.7015826
0.9784259	269	0.2105731	0.8194269	164	0.1835369	0.5437571	434	10.7015889
0.9784322	269	0.2105668	0.8194332	164	0.1835432	0.5437508	434	10.7015952
0.9784385	269	0.2105605	0.8194395	164	0.1835495	0.5437445	434	10.7016015
0.9784448	269	0.2105542	0.8194458	164	0.1835558	0.5437382	434	10.7016078
0.9784511	269	0.2105479	0.8194521	164	0.1835621	0.5437319	434	10.7016141
0.9784574	269	0.2105416	0.8194584	164	0.1835684	0.5437256	434	10.7016204
0.9784637	269	0.2105353	0.8194647	164	0.1835747	0.5437193	434	10.7016267
0.9784700	269	0.2105290	0.8194710	164	0.1835810	0.5437130	434	10.7016330
0.9784763	269	0.2105227	0.8194773	164	0.1835873	0.5437067	434	10.7016393
0.9784826	269	0.2105164	0.8194836	164	0.1835936	0.5437004	434	10.7016456
0.9784889	269	0.2105101	0.8194899	164	0.1836000	0.5436941	434	10.7016519
0.9784952	269	0.2105038	0.8194962	164	0.1836063	0.5436878	434	10.7016582
0.9785015	269	0.2104975	0.8195025	164	0.1836126	0.5436815	434	10.7016645
0.9785078	269	0.2104912	0.8195088	164	0.1836189	0.5436752	434	10.7016708
0.9785141	269	0.2104849	0.8195151	164	0.1836252	0.5436689	434	10.7016771
0.9785204	269	0.2104786	0.8195214	164	0.1836315	0.5436626	434	10.7016834
0.9785267	269	0.2104723	0.8195277	164	0.1836378	0.5436563	434	10.7016897
0.9785330	269	0.2104660	0.8195340	164	0.1836441	0.5436500	434	10.7016960
0.9785393	269	0.2104597	0.8195403	164	0.1836504	0.5436437	434	10.7017023
0.9785456	269	0.2104534	0.8195466	164	0.1836567	0.5436374	434	10.7017086
0.9785519	269	0.2104471	0.8195529	164	0.1836630	0.5436311	434	10.7017149
0.9785582	269	0.2104408	0.8195592	164	0.1836693	0.5436248	434	10.7017212
0.9785645	269	0.2104345	0.8195655	164	0.1836756	0.5436185	434	10.7017275
0.9785708	269	0.2104282	0.8195718	164	0.1836819	0.5436122	434	10.7017338
0.9785771	269	0.2104219	0.8195781	164	0.1836882	0.5436059	434	10.7017401
0.9785834	269	0.2104156	0.8195844	164	0.1836945	0.5435996	434	10.7017464
0.9785897	269	0.2104093	0.8195907	164	0.1837008	0.5435933	434	10.7017527
0.9785960	269	0.2104030	0.8195970	164	0.1837071	0.5435870	434	10.7017590
0.9786023	269	0.2103967	0.8196033	164	0.1837134	0.5435807	434	10.7017653
0.9786086	269	0.2103904	0.8196096	164	0.1837197	0.5435744	434	10.7017716
0.9786149	269	0.2103841	0.8196159	164	0.1837260	0.5435681	434	10.7017779
0.9786212	269	0.2103778	0.8196222	164	0.1837323	0.5435618	434	10.7017842
0.9786275	269	0.2103715	0.8196285	164	0.1837386	0.5435555	434	10.7017905
0.9786338	269	0.2103652	0.8196348	164	0.1837449	0.5435492	434	10.7017968
0.9786401	269	0.2103589	0.8196411	164	0.1837512	0.5435429	434	10.7018031
0.9786464	269	0.2103526	0.8196474	164	0.1837575	0.5435366	434	10.7018094
0.9786527	269	0.2103463	0.8196537	164	0.1837638	0.5435303	434	10.7018157
0.9786590	269	0.2103400	0.8196600	164	0.1837701	0.5435240	434	10.7018220
0.9786653	269	0.2103337	0.8196663	164	0.1837764	0.5435177	434	10.7018283
0.9786716	269	0.2103274	0.8196726	164	0.1837827	0.5435114	434	10.7018346
0.9786779	269	0.2103211	0.8196789	164	0.1837890	0.5435051	434	10.7018409
0.9786842	269	0.2103148	0.8196852	164	0.1837953	0.5434988	434	10.7018472
0.9786905	269	0.2103085	0.8196915	164	0.1838016	0.5434925	434	10.7018535
0.9786968	269	0.2103022	0.8196978	164	0.1838079	0.5434862	434	10.7018598
0.9787031	269	0.2102959	0.8197041	164	0.1838142	0.5434799	434	10.7018661
0.9787094	269	0.2102896	0.8197104	164	0.1838205	0.5434736	434	10.7018724
0.9787157	269	0.2102833	0.8197167	164	0.1838268	0.5434673	434	10.7018787
0.9787220	269	0.2102770	0.8197230	164	0.1838331	0.5434610	434	10.7018850
0.9787283	269	0.2102707	0.8197293	164	0.1838394	0.5434547	434	10.7018913
0.9787346	269	0.2102644	0.8197356	164	0.1838457	0.5434484	434	10.7018976
0.9787409	269	0.2102581	0.8197419	164	0.1838520	0.5434421	434	10.7019039
0.9787472	269	0.2102518	0.8197482	164	0.1838583	0.5434358	434	10.7019102
0.9787535	269	0.2102455	0.8197545	164	0.1838646	0.5434295	434	10.7019165
0.9787598	269	0.2102392	0.8197608	164	0.1838709	0.5434232	434	10.7019228
0.9787661	269	0.2102329	0.8197671	164	0.1838772	0.5434169	434	10.7019291
0.9787724	269	0.2102266	0.8197734	164	0.1838835	0.5434106	434	10.7019354
0.9787787	269	0.2102203	0.8197797	164	0.1838898	0.5434043	434	10.7019417
0.9787850	269	0.2102140	0.8197860	164	0.1838961	0.5433980	434	10.7019480
0.9787913	269	0.2102077	0.8197923	164	0.1839024	0.5433917	434	10.7019543
0.9787976	269	0.2102014	0.8197986	164	0.1839087	0.5433854	434	10.7019606
0.9788039	269	0.2101951	0.8198049	164	0.1839150	0.5433791	434	10.7019669
0.9788102	269	0.2101888	0.8198112	164	0.1839213	0.5433728	434	10.7019732
0.9788165	269	0.2101825	0.8198175	164	0.1839276	0.5433665	434	10.7019795
0.9788228	269	0.2101762	0.8198238	164	0.1839339	0.5433602	434	10.7019858
0.9788291	269	0.2101699	0.8198301	164	0.1839402	0.5433539	434	10.7019921
0.9788354	269	0.2101636	0.8198364	164	0.1839465	0.5433476	434	10.7019984
0.9788417	269	0.2101573	0.8198427	164	0.1839528	0.5433413	434	10.7020047
0.9788480	269	0.2101510	0.8198490	164	0.1839591	0.5433350	434	10.7020110
0.9788543	269	0.2101447	0.8198553	164	0.1839654	0.5433287	434	10.7020173
0.9788606	269	0.2101384	0.8198616	164	0.1839717	0.5433224	434	10.7020236
0.9788669	269	0.2101321	0.8198679	164	0.1839780	0.5433161	434	10.7020299
0.9788732	269	0.2101258	0.8198742	164	0.1839843	0.5433098	434	10.7020362
0.9788795	269	0.2101195	0.8198805	164	0.1839906	0.5433035	434	10.7020425
0.9788858	269	0.2101132	0.8198868	164	0.1839969	0.5432972	434	10.7020488
0.9788921	269	0.2101069	0.8198931	164	0.1840032	0.5432909	434	10.7020551
0.9788984	269	0.2101006	0.8198994	164	0.1840095	0.5432846	434	10.7020614
0.9789047	269	0.2100943	0.8199057	164	0.1840158	0.5432783	434	10.7020677
0.9789110	269	0.2100880	0.8199120	164	0.1840221	0.5432720	434	10.7020740
0.9789173	269	0.2100817	0.8199183	164	0.1840284	0.5432657	434	10.7020803
0.9789236	269	0.2100754	0.8199246	164	0.1840347	0.5432594	434	10.7020866
0.9789299	269	0.2100691	0.8199309	164	0.1840410	0.5432531	434	10.7020929
0.9789362	269	0.2100628	0.8199372	164	0.1840473	0.5432468	434	10.7021000
0.9789425	269	0.2100565	0.8199435	164	0.1840536	0.5432405	434	10.7021063
0.9789488	269	0.2100502	0.8199498	164	0.1840599	0.5432342	434	10.7021126
0.9789551	269	0.2100439	0.8199561	164	0.1840662	0.5432279	434	10.7021189
0.9789614	269	0.2100376	0.8199624	164	0.1840725	0.5432216	434	10.7021252
0.9789677	269	0.2100313	0.8199687	164	0.1840788	0.5432153	434	10.7021315
0.9789740	269	0.2100250	0.8199750	164	0.1840851	0.5432090	434	10.7021378
0.9789803	269	0.2100187	0.8199813	164	0.1840914	0.5432027	434	10.7021441
0.9789866	269	0.2100124	0.8199876	164	0.1840977	0.5431964	434	10.7021504
0.9789929	269	0.2100061	0.8199939	164	0.1841040	0.5431901	434	10.7021567
0.9789992	269	0.2100000	0.8200000	164	0.1841103	0.5431838	434	10.7021630

Tab. 18.

Dec. 51.

29 Dec.

Tab. 15.

Sine	D10	Comp. sin.	Cosine	D10	Comp. cos.	Tangent	D10	Cotangent
19° 7' 2887 19	260	0.9901128	9.8905028	170	0.1094974	9.9085692	430	10.0916302
19° 7' 2990 78	260	0.9900973	9.8904003	171	0.1093937	9.9086273	430	10.0917325
19° 7' 3091 83	260	0.9900816	9.8902974	171	0.1092901	9.9086853	430	10.0918348
19° 7' 3192 94	260	0.9900659	9.8901933	171	0.1091864	9.9087434	430	10.0919371
19° 7' 3293 51	259	0.9900502	9.8900892	171	0.1090827	9.9088014	430	10.0920394
19° 7' 3394 30	259	0.9900345	9.8899851	171	0.1089790	9.9088594	430	10.0921417
19° 7' 3495 16	259	0.9900188	9.8898810	171	0.1088753	9.9089174	430	10.0922440
19° 7' 3596 01	259	0.9900031	9.8897769	171	0.1087716	9.9089754	430	10.0923463
19° 7' 3696 46	259	0.9899874	9.8896728	171	0.1086679	9.9090334	430	10.0924486
19° 7' 3797 31	258	0.9899717	9.8895687	171	0.1085642	9.9090914	430	10.0925509
19° 7' 3898 16	258	0.9899560	9.8894646	171	0.1084605	9.9091494	430	10.0926532
19° 7' 3999 01	258	0.9899403	9.8893605	171	0.1083568	9.9092074	430	10.0927555
19° 7' 4099 46	258	0.9899246	9.8892564	171	0.1082531	9.9092654	430	10.0928578
19° 7' 4100 31	258	0.9899089	9.8891523	171	0.1081494	9.9093234	430	10.0929601
19° 7' 4200 76	258	0.9898932	9.8890482	171	0.1080457	9.9093814	430	10.0930624
19° 7' 4301 61	258	0.9898775	9.8889441	171	0.1079420	9.9094394	430	10.0931647
19° 7' 4402 46	258	0.9898618	9.8888400	171	0.1078383	9.9094974	430	10.0932670
19° 7' 4503 31	257	0.9898461	9.8887359	171	0.1077346	9.9095554	430	10.0933693
19° 7' 4604 16	257	0.9898304	9.8886318	171	0.1076309	9.9096134	430	10.0934716
19° 7' 4705 01	257	0.9898147	9.8885277	171	0.1075272	9.9096714	430	10.0935739
19° 7' 4805 46	257	0.9897990	9.8884236	171	0.1074235	9.9097294	430	10.0936762
19° 7' 4906 31	257	0.9897833	9.8883195	171	0.1073198	9.9097874	430	10.0937785
19° 7' 5007 16	257	0.9897676	9.8882154	171	0.1072161	9.9098454	430	10.0938808
19° 7' 5108 01	256	0.9897519	9.8881113	171	0.1071124	9.9099034	430	10.0939831
19° 7' 5208 46	256	0.9897362	9.8880072	171	0.1070087	9.9099614	430	10.0940854
19° 7' 5309 31	256	0.9897205	9.8879031	171	0.1069050	9.9100194	430	10.0941877
19° 7' 5410 16	256	0.9897048	9.8877990	171	0.1068013	9.9100774	430	10.0942900
19° 7' 5511 01	256	0.9896891	9.8876949	171	0.1066976	9.9101354	430	10.0943923
19° 7' 5611 46	256	0.9896734	9.8875908	171	0.1065939	9.9101934	430	10.0944946
19° 7' 5712 31	255	0.9896577	9.8874867	171	0.1064902	9.9102514	430	10.0945969
19° 7' 5813 16	255	0.9896420	9.8873826	171	0.1063865	9.9103094	430	10.0946992
19° 7' 5914 01	255	0.9896263	9.8872785	171	0.1062828	9.9103674	430	10.0948015
19° 7' 6014 46	255	0.9896106	9.8871744	171	0.1061791	9.9104254	430	10.0949038
19° 7' 6115 31	255	0.9895949	9.8870703	171	0.1060754	9.9104834	430	10.0950061
19° 7' 6216 16	255	0.9895792	9.8869662	171	0.1059717	9.9105414	430	10.0951084
19° 7' 6317 01	255	0.9895635	9.8868621	171	0.1058680	9.9105994	430	10.0952107
19° 7' 6417 46	255	0.9895478	9.8867580	171	0.1057643	9.9106574	430	10.0953130
19° 7' 6518 31	255	0.9895321	9.8866539	171	0.1056606	9.9107154	430	10.0954153
19° 7' 6619 16	255	0.9895164	9.8865498	171	0.1055569	9.9107734	430	10.0955176
19° 7' 6720 01	255	0.9895007	9.8864457	171	0.1054532	9.9108314	430	10.0956199
19° 7' 6820 46	255	0.9894850	9.8863416	171	0.1053495	9.9108894	430	10.0957222
19° 7' 6921 31	255	0.9894693	9.8862375	171	0.1052458	9.9109474	430	10.0958245
19° 7' 7022 16	255	0.9894536	9.8861334	171	0.1051421	9.9110054	430	10.0959268
19° 7' 7123 01	255	0.9894379	9.8860293	171	0.1050384	9.9110634	430	10.0960291
19° 7' 7223 46	255	0.9894222	9.8859252	171	0.1049347	9.9111214	430	10.0961314
19° 7' 7324 31	255	0.9894065	9.8858211	171	0.1048310	9.9111794	430	10.0962337
19° 7' 7425 16	255	0.9893908	9.8857170	171	0.1047273	9.9112374	430	10.0963360
19° 7' 7526 01	255	0.9893751	9.8856129	171	0.1046236	9.9112954	430	10.0964383
19° 7' 7626 46	255	0.9893594	9.8855088	171	0.1045199	9.9113534	430	10.0965406
19° 7' 7727 31	255	0.9893437	9.8854047	171	0.1044162	9.9114114	430	10.0966429
19° 7' 7828 16	255	0.9893280	9.8853006	171	0.1043125	9.9114694	430	10.0967452
19° 7' 7929 01	255	0.9893123	9.8851965	171	0.1042088	9.9115274	430	10.0968475
19° 7' 8030 46	255	0.9892966	9.8850924	171	0.1041051	9.9115854	430	10.0969498
19° 7' 8131 31	255	0.9892809	9.8849883	171	0.1040014	9.9116434	430	10.0970521
19° 7' 8232 16	255	0.9892652	9.8848842	171	0.1038977	9.9117014	430	10.0971544
19° 7' 8333 01	255	0.9892495	9.8847801	171	0.1037940	9.9117594	430	10.0972567
19° 7' 8433 46	255	0.9892338	9.8846760	171	0.1036903	9.9118174	430	10.0973590
19° 7' 8534 31	255	0.9892181	9.8845719	171	0.1035866	9.9118754	430	10.0974613
19° 7' 8635 16	255	0.9892024	9.8844678	171	0.1034829	9.9119334	430	10.0975636
19° 7' 8736 01	255	0.9891867	9.8843637	171	0.1033792	9.9119914	430	10.0976659
19° 7' 8836 46	255	0.9891710	9.8842596	171	0.1032755	9.9120494	430	10.0977682
19° 7' 8937 31	255	0.9891553	9.8841555	171	0.1031718	9.9121074	430	10.0978705
19° 7' 9038 16	255	0.9891396	9.8840514	171	0.1030681	9.9121654	430	10.0979728
19° 7' 9139 01	255	0.9891239	9.8839473	171	0.1029644	9.9122234	430	10.0980751
19° 7' 9240 46	255	0.9891082	9.8838432	171	0.1028607	9.9122814	430	10.0981774
19° 7' 9341 31	255	0.9890925	9.8837391	171	0.1027570	9.9123394	430	10.0982797
19° 7' 9442 16	255	0.9890768	9.8836350	171	0.1026533	9.9123974	430	10.0983820
19° 7' 9543 01	255	0.9890611	9.8835309	171	0.1025496	9.9124554	430	10.0984843
19° 7' 9643 46	255	0.9890454	9.8834268	171	0.1024459	9.9125134	430	10.0985866
19° 7' 9744 31	255	0.9890297	9.8833227	171	0.1023422	9.9125714	430	10.0986889
19° 7' 9845 16	255	0.9890140	9.8832186	171	0.1022385	9.9126294	430	10.0987912
19° 7' 9946 01	255	0.9889983	9.8831145	171	0.1021348	9.9126874	430	10.0988935
19° 7' 1000 46	255	0.9889826	9.8830104	171	0.1020311	9.9127454	430	10.0989958
19° 7' 1001 31	255	0.9889669	9.8829063	171	0.1019274	9.9128034	430	10.0990981
19° 7' 1002 16	255	0.9889512	9.8828022	171	0.1018237	9.9128614	430	10.0992004
19° 7' 1003 01	255	0.9889355	9.8826981	171	0.1017200	9.9129194	430	10.0993027
19° 7' 1004 46	255	0.9889198	9.8825940	171	0.1016163	9.9129774	430	10.0994050
19° 7' 1005 31	255	0.9889041	9.8824899	171	0.1015126	9.9130354	430	10.0995073
19° 7' 1006 16	255	0.9888884	9.8823858	171	0.1014089	9.9130934	430	10.0996096
19° 7' 1007 01	255	0.9888727	9.8822817	171	0.1013052	9.9131514	430	10.0997119
19° 7' 1008 46	255	0.9888570	9.8821776	171	0.1012015	9.9132094	430	10.0998142
19° 7' 1009 31	255	0.9888413	9.8820735	171	0.1010978	9.9132674	430	10.0999165
19° 7' 1010 16	255	0.9888256	9.8819694	171	0.1009941	9.9133254	430	10.1000188
19° 7' 1011 01	255	0.9888099	9.8818653	171	0.1008904	9.9133834	430	10.1001211
19° 7' 1012 46	255	0.9887942	9.8817612	171	0.1007867	9.9134414	430	10.1002234
19° 7' 1013 31	255	0.9887785	9.8816571	171	0.1006830	9.9134994	430	10.1003257
19° 7' 1014 16	255	0.9887628	9.8815530	171	0.1005793	9.9135574	430	10.1004280
19° 7' 1015 01	255	0.9887471	9.8814489	171	0.1004756	9.9136154	430	10.1005303
19° 7' 1016 46	255	0.9887314	9.8813448	171	0.1003719	9.9136734	430	10.1006326
19° 7' 1017 31	255	0.9887157	9.8812407	171	0.1002682	9.9137314	430	10.1007349
19° 7' 1018 16	255	0.9887000	9.8811366	171	0.1001645	9.9137894	430	10.1008372
19° 7' 1019 01	255	0.9886843	9.8810325	171	0.1000608	9.9138474	430	10.1009395
19° 7' 1020 46	255	0.9886686	9.8809284	171	0.0999571	9.9139054	430	10.1010418
19° 7' 1021 31	255	0.9886529	9.8808243	171	0.0998534	9.9139634	430	10.1011441
19° 7' 1022 16	255	0.9886372	9.8807202	171	0.0997497	9.9140214	430	10.1012464
19° 7' 1023 01	255	0.9886215	9.8806161	171	0.0996460	9.9140794	430	10.1013487
19° 7' 1024 46	255	0.9886058	9.8805120	171	0.0995423	9.9141374	430	10.1014510
19° 7' 1025 31	255	0.9885901	9.8804079	171	0.0994386	9.9141954	430	10.1015533
19° 7' 1026 16	255	0.9885744	9.8803038	171	0.0993349	9.9142534	430	10.1016556
19° 7' 1027 01	255	0.9885587	9.8801997	171	0.0992312	9.9143114	430	10.1017579
19° 7' 1028 46	255	0.9885430	9.8800956	171	0.0991275	9.9143694	430	10.1018602
19° 7' 1029 31	255	0.9885273	9.8800015	171	0.0990238	9.9144274	430	10.1019625
19° 7' 1								

40 Dc.

Tab. 18.

	Cosine	D10	Comp. Cosine	Sine	D10	Comp. Sine	Tangent	D10	Cotangent
0-8080675	251	0-1915	8842540	177	0-1157450	9-9234153	428	10-0761665	40
1-8082180	251	0-1915	8841479	177	0-1158311	9-9241070	427	10-0758209	41
2-8083684	251	0-1915	8840418	177	0-1159172	9-9247986	427	10-0754753	42
3-8085188	251	0-1915	8839357	177	0-1160033	9-9254903	427	10-0751297	43
4-8086692	250	0-1915	8838296	177	0-1160894	9-9261820	427	10-0747841	44
5-8088196	250	0-1915	8837235	177	0-1161755	9-9268737	427	10-0744385	45
6-8089699	250	0-1915	8836174	177	0-1162616	9-9275654	427	10-0740929	46
7-8091192	250	0-1915	8835113	177	0-1163477	9-9282571	427	10-0737473	47
8-8092695	250	0-1915	8834052	177	0-1164338	9-9289488	427	10-0734017	48
9-8094198	250	0-1915	8832991	178	0-1165199	9-9296405	427	10-0730561	49
10-8095696	250	0-1915	8831930	178	0-1166060	9-9303322	427	10-0727105	50
11-8097192	249	0-1915	8830869	178	0-1166921	9-9310239	427	10-0723649	51
12-8098687	249	0-1915	8829808	178	0-1167782	9-9317156	427	10-0720193	52
13-8100172	249	0-1915	8828747	178	0-1168643	9-9324073	427	10-0716737	53
14-8101656	249	0-1915	8827686	178	0-1169504	9-9330990	427	10-0713281	54
15-8103150	249	0-1915	8826625	178	0-1170365	9-9337907	427	10-0709825	55
16-8104650	249	0-1915	8825564	178	0-1171226	9-9344824	427	10-0706369	56
17-8106141	249	0-1915	8824503	178	0-1172087	9-9351741	427	10-0702913	57
18-8107631	249	0-1915	8823442	179	0-1172948	9-9358658	427	10-0699457	58
19-8109121	249	0-1915	8822381	179	0-1173809	9-9365575	427	10-0696001	59
20-8110609	249	0-1915	8821320	179	0-1174670	9-9372492	427	10-0692545	60
21-8112094	249	0-1915	8820259	179	0-1175531	9-9379409	427	10-0689089	61
22-8113584	249	0-1915	8819198	179	0-1176392	9-9386326	427	10-0685633	62
23-8115074	249	0-1915	8818137	179	0-1177253	9-9393243	427	10-0682177	63
24-8116564	249	0-1915	8817076	179	0-1178114	9-9400160	427	10-0678721	64
25-8118054	249	0-1915	8816015	179	0-1178975	9-9407077	427	10-0675265	65
26-8119544	249	0-1915	8814954	179	0-1179836	9-9413994	427	10-0671809	66
27-8121034	249	0-1915	8813893	179	0-1180697	9-9420911	427	10-0668353	67
28-8122524	249	0-1915	8812832	180	0-1181558	9-9427828	427	10-0664897	68
29-8124014	249	0-1915	8811771	180	0-1182419	9-9434745	427	10-0661441	69
30-8125504	249	0-1915	8810710	180	0-1183280	9-9441662	427	10-0657985	70
31-8126994	249	0-1915	8809649	180	0-1184141	9-9448579	427	10-0654529	71
32-8128484	249	0-1915	8808588	180	0-1185002	9-9455496	427	10-0651073	72
33-8129974	249	0-1915	8807527	180	0-1185863	9-9462413	427	10-0647617	73
34-8131464	249	0-1915	8806466	180	0-1186724	9-9469330	427	10-0644161	74
35-8132954	249	0-1915	8805405	180	0-1187585	9-9476247	427	10-0640705	75
36-8134444	249	0-1915	8804344	180	0-1188446	9-9483164	427	10-0637249	76
37-8135934	249	0-1915	8803283	181	0-1189307	9-9490081	427	10-0633793	77
38-8137424	249	0-1915	8802222	181	0-1190168	9-9497000	427	10-0630337	78
39-8138914	249	0-1915	8801161	181	0-1191029	9-9503917	427	10-0626881	79
40-8140404	249	0-1915	8800100	181	0-1191890	9-9510834	427	10-0623425	80
41-8141894	249	0-1915	8799039	181	0-1192751	9-9517751	427	10-0620000	81
42-8143384	249	0-1915	8797978	181	0-1193612	9-9524668	427	10-0616544	82
43-8144874	249	0-1915	8796917	181	0-1194473	9-9531585	427	10-0613088	83
44-8146364	249	0-1915	8795856	181	0-1195334	9-9538502	427	10-0609632	84
45-8147854	249	0-1915	8794795	181	0-1196195	9-9545419	427	10-0606176	85
46-8149344	249	0-1915	8793734	181	0-1197056	9-9552336	427	10-0602720	86
47-8150834	249	0-1915	8792673	182	0-1197917	9-9559253	427	10-0599264	87
48-8152324	249	0-1915	8791612	182	0-1198778	9-9566170	427	10-0595808	88
49-8153814	249	0-1915	8790551	182	0-1199639	9-9573087	427	10-0592352	89
50-8155304	249	0-1915	8789490	182	0-1200500	9-9580004	427	10-0588896	90
51-8156794	249	0-1915	8788429	182	0-1201361	9-9586921	427	10-0585440	91
52-8158284	249	0-1915	8787368	182	0-1202222	9-9593838	427	10-0581984	92
53-8159774	249	0-1915	8786307	182	0-1203083	9-9600755	427	10-0578528	93
54-8161264	249	0-1915	8785246	182	0-1203944	9-9607672	427	10-0575072	94
55-8162754	249	0-1915	8784185	182	0-1204805	9-9614589	427	10-0571616	95
56-8164244	249	0-1915	8783124	183	0-1205666	9-9621506	427	10-0568160	96
57-8165734	249	0-1915	8782063	183	0-1206527	9-9628423	427	10-0564704	97
58-8167224	249	0-1915	8781002	183	0-1207388	9-9635340	427	10-0561248	98
59-8168714	249	0-1915	8779941	183	0-1208249	9-9642257	427	10-0557792	99
60-8170204	249	0-1915	8778880	183	0-1209110	9-9649174	427	10-0554336	100

Tab. 18.

Dep. 18.

Sine	D10	Comp. sine	Cosine	D10	Comp. cosine	Tan	D10	Cotangent
0 8109429	243	0 1830571	9 8777750	183	0 1232301	9 94933	425	10 0498369
1 8109882	243	0 1829118	9 8776710	183	0 1232300	9 94933	425	10 0498369
2 8110334	243	0 1827666	9 8775670	183	0 1232300	9 94933	425	10 0498369
3 8110787	243	0 1826215	9 8774630	183	0 1232300	9 94933	425	10 0498369
4 8111240	243	0 1824763	9 8773590	183	0 1232300	9 94933	425	10 0498369
5 8111693	243	0 1823311	9 8772550	183	0 1232300	9 94933	425	10 0498369
6 8112146	243	0 1821859	9 8771510	183	0 1232300	9 94933	425	10 0498369
7 8112599	243	0 1820407	9 8770470	183	0 1232300	9 94933	425	10 0498369
8 8113052	243	0 1818955	9 8769430	183	0 1232300	9 94933	425	10 0498369
9 8113505	243	0 1817503	9 8768390	183	0 1232300	9 94933	425	10 0498369
10 8113958	243	0 1816051	9 8767350	183	0 1232300	9 94933	425	10 0498369
11 8114411	243	0 1814599	9 8766310	183	0 1232300	9 94933	425	10 0498369
12 8114864	243	0 1813147	9 8765270	183	0 1232300	9 94933	425	10 0498369
13 8115317	243	0 1811695	9 8764230	183	0 1232300	9 94933	425	10 0498369
14 8115770	243	0 1810243	9 8763190	183	0 1232300	9 94933	425	10 0498369
15 8116223	243	0 1808791	9 8762150	183	0 1232300	9 94933	425	10 0498369
16 8116676	243	0 1807339	9 8761110	183	0 1232300	9 94933	425	10 0498369
17 8117129	243	0 1805887	9 8760070	183	0 1232300	9 94933	425	10 0498369
18 8117582	243	0 1804435	9 8759030	183	0 1232300	9 94933	425	10 0498369
19 8118035	243	0 1802983	9 8757990	183	0 1232300	9 94933	425	10 0498369
20 8118488	243	0 1801531	9 8756950	183	0 1232300	9 94933	425	10 0498369
21 8118941	243	0 1800079	9 8755910	183	0 1232300	9 94933	425	10 0498369
22 8119394	243	0 1798627	9 8754870	183	0 1232300	9 94933	425	10 0498369
23 8119847	243	0 1797175	9 8753830	183	0 1232300	9 94933	425	10 0498369
24 8120300	243	0 1795723	9 8752790	183	0 1232300	9 94933	425	10 0498369
25 8120753	243	0 1794271	9 8751750	183	0 1232300	9 94933	425	10 0498369
26 8121206	243	0 1792819	9 8750710	183	0 1232300	9 94933	425	10 0498369
27 8121659	243	0 1791367	9 8749670	183	0 1232300	9 94933	425	10 0498369
28 8122112	243	0 1789915	9 8748630	183	0 1232300	9 94933	425	10 0498369
29 8122565	243	0 1788463	9 8747590	183	0 1232300	9 94933	425	10 0498369
30 8123018	243	0 1787011	9 8746550	183	0 1232300	9 94933	425	10 0498369
31 8123471	243	0 1785559	9 8745510	183	0 1232300	9 94933	425	10 0498369
32 8123924	243	0 1784107	9 8744470	183	0 1232300	9 94933	425	10 0498369
33 8124377	243	0 1782655	9 8743430	183	0 1232300	9 94933	425	10 0498369
34 8124830	243	0 1781203	9 8742390	183	0 1232300	9 94933	425	10 0498369
35 8125283	243	0 1779751	9 8741350	183	0 1232300	9 94933	425	10 0498369
36 8125736	243	0 1778299	9 8740310	183	0 1232300	9 94933	425	10 0498369
37 8126189	243	0 1776847	9 8739270	183	0 1232300	9 94933	425	10 0498369
38 8126642	243	0 1775395	9 8738230	183	0 1232300	9 94933	425	10 0498369
39 8127095	243	0 1773943	9 8737190	183	0 1232300	9 94933	425	10 0498369
40 8127548	243	0 1772491	9 8736150	183	0 1232300	9 94933	425	10 0498369
41 8128001	243	0 1771039	9 8735110	183	0 1232300	9 94933	425	10 0498369
42 8128454	243	0 1769587	9 8734070	183	0 1232300	9 94933	425	10 0498369
43 8128907	243	0 1768135	9 8733030	183	0 1232300	9 94933	425	10 0498369
44 8129360	243	0 1766683	9 8731990	183	0 1232300	9 94933	425	10 0498369
45 8129813	243	0 1765231	9 8730950	183	0 1232300	9 94933	425	10 0498369
46 8130266	243	0 1763779	9 8729910	183	0 1232300	9 94933	425	10 0498369
47 8130719	243	0 1762327	9 8728870	183	0 1232300	9 94933	425	10 0498369
48 8131172	243	0 1760875	9 8727830	183	0 1232300	9 94933	425	10 0498369
49 8131625	243	0 1759423	9 8726790	183	0 1232300	9 94933	425	10 0498369
50 8132078	243	0 1757971	9 8725750	183	0 1232300	9 94933	425	10 0498369
51 8132531	243	0 1756519	9 8724710	183	0 1232300	9 94933	425	10 0498369
52 8132984	243	0 1755067	9 8723670	183	0 1232300	9 94933	425	10 0498369
53 8133437	243	0 1753615	9 8722630	183	0 1232300	9 94933	425	10 0498369
54 8133890	243	0 1752163	9 8721590	183	0 1232300	9 94933	425	10 0498369
55 8134343	243	0 1750711	9 8720550	183	0 1232300	9 94933	425	10 0498369
56 8134796	243	0 1749259	9 8719510	183	0 1232300	9 94933	425	10 0498369
57 8135249	243	0 1747807	9 8718470	183	0 1232300	9 94933	425	10 0498369
58 8135702	243	0 1746355	9 8717430	183	0 1232300	9 94933	425	10 0498369
59 8136155	243	0 1744903	9 8716390	183	0 1232300	9 94933	425	10 0498369
60 8136608	243	0 1743451	9 8715350	183	0 1232300	9 94933	425	10 0498369
61 8137061	243	0 1742000	9 8714310	183	0 1232300	9 94933	425	10 0498369
62 8137514	243	0 1740548	9 8713270	183	0 1232300	9 94933	425	10 0498369
63 8137967	243	0 1739096	9 8712230	183	0 1232300	9 94933	425	10 0498369
64 8138420	243	0 1737644	9 8711190	183	0 1232300	9 94933	425	10 0498369
65 8138873	243	0 1736192	9 8710150	183	0 1232300	9 94933	425	10 0498369
66 8139326	243	0 1734740	9 8709110	183	0 1232300	9 94933	425	10 0498369
67 8139779	243	0 1733288	9 8708070	183	0 1232300	9 94933	425	10 0498369
68 8140232	243	0 1731836	9 8707030	183	0 1232300	9 94933	425	10 0498369
69 8140685	243	0 1730384	9 8705990	183	0 1232300	9 94933	425	10 0498369
70 8141138	243	0 1728932	9 8704950	183	0 1232300	9 94933	425	10 0498369
71 8141591	243	0 1727480	9 8703910	183	0 1232300	9 94933	425	10 0498369
72 8142044	243	0 1726028	9 8702870	183	0 1232300	9 94933	425	10 0498369
73 8142497	243	0 1724576	9 8701830	183	0 1232300	9 94933	425	10 0498369
74 8142950	243	0 1723124	9 8700790	183	0 1232300	9 94933	425	10 0498369
75 8143403	243	0 1721672	9 8699750	183	0 1232300	9 94933	425	10 0498369
76 8143856	243	0 1720220	9 8698710	183	0 1232300	9 94933	425	10 0498369
77 8144309	243	0 1718768	9 8697670	183	0 1232300	9 94933	425	10 0498369
78 8144762	243	0 1717316	9 8696630	183	0 1232300	9 94933	425	10 0498369
79 8145215	243	0 1715864	9 8695590	183	0 1232300	9 94933	425	10 0498369
80 8145668	243	0 1714412	9 8694550	183	0 1232300	9 94933	425	10 0498369
81 8146121	243	0 1712960	9 8693510	183	0 1232300	9 94933	425	10 0498369
82 8146574	243	0 1711508	9 8692470	183	0 1232300	9 94933	425	10 0498369
83 8147027	243	0 1710056	9 8691430	183	0 1232300	9 94933	425	10 0498369
84 8147480	243	0 1708604	9 8690390	183	0 1232300	9 94933	425	10 0498369
85 8147933	243	0 1707152	9 8689350	183	0 1232300	9 94933	425	10 0498369
86 8148386	243	0 1705700	9 8688310	183	0 1232300	9 94933	425	10 0498369
87 8148839	243	0 1704248	9 8687270	183	0 1232300	9 94933	425	10 0498369
88 8149292	243	0 1702796	9 8686230	183	0 1232300	9 94933	425	10 0498369
89 8149745	243	0 1701344	9 8685190	183	0 1232300	9 94933	425	10 0498369
90 8150198	243	0 1700000	9 8684150	183	0 1232300	9 94933	425	10 0498369
91 8150651	243	0 1698548	9 8683110	183	0 1232300	9 94933	425	10 0498369
92 8151104	243	0 1697096	9 8682070	183	0 1232300	9 94933	425	10 0498369
93 8151557	243	0 1695644	9 8681030	183	0 1232300	9 94933	425	10 0498369
94 8152010	243	0 1694192	9 8680000	183	0 1232300	9 94933	425	10 0498369
95 8152463	243	0 1692740	9 8678960	183	0 1232300	9 94933	425	10 0498369
96 8152916	243	0 1691288	9 8677920	183	0 1232300	9 94933	425	10 0498369
97 8153369	243	0 1689836	9 8676880	183	0 1232300	9 94933	425	10 0498369
98 8153822	243	0 1688384	9 8675840	183	0 1232300	9 94933	425	10 0498369
99 8154275	243	0 1686932	9 8674800	183	0 1232300	9 94933	425	10 0498369
100 8154728	243	0 1685480	9 8673760	183	0 1232300	9 94933	425	10 0498369

42 Deg.

Tab. 46.

Sine	D10'	Comp. sin	Cosine	D10'	Comp. cos	Tangent	D10'	Cotangent
0 9 8255 109	234	0 1744889	9 8719735	190	0 1258926	9 3344378	423	10 0655621
1 9 8256 511	234	0 1744888	9 8719735	190	0 1258926	9 3344378	423	10 0655621
2 9 8257 911	233	0 1744887	9 8719735	190	0 1258926	9 3344378	423	10 0655621
3 9 8258 911	233	0 1744886	9 8719735	190	0 1258926	9 3344378	423	10 0655621
4 9 8259 911	233	0 1744885	9 8719735	190	0 1258926	9 3344378	423	10 0655621
5 9 8260 715	233	0 1744884	9 8719735	190	0 1258926	9 3344378	423	10 0655621
6 9 8261 114	233	0 1744883	9 8719735	190	0 1258926	9 3344378	423	10 0655621
7 9 8262 354	233	0 1744882	9 8719735	190	0 1258926	9 3344378	423	10 0655621
8 9 8263 490	233	0 1744881	9 8719735	190	0 1258926	9 3344378	423	10 0655621
9 9 8264 703	232	0 1744880	9 8719735	190	0 1258926	9 3344378	423	10 0655621
10 9 8265 909	232	0 1744879	9 8719735	190	0 1258926	9 3344378	423	10 0655621
11 9 8270 499	232	0 1744878	9 8719735	190	0 1258926	9 3344378	423	10 0655621
12 9 8271 887	232	0 1744877	9 8719735	190	0 1258926	9 3344378	423	10 0655621
13 9 8273 277	232	0 1744876	9 8719735	190	0 1258926	9 3344378	423	10 0655621
14 9 8274 671	232	0 1744875	9 8719735	190	0 1258926	9 3344378	423	10 0655621
15 9 8276 063	232	0 1744874	9 8719735	190	0 1258926	9 3344378	423	10 0655621
16 9 8277 453	232	0 1744873	9 8719735	190	0 1258926	9 3344378	423	10 0655621
17 9 8278 843	232	0 1744872	9 8719735	190	0 1258926	9 3344378	423	10 0655621
18 9 8280 231	231	0 1744871	9 8719735	190	0 1258926	9 3344378	423	10 0655621
19 9 8281 619	231	0 1744870	9 8719735	190	0 1258926	9 3344378	423	10 0655621
20 9 8283 006	231	0 1744869	9 8719735	190	0 1258926	9 3344378	423	10 0655621
21 9 8284 393	231	0 1744868	9 8719735	190	0 1258926	9 3344378	423	10 0655621
22 9 8285 778	231	0 1744867	9 8719735	190	0 1258926	9 3344378	423	10 0655621
23 9 8287 161	231	0 1744866	9 8719735	190	0 1258926	9 3344378	423	10 0655621
24 9 8288 543	230	0 1744865	9 8719735	190	0 1258926	9 3344378	423	10 0655621
25 9 8289 924	230	0 1744864	9 8719735	190	0 1258926	9 3344378	423	10 0655621
26 9 8291 302	230	0 1744863	9 8719735	190	0 1258926	9 3344378	423	10 0655621
27 9 8292 679	230	0 1744862	9 8719735	190	0 1258926	9 3344378	423	10 0655621
28 9 8294 054	230	0 1744861	9 8719735	190	0 1258926	9 3344378	423	10 0655621
29 9 8295 428	230	0 1744860	9 8719735	190	0 1258926	9 3344378	423	10 0655621
30 9 8296 801	230	0 1744859	9 8719735	190	0 1258926	9 3344378	423	10 0655621
31 9 8298 172	229	0 1744858	9 8719735	190	0 1258926	9 3344378	423	10 0655621
32 9 8299 542	229	0 1744857	9 8719735	190	0 1258926	9 3344378	423	10 0655621
33 9 8300 910	229	0 1744856	9 8719735	190	0 1258926	9 3344378	423	10 0655621
34 9 8302 276	229	0 1744855	9 8719735	190	0 1258926	9 3344378	423	10 0655621
35 9 8303 641	229	0 1744854	9 8719735	190	0 1258926	9 3344378	423	10 0655621
36 9 8305 004	229	0 1744853	9 8719735	190	0 1258926	9 3344378	423	10 0655621
37 9 8306 366	229	0 1744852	9 8719735	190	0 1258926	9 3344378	423	10 0655621
38 9 8307 727	229	0 1744851	9 8719735	190	0 1258926	9 3344378	423	10 0655621
39 9 8309 087	228	0 1744850	9 8719735	190	0 1258926	9 3344378	423	10 0655621
40 9 8310 446	228	0 1744849	9 8719735	190	0 1258926	9 3344378	423	10 0655621
41 9 8311 804	228	0 1744848	9 8719735	190	0 1258926	9 3344378	423	10 0655621
42 9 8313 160	228	0 1744847	9 8719735	190	0 1258926	9 3344378	423	10 0655621
43 9 8314 515	228	0 1744846	9 8719735	190	0 1258926	9 3344378	423	10 0655621
44 9 8315 868	228	0 1744845	9 8719735	190	0 1258926	9 3344378	423	10 0655621
45 9 8317 219	227	0 1744844	9 8719735	190	0 1258926	9 3344378	423	10 0655621
46 9 8318 568	227	0 1744843	9 8719735	190	0 1258926	9 3344378	423	10 0655621
47 9 8320 015	227	0 1744842	9 8719735	190	0 1258926	9 3344378	423	10 0655621
48 9 8321 460	227	0 1744841	9 8719735	190	0 1258926	9 3344378	423	10 0655621
49 9 8322 903	227	0 1744840	9 8719735	190	0 1258926	9 3344378	423	10 0655621
50 9 8324 344	227	0 1744839	9 8719735	190	0 1258926	9 3344378	423	10 0655621
51 9 8325 783	227	0 1744838	9 8719735	190	0 1258926	9 3344378	423	10 0655621
52 9 8327 220	226	0 1744837	9 8719735	190	0 1258926	9 3344378	423	10 0655621
53 9 8328 655	226	0 1744836	9 8719735	190	0 1258926	9 3344378	423	10 0655621
54 9 8330 088	226	0 1744835	9 8719735	190	0 1258926	9 3344378	423	10 0655621
55 9 8331 519	226	0 1744834	9 8719735	190	0 1258926	9 3344378	423	10 0655621
56 9 8332 948	226	0 1744833	9 8719735	190	0 1258926	9 3344378	423	10 0655621
57 9 8334 375	226	0 1744832	9 8719735	190	0 1258926	9 3344378	423	10 0655621
58 9 8335 800	226	0 1744831	9 8719735	190	0 1258926	9 3344378	423	10 0655621
59 9 8337 223	225	0 1744830	9 8719735	190	0 1258926	9 3344378	423	10 0655621
60 9 8338 644	225	0 1744829	9 8719735	190	0 1258926	9 3344378	423	10 0655621

Tab. 18.

Deg. 47.

43 Deg.

Sine	D10'	Comp. sin.	Cosine	D10'	Comp. cos.	Tangent	D10'	Cotangent
0 9-8387833	226	0-1662167	9-8641275	196	0-1358723	9-9696559	422	10-0303441
1 9-8389188	226	0-1660812	9-8640096	196	0-1359904	9-9699091	422	10-0300909
2 9-8340541	225	0-1659459	9-8638917	197	0-1361085	9-9701624	422	10-0298376
3 9-8341894	225	0-1658106	9-8637737	197	0-1362266	9-9704157	422	10-0295843
4 9-8343246	225	0-1656754	9-8636557	197	0-1363443	9-9706689	422	10-0293311
5 9-8344597	225	0-1655403	9-8635376	197	0-1364624	9-9709221	422	10-0290779
6 9-8345948	225	0-1654052	9-8634194	197	0-1365804	9-9711754	422	10-0288246
7 9-8347297	225	0-1652703	9-8633011	197	0-1366985	9-9714286	422	10-0285714
8 9-8348646	225	0-1651354	9-8631828	197	0-1368171	9-9716818	422	10-0283182
9 9-8349994	224	0-1650006	9-8630644	197	0-1369356	9-9719350	422	10-0280650
10 9-8351341	224	0-1648657	9-8629460	198	0-1370540	9-9721882	422	10-0278118
11 9-8352688	224	0-1647312	9-8628274	198	0-1371726	9-9724413	422	10-0275587
12 9-8354033	224	0-1645967	9-8627088	198	0-1372912	9-9726945	422	10-0273055
13 9-8355378	224	0-1644622	9-8625902	198	0-1374098	9-9729477	422	10-0270523
14 9-8356722	224	0-1643278	9-8624714	198	0-1375286	9-9732008	422	10-0267992
15 9-8358066	224	0-1641934	9-8623526	198	0-1376474	9-9734539	422	10-0265461
16 9-8359408	224	0-1640592	9-8622338	198	0-1377662	9-9737071	422	10-0262929
17 9-8360750	224	0-1639250	9-8621148	198	0-1378850	9-9739602	422	10-0260398
18 9-8362091	223	0-1637909	9-8619958	198	0-1380042	9-9742133	422	10-0257867
19 9-8363431	223	0-1636569	9-8618767	198	0-1381233	9-9744664	422	10-0255336
20 9-8364771	223	0-1635229	9-8617576	199	0-1382424	9-9747195	422	10-0252805
21 9-8366109	223	0-1633889	9-8616383	199	0-1383617	9-9749726	422	10-0250274
22 9-8367447	223	0-1632553	9-8615190	199	0-1384810	9-9752257	422	10-0247743
23 9-8368784	223	0-1631216	9-8613997	199	0-1386003	9-9754787	422	10-0245213
24 9-8370121	223	0-1629879	9-8612803	199	0-1387197	9-9757318	422	10-0242682
25 9-8371456	222	0-1628544	9-8611608	199	0-1388392	9-9759849	422	10-0240151
26 9-8372791	222	0-1627209	9-8610412	199	0-1389588	9-9762379	422	10-0237621
27 9-8374125	222	0-1625875	9-8609215	199	0-1390785	9-9764909	422	10-0235091
28 9-8375458	222	0-1624542	9-8608018	199	0-1391982	9-9767440	422	10-0232560
29 9-8376790	222	0-1623210	9-8606821	200	0-1393179	9-9769970	422	10-0230030
30 9-8378122	222	0-1621878	9-8605622	200	0-1394376	9-9772500	422	10-0227500
31 9-8379453	222	0-1620547	9-8604423	200	0-1395577	9-9775030	422	10-0224970
32 9-8380783	221	0-1619217	9-8603223	200	0-1396777	9-9777560	422	10-0222440
33 9-8382112	221	0-1617888	9-8602022	200	0-1397978	9-9780090	422	10-0219910
34 9-8383441	221	0-1616559	9-8600821	200	0-1399179	9-9782620	422	10-0217380
35 9-8384769	221	0-1615231	9-8599619	200	0-1400381	9-9785150	422	10-0214851
36 9-8386096	221	0-1613904	9-8598416	200	0-1401584	9-9787679	422	10-0212321
37 9-8387422	221	0-1612578	9-8597213	201	0-1402787	9-9790209	422	10-0209791
38 9-8388747	221	0-1611255	9-8596009	201	0-1403991	9-9792738	422	10-0207262
39 9-8390072	221	0-1609928	9-8594801	201	0-1405196	9-9795268	422	10-0204732
40 9-8391396	220	0-1608604	9-8593599	201	0-1406401	9-9797797	421	10-0202203
41 9-8392719	220	0-1607281	9-8592393	201	0-1407607	9-9800326	421	10-0199674
42 9-8394041	220	0-1605959	9-8591186	201	0-1408814	9-9802856	421	10-0197144
43 9-8395363	220	0-1604637	9-8589978	201	0-1410022	9-9805385	421	10-0194615
44 9-8396684	220	0-1603316	9-8588770	201	0-1411231	9-9807914	421	10-0192086
45 9-8398004	220	0-1601996	9-8587561	202	0-1412439	9-9810443	421	10-0189557
46 9-8399328	220	0-1600677	9-8586351	202	0-1413648	9-9812972	421	10-0187028
47 9-8400649	219	0-1599358	9-8585141	202	0-1414859	9-9815501	421	10-0184499
48 9-8401959	219	0-1598041	9-8583929	202	0-1416071	9-9818030	421	10-0181970
49 9-8403276	219	0-1596724	9-8582718	202	0-1417282	9-9820559	421	10-0179441
50 9-8404593	219	0-1595407	9-8581505	202	0-1418495	9-9823087	421	10-0176913
51 9-8405909	219	0-1594092	9-8580292	202	0-1419708	9-9825616	421	10-0174384
52 9-8407223	219	0-1592777	9-8579078	202	0-1420922	9-9828145	421	10-0171855
53 9-8408537	219	0-1591463	9-8577863	202	0-1422137	9-9830673	421	10-0169327
54 9-8409850	219	0-1590150	9-8576648	202	0-1423352	9-9833202	421	10-0166798
55 9-8411162	219	0-1588838	9-8575432	203	0-1424568	9-9835730	421	10-0164270
56 9-8412474	218	0-1587526	9-8574215	203	0-1425785	9-9838259	421	10-0161741
57 9-8413785	218	0-1586215	9-8572998	203	0-1427002	9-9840787	421	10-0159213
58 9-8415093	218	0-1584903	9-8571779	203	0-1428221	9-9843315	421	10-0156685
59 9-8416404	218	0-1583596	9-8570561	203	0-1429439	9-9845844	421	10-0154156
60 9-8417713	218	0-1582287	9-8569341	203	0-1430659	9-9848372	421	10-0151628
Sine	D10'	Comp. cos.	Sine	D10'	Comp. sin.	Cotang.	D10'	Tangent

LOGARITHMIC SINES, &c.

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Sine	D10	Comp sin	Cosine	D10	Comp cos	Tangent	D10	Cotangent
0 8417718	218	0 1562287	9 8569343	203	0 1430658	9 9848372	421	10 0131628
1 8419071	218	0 1560979	9 8568121	203	0 1431879	9 9849090	421	10 0130910
2 8420328	218	0 1559672	9 8566900	204	0 1433100	9 9849808	421	10 0130192
3 8421583	217	0 1558366	9 8565678	204	0 1434322	9 9850526	421	10 0129474
4 8422839	217	0 1557061	9 8564455	204	0 1435543	9 9851244	421	10 0128756
5 8424094	217	0 1555756	9 8563232	204	0 1436765	9 9851962	421	10 0128038
6 8425348	217	0 1554450	9 8562008	204	0 1437987	9 9852680	421	10 0127320
7 8426601	217	0 1553145	9 8560783	204	0 1439209	9 9853398	421	10 0126602
8 8427854	217	0 1551840	9 8559558	204	0 1440432	9 9854116	421	10 0125884
9 8429106	217	0 1550534	9 8558333	204	0 1441654	9 9854834	421	10 0125166
10 8430357	217	0 1549229	9 8557106	205	0 1442877	9 9855552	421	10 0124448
11 8431607	216	0 1547924	9 8555878	205	0 1444100	9 9856270	421	10 0123730
12 8432857	216	0 1546619	9 8554650	205	0 1445322	9 9856988	421	10 0123012
13 8434106	216	0 1545314	9 8553422	205	0 1446545	9 9857706	421	10 0122294
14 8435355	216	0 1544009	9 8552194	205	0 1447767	9 9858424	421	10 0121576
15 8436603	216	0 1542704	9 8550966	205	0 1448990	9 9859142	421	10 0120858
16 8437851	216	0 1541399	9 8549738	205	0 1450212	9 9859860	421	10 0120140
17 8439100	216	0 1540094	9 8548510	205	0 1451435	9 9860578	421	10 0119422
18 8440348	216	0 1538789	9 8547282	205	0 1452657	9 9861296	421	10 0118704
19 8441596	215	0 1537484	9 8546054	206	0 1453880	9 9862014	421	10 0117986
20 8442843	215	0 1536179	9 8544826	206	0 1455102	9 9862732	421	10 0117268
21 8444090	215	0 1534874	9 8543598	206	0 1456325	9 9863450	421	10 0116550
22 8445337	215	0 1533569	9 8542370	206	0 1457547	9 9864168	421	10 0115832
23 8446584	215	0 1532264	9 8541142	206	0 1458770	9 9864886	421	10 0115114
24 8447831	215	0 1530959	9 8539914	206	0 1460000	9 9865604	421	10 0114396
25 8449078	215	0 1529654	9 8538686	206	0 1461222	9 9866322	421	10 0113678
26 8450325	214	0 1528349	9 8537458	207	0 1462445	9 9867040	421	10 0112960
27 8451571	214	0 1527044	9 8536230	207	0 1463667	9 9867758	421	10 0112242
28 8452818	214	0 1525739	9 8535002	207	0 1464890	9 9868476	421	10 0111524
29 8454064	214	0 1524434	9 8533774	207	0 1466112	9 9869194	421	10 0110806
30 8455311	214	0 1523129	9 8532546	207	0 1467335	9 9869912	421	10 0110088
31 8456557	214	0 1521824	9 8531318	207	0 1468557	9 9870630	421	10 0109370
32 8457803	213	0 1520519	9 8530090	208	0 1469780	9 9871348	421	10 0108652
33 8459049	213	0 1519214	9 8528862	208	0 1471002	9 9872066	421	10 0107934
34 8460295	213	0 1517909	9 8527634	208	0 1472225	9 9872784	421	10 0107216
35 8461541	213	0 1516604	9 8526406	208	0 1473447	9 9873502	421	10 0106498
36 8462787	213	0 1515299	9 8525178	208	0 1474670	9 9874220	421	10 0105780
37 8464033	212	0 1513994	9 8523950	209	0 1475892	9 9874938	421	10 0105062
38 8465279	212	0 1512689	9 8522722	209	0 1477115	9 9875656	421	10 0104344
39 8466525	212	0 1511384	9 8521494	209	0 1478337	9 9876374	421	10 0103626
40 8467771	212	0 1510079	9 8520266	209	0 1479560	9 9877092	421	10 0102908
41 8469017	212	0 1508774	9 8519038	209	0 1480782	9 9877810	421	10 0102190
42 8470263	212	0 1507469	9 8517810	209	0 1482005	9 9878528	421	10 0101472
43 8471509	212	0 1506164	9 8516582	209	0 1483227	9 9879246	421	10 0100754
44 8472755	212	0 1504859	9 8515354	209	0 1484450	9 9879964	421	10 0100036
45 8474001	212	0 1503554	9 8514126	209	0 1485672	9 9880682	421	10 0099318
46 8475247	212	0 1502249	9 8512898	209	0 1486895	9 9881400	421	10 0098600
47 8476493	212	0 1500944	9 8511670	209	0 1488117	9 9882118	421	10 0097882
48 8477739	212	0 1500000	9 8510442	209	0 1489340	9 9882836	421	10 0097164
49 8478985	212	0 1499055	9 8509214	209	0 1490562	9 9883554	421	10 0096446
50 8480231	212	0 1498111	9 8507986	209	0 1491785	9 9884272	421	10 0095728
51 8481477	212	0 1497166	9 8506758	209	0 1493007	9 9884990	421	10 0095010
52 8482723	211	0 1496222	9 8505530	210	0 1494230	9 9885708	421	10 0094292
53 8483969	211	0 1495277	9 8504302	210	0 1495452	9 9886426	421	10 0093574
54 8485215	211	0 1494333	9 8503074	210	0 1496675	9 9887144	421	10 0092856
55 8486461	211	0 1493388	9 8501846	210	0 1497897	9 9887862	421	10 0092138
56 8487707	211	0 1492444	9 8500618	210	0 1499120	9 9888580	421	10 0091420
57 8488953	211	0 1491499	9 8499390	210	0 1500342	9 9889298	421	10 0090702
58 8490199	211	0 1490555	9 8498162	210	0 1501565	9 9889916	421	10 0090084
59 8491445	211	0 1489610	9 8496934	210	0 1502787	9 9890634	421	10 0089366
60 8492691	211	0 1488666	9 8495706	210	0 1504010	9 9891352	421	10 0088648
61 8493937	211	0 1487721	9 8494478	210	0 1505232	9 9892070	421	10 0087930
62 8495183	211	0 1486777	9 8493250	210	0 1506455	9 9892788	421	10 0087212
63 8496429	211	0 1485832	9 8492022	210	0 1507677	9 9893506	421	10 0086494
64 8497675	211	0 1484888	9 8490794	210	0 1508900	9 9894224	421	10 0085776
65 8498921	211	0 1483943	9 8489566	210	0 1510122	9 9894942	421	10 0085058
66 8500167	211	0 1483000	9 8488338	210	0 1511345	9 9895660	421	10 0084340
67 8501413	211	0 1482055	9 8487110	210	0 1512567	9 9896378	421	10 0083622
68 8502659	211	0 1481111	9 8485882	210	0 1513790	9 9897096	421	10 0082904
69 8503905	211	0 1480166	9 8484654	210	0 1515012	9 9897814	421	10 0082186
70 8505151	211	0 1479222	9 8483426	210	0 1516235	9 9898532	421	10 0081468
71 8506397	211	0 1478277	9 8482198	210	0 1517457	9 9899250	421	10 0080750
72 8507643	211	0 1477333	9 8480970	210	0 1518680	9 9900000	421	10 0080032
73 8508889	211	0 1476388	9 8479742	210	0 1519902		421	10 0079314
74 8510135	211	0 1475444	9 8478514	210	0 1521125		421	10 0078596
75 8511381	211	0 1474499	9 8477286	210	0 1522347		421	10 0077878
76 8512627	211	0 1473555	9 8476058	210	0 1523570		421	10 0077160
77 8513873	211	0 1472610	9 8474830	210	0 1524792		421	10 0076442
78 8515119	211	0 1471666	9 8473602	210	0 1526015		421	10 0075724
79 8516365	211	0 1470721	9 8472374	210	0 1527237		421	10 0075006
80 8517611	211	0 1469777	9 8471146	210	0 1528460		421	10 0074288
81 8518857	211	0 1468832	9 8469918	210	0 1529682		421	10 0073570
82 8520103	211	0 1467888	9 8468690	210	0 1530905		421	10 0072852
83 8521349	211	0 1466943	9 8467462	210	0 1532127		421	10 0072134
84 8522595	211	0 1466000	9 8466234	210	0 1533350		421	10 0071416
85 8523841	211	0 1465055	9 8465006	210	0 1534572		421	10 0070698
86 8525087	211	0 1464111	9 8463778	210	0 1535795		421	10 0070000
87 8526333	211	0 1463166	9 8462550	210	0 1537017		421	10 0069282
88 8527579	211	0 1462222	9 8461322	210	0 1538240		421	10 0068564
89 8528825	211	0 1461277	9 8460094	210	0 1539462		421	10 0067846
90 8530071	211	0 1460333	9 8458866	210	0 1540685		421	10 0067128
91 8531317	211	0 1459388	9 8457638	210	0 1541907		421	10 0066410
92 8532563	211	0 1458444	9 8456410	210	0 1543130		421	10 0065692
93 8533809	211	0 1457499	9 8455182	210	0 1544352		421	10 0064974
94 8535055	211	0 1456555	9 8453954	210	0 1545575		421	10 0064256
95 8536301	211	0 1455610	9 8452726	210	0 1546797		421	10 0063538
96 8537547	211	0 1454666	9 8451498	210	0 1548020		421	10 0062820
97 8538793	211	0 1453721	9 8450270	210	0 1549242		421	10 0062102
98 8540039	211	0 1452777	9 8449042	210	0 1550465		421	10 0061384
99 8541285	211	0 1451832	9 8447814	210	0 1551687		421	10 0060666
100 8542531	211	0 1450888	9 8446586	210	0 1552910		421	10 0059948

Tab 11.

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